면외전단하중이 작용하는 기능경사재료 접합면 균열의 동적전파에 관한 연구

Dynamic Propagation of a Interface Crack in Functionally Graded Layers under Anti-plane Shear

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요 약

The dynamic propagation of an interface crack between two dissimilar functionally graded layers under anti-plane shear is analyzed using the integral transform method. The properties of the functionally graded layers vary continuously along the thickness. A constant velocity Yoffe-type moving crack is considered. Fourier transform is used to reduce the problem to a dual integral equation, which is then expressed to a Fredholm integral equation of the second kind. Numerical values on the dynamic energy release rate (DERR) are presented. Followings are helpful to increase of the resistance of the interface crack propagation of FGM: a) increase of the gradient of material properties; b) increase of the material properties from the interface to the upper and lower free surface; c) increase of the thickness of FGM layer. The DERR increases or decreases with increase of the crack moving velocity.

keywords : Functionally graded material, Interface, Moving crack, Dynamic energy release rate

1. Introduction

Functionally graded materials (FGMs) have been recently attracted extensive attention for use in high temperature applications and wear-protective coatings. The FGMs are microscopically non-homogeneous because the mechanical properties of the FGM vary smoothly and continuously. The fracture behavior of the FGM is important design issue. A crack in the FGM may exhibit complex behavior because of the variation of the mechanical properties of the material. The fracture behaviors of the FGMs have been studied widely for both static and dynamic problems. But, solution of the dynamic crack propagation of an interface crack between two dissimilar functionally graded layers has not been presented.

In this paper, dynamic propagation of an interface Griffith crack between two dissimilar functionally graded layers under anti-plane shear is analyzed. The properties of the functionally graded layers vary

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continuously along the thickness. The properties of the two functionally graded layers vary differently and the two layers are connected weak-discontinuously (Li et al., 2006). The Yoffe-type model (Yoffe, 1951) for crack propagation is adopted. Fourier transform is used to reduce the problem to a dual integral equation, which is then expressed in a Fredholm integral equation of the second kind. Numerical results of the dynamic energy release rate are presented graphically to show the effect of gradient of material properties, crack moving velocity, and thickness of layers.

2. Problem statement and formulation

Consider two dissimilar functionally graded layers containing a finite interface crack subjected to anti-plane shear loading, as shown in Fig. 1. The cartesian coordinates (X, Y, Z) are fixed for the reference. The functionally graded layers occupy the region, $-\infty < X < \infty, -h_2 \leq Y \leq h_1$, and are thick enough in the Z-direction. The crack is situated along the interface line $(-a \leqq X \leqq a, Y=0)$.

We assume that the properties of the two functionally graded layers vary continuously along the thickness and are simplified as follows (Delale and Erdogan, 1983):

$$\mu_i = \mu_0 e^{\beta_i Y}, \ \rho_i = \rho_0 e^{\beta_i Y} \tag{1}$$

where μ_i and ρ_i are the shear modulus and material density, respectively. μ_0 and ρ_0 are the material properties at the interface and β_i is the non-homogeneous material constant. Subscript $i\,(i=1,2)$ stands for the upper and lower layers, respectively.

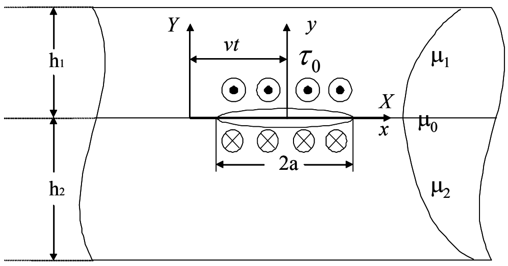

Fig. 1 Geometry of a constant moving interface crack between two functionally graded layers

The boundary value problem is simplified if we consider only the out-of-plane displacement such that

$$u_{Xi} = u_{Yi} = 0, \quad u_{Zi} = w_i(X, Y, t) \tag{2}$$

where u_{ki} $(k = X, Y, Z)$ is the displacements.

The dynamic anti-plane governing equation for FGM is simplified to

$$\sigma_{Zji}(X, Y, t) = \mu_i w_{i,j} \tag{3}$$

where σ_{Zji} $(j = X, Y)$.

By substituting Eq. (1) into the Eq. (3), the dynamic governing equation is transformed into the following equation:

$$\nabla^2 w_i + \beta_i \frac{\partial w_i}{\partial Y} = \frac{1}{c_2^2} \frac{\partial^2 w_i}{\partial t^2} \qquad (4)$$

where $c_2 = \sqrt{\mu_0 / \rho_0}$.

For the problem of a moving crack with constant velocity "v" along the X-direction, it is convenient to introduce a Galilean transformation such as

$$x = X - vt,\ y = Y,\ z = Z,\ t = t \qquad (5)$$

where (x, y, z) is the translating coordinate system attached to the center of the moving crack.

In the transformed coordinate system, the dynamic anti-plane governing equation for FGM can be simplified to the following form:

$$\alpha^2 \frac{\partial^2 w_i(x,y)}{\partial x^2} + \frac{\partial^2 w_i(x,y)}{\partial y^2} + \beta_i \frac{\partial w_i(x,y)}{\partial y} = 0 \qquad (6)$$

where

$$\alpha = \sqrt{1 - (v/c_2)^2} \qquad (7)$$

A Fourier transform is applied to Eq. (6), and the results are as follows:

$$w_i(x,y) = \frac{2}{\pi} \int_0^\infty \left[A_{1i}(s) e^{-q_{1i} y} + A_{2i}(s) e^{q_{2i} y} \right] \cos(sx)\, ds \qquad (8)$$

$$\sigma_{yzi}(x,y) = \mu_0 \frac{2}{\pi} \int_0^\infty \left[-q_{1i} A_{1i}(s) e^{-q_{2i} y} + q_{2i} A_{2i}(s) e^{q_{1i} y} \right] \cos(sx)\, ds \qquad (9)$$

where

$$q_{1i} = \delta_i + \beta_i/2,\ q_{2i} = \delta_i - \beta_i/2,\ \delta_i = \sqrt{\alpha^2 s^2 + \beta_i^2/4} \qquad (10)$$

σ_{yzi} is the stress component, and A_{1i} and A_{2i} are the unknowns to be solved.

The boundary conditions can be written as

$$\sigma_{zyi}(x,0) = -\tau_0 \qquad (0 \le x < a) \qquad (11)$$

$$w_1(x,0^+) = w_2(x,0^-) \qquad (a \le x < \infty) \qquad (12)$$

$$\sigma_{yz1}(x,0^+) = \sigma_{yz2}(x,0^-) \qquad (a \le x < \infty) \qquad (13)$$

$$\sigma_{yz1}(x,h_1) = \sigma_{yz2}(x,-h_2) = 0 \qquad (0 < x \le \infty) \qquad (14)$$

where τ_0 is the uniform shear traction.

It is convenient to use the following definitions:

$$A_{11}(s) - A_{12}(s) + A_{21}(s) - A_{22}(s) = A(s) \qquad (15)$$

The mixed boundary conditions of Eqs. (11) and (12), continuous condition of Eq. (13), and edge loading condition of Eq. (14) lead to a dual integral equations in the following form:

$$\int_0^\infty sF(s) A(s) \cos(sx)\, ds = \frac{\pi}{2} \frac{2}{\alpha} \frac{\tau_0}{\mu_0} \qquad (0 \le x < a)$$

$$\int_0^\infty A(s) \cos(sx)\, ds = 0 \qquad (a < x \le \infty) \qquad (16)$$

where

$$F(s) = \frac{2}{\alpha}\frac{1}{s}\left[\frac{q_{11}q_{12}q_{21}q_{22}(1-e^{-2\delta_1 h_1})(1-e^{-2\delta_2 h_2})}{q_{11}q_{21}(1-e^{-2\delta_1 h_1})(q_{11}+q_{22}e^{-2\delta_2 h_2})+q_{12}q_{22}(1-e^{-2\delta_2 h_2})(q_{21}+q_{22}e^{-2\delta_1 h_1})}\right] \quad (17)$$

The dual integral Eq. (16) may be solved by using new function $\Omega(\xi)$ defined by

$$A(s) = \int \xi\Omega(\xi)J_0(s\xi)d\xi \quad (18)$$

where J_0 is the zero-order Bessel function of the first kind.

By inserting Eq. (18) into Eq. (16), we can obtain a Fredholm integral equation of the second kind in the following form:

$$\Psi(\Xi)+\int_0^1 L(\Xi,H)\Psi(H)dH = \sqrt{\Xi} \quad (19)$$

where

$$L(\Xi,H) = \sqrt{\Xi H}\int_0^\infty S[F(S/a)-1]J_0(SH)J_0(S\Xi)dS \quad (20)$$

$$F(S/a) = \frac{2}{\alpha}\frac{1}{s}\left[\frac{Q_{11}Q_{12}Q_{21}Q_{22}(1-e^{-2\Delta_1\frac{h_1}{a}})(1-e^{-2\Delta_2\frac{h_2}{a}})}{Q_{11}Q_{21}(1-e^{-2\Delta_1\frac{h_1}{a}})(Q_{11}+Q_{22}e^{-2\Delta_2\frac{h_2}{a}})+Q_{12}Q_{22}(1-e^{-2\Delta_2\frac{h_2}{a}})(Q_{21}+Q_{22}e^{-2\Delta_1\frac{h_1}{a}})}\right] \quad (21)$$

$$Q_{11} = \Delta_1+B_1/2,\ Q_{12} = \Delta_2+B_2/2,\ Q_{21} = \Delta_1-B_1/2,\ Q_{22} = \Delta_2-B_2/2 \quad (22)$$

$$S = as,\ B_1 = a\beta_1,\ B_2 = a\beta_2,\ \Delta_1 = a\delta_1,\ \Delta_2 = a\delta_2 \quad (23)$$

$$\eta = aH,\ \xi = a\Xi,\ \Omega(\xi) = \frac{\pi}{2}\frac{2}{\alpha}\frac{\tau_0}{\mu_0}\frac{\Psi(\Xi)}{\sqrt{\Xi}} \quad (24)$$

The mode Ⅲ dynamic energy release rate is defined and determined in the following forms:

$$G_{III}(v) = \frac{\pi a}{2\mu_0}\tau_0^2\Psi^2(1) \quad (25)$$

in which the function $\Psi(1)$ can be calculated from Eq. (19).

3. Discussions

To investigate the effect of the gradient of material properties, crack moving velocity and thickness of layers on the dynamic energy release rate (DERR), numerical analyses are carried out.

Fig. 2 displays the variation of the normalized DERR against the normalized non-homogeneous material constant of the upper layer with various normalized non-homogeneous material constants of the lower layer at $v/c_2 = 0.4$ and $h_1/a = h_2/a = 10.0$. The DERR decreases when the gradients of material properties of the upper and lower layers increase. For the upper layer, the gradient of material properties increases as the non-homogeneous material constant increases. But for the lower layer, the gradient of material properties increases as the non-homogeneous material constant decreases because value of the y-axis is negative. Increase of the gradient of material properties from the interface is beneficial to increase of the resistance of the interface crack propagation of FGM.

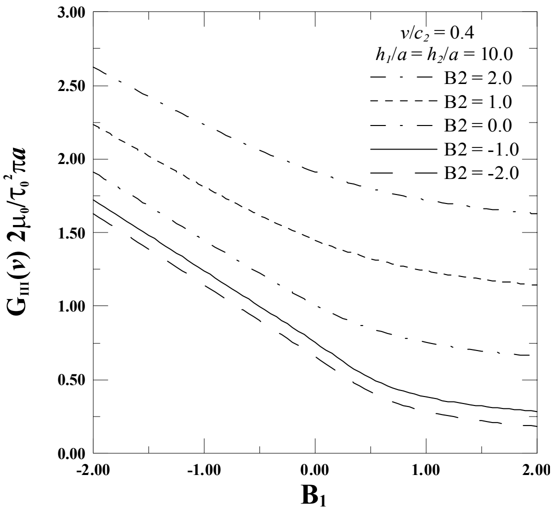

Fig. 2 Variation of the normalized DERR $G_{III}(v)\ 2\mu_0/\tau_0^2\pi a$ with B_1

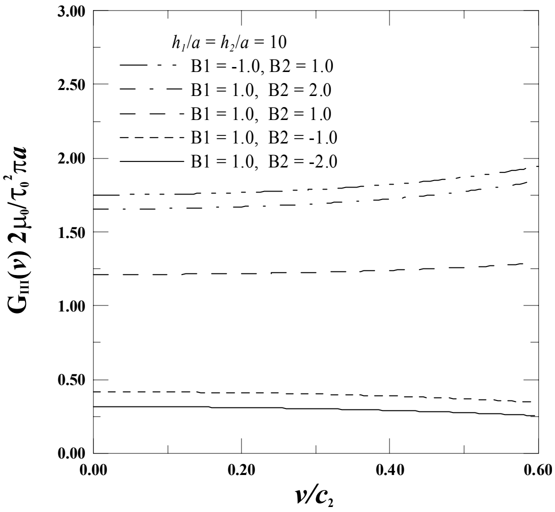

Fig. 3 Variation of the normalized DERR $G_{III}(v)\ 2\mu_0/\tau_0^2\pi a$ with v/c_2

Fig. 3 shows the variation of the normalized DERR against the normalized crack moving velocity with the various normalized non-homogeneous material constants. According to the values of the gradient of material properties of the upper and lower layers, we can classify into three categories as follows:

Case I : material properties increase when the thickness of upper and lower layers increases from the interface,

Case II : material properties decrease when the thickness of upper and lower layers increases from the interface,

Case III : material properties increase monotonically from the lower surface $(y=-h_2)$ to upper surface $(y=h_1)$, and vice verse.

For the Case II and III, the DERR increases as the crack moving velocity increases. But for the Case I, the trend is opposite. The DERR decreases when the crack moving velocity increases. That is, increase of the stiffness from the interface to the upper and lower surface is helpful to increase of the resistance of the interface crack propagation of FGM.

The effect of the crack moving velocity on the variation of the normalized DERR is shown in Fig. 4 with various thicknesses of the layers. The DERR increases with the increase of the crack moving velocity. But the DERR decreases when the thickness of layer increases. Increase of the thickness of FGM layer is also beneficial to increase of the resistance of the interface crack propagation of FGM.

Fig. 5 presents the variation of the normalized DERR against the normalized thickness of the lower FGM layer with the various non-homogeneous material constants. As similar to Fig. 4, the DERR decreases as the thickness of the lower layer increases. But, over certain value of the thickness of the lower layer (about 3.00), the effect of decrease of the DERR is negligible. As seen in Case I of Fig. 3, Fig. 5 also shows that increase of the stiffness from the interface to the upper and lower surface is helpful to increase of the resistance of the interface crack propagation of FGM.

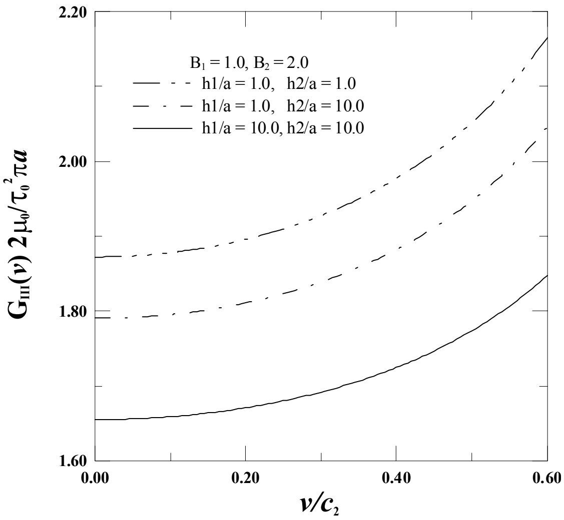

Fig. 4 Variation of the normalized DERR $G_{III}(v)\ 2\mu_0/\tau_0^2\pi a$ with v/c_2

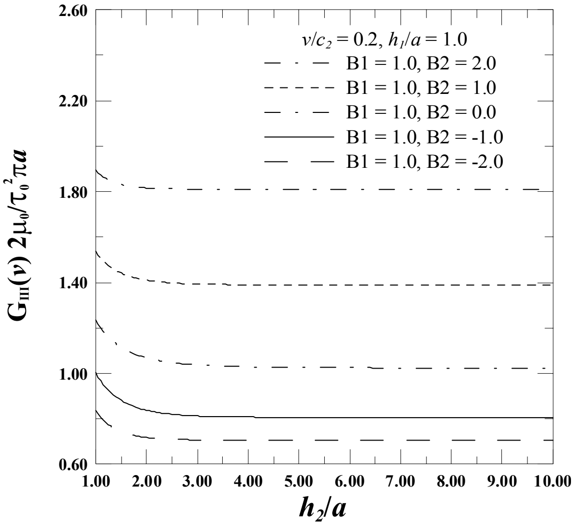

Fig. 5 Variation of the normalized DERR $G_{III}(v)\ 2\mu_0/\tau_0^2\pi a$ with h_2/a

4. Conclusions

The problem of dynamic propagation of a weak-discontinuous interface crack between two functionally graded layers under anti-plane shear loading was analyzed by the integral transform approach. The shear modulus and mass density of the FGM vary continuously along the thickness. A Fredholm integral equation is solved numerically. The computed results show that the followings are helpful to increase of the resistance of the interface crack propagation of FGM:

a) Increase of the gradient of material properties,
b) Increase of the material properties from the interface to the upper and lower free surface,
c) Increase of the thickness of FGM layer.

The normalized DERR increases or decreases with increase of crack moving velocity.

참고문헌

Delale, F., Erdogan, F. (1983) The crack problem for a nonhomgeneous plane, *ASME Journal of Applied Mechanics,* 50, p.p.609~614.

Li, Y.D., Jia, B., Zhang, N., Tang, L.Q., Dai, Y. (2006) Dynamic stress intensity factor of the weak/micro-discontinuous interface crack of a FGM coating, *International Journal of Solids Structures*, 43, p.p.4795~4809.

Yoffe, E.H. (1951) The moving Griffith crack, *Philos. Magazine*, 7, p.p.739~750.