Low Complexity ML Detection Based on Linear Detectors in MIMO Systems

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Abstract

This paper studies about reducing the complexity of ML detection in MIMO/V-blast system, based on MMSE and ZF linear detectors. Beforehand, the receiver detects the signal using the linear detector such as ZF or MMSE. Moreover, the next step is to assess whether the signal is reliable or not by verifying the reliability condition, if the latter is reliable then it is the output if not it has to be detected by the advanced detector until the reliability condition is verified.

Keywords
Low complexity ML, linear detector, V-blast detectors

Ⅰ. Introduction

In the MIMO system with V-BLAST architecture, the receiver equalization is done by linear detectors such as ZF and MMSE [2], though simple to implement, they are poor in performance. However, advanced detector such as ML optimizes the performance but with a high computational complexity. The complexity increases exponentially as the number of antenna increases and modulation scheme respectively.

To achieve a tradeoff between complexity and BER performance, this paper focused on combining linear detectors and ML, in between both detectors there is a step of assessing the reliability of the estimated transmitted signal. Once the estimated is reliable, then it is the final output. If not, it passes through ML detector and thereafter it is assessed as well, if it is reliable then it is the final output if not the ML detects until the reliability condition is verified. This paper considers MIMO 2x2 in Rayleigh environment and BPSK as a modulation scheme.

The rest of the paper is as follows, section2 gives the system overview, section 3 explains the reliability condition, section4 covers the simulation as well as the result interpretation finally section5 is the conclusion.

Ⅱ. System overview

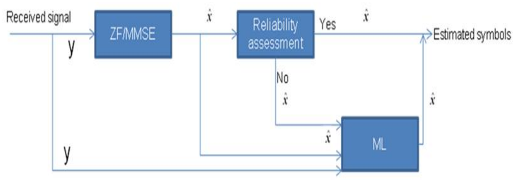

Figure 1. Detection architecture.

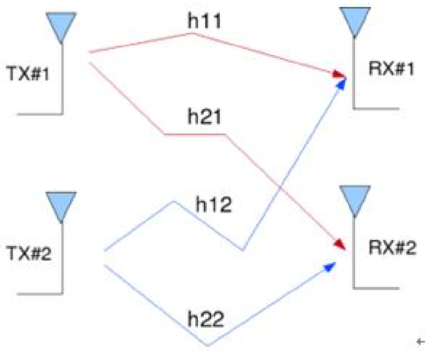

Figure 2. (2×2) MIMO channel.

Consider that we have a transmission sequence, $\{x_1,x_2,x_3\cdots x_n\}$, we now have 2 transmit antennas, and let's group the symbols into groups of two. In the first time slot, send x_1 and x_2 from the first and second antenna.

In second time slot, send x_3 and x_4 respectively; send x_5 and x_6 in the third time slot and so on.

As we are grouping two symbols and sending them in one time slot, we need only time slots to complete the transmission.

$$X=(x_1,x_2,\cdots x_{nt})^T \text{ : transmitted vector}$$
$$y=(y_1,y_2,\cdots x_{nr})^T\text{: received vector}$$

Therefore **y=Hx+n.** where H is the channel matrix of dimension **Nr x Nt** with the element h_{ij} representing the channel between transmit antenna j and receive antenna I, and n is **nr x1** noise vector with variance σ^2.

1. Zero Force

The Zero force (ZF) approach tries to find a matrix **W** which satisfies **WH=I** to meet this constraint,

$$W=(HH^*H)^{-1}H^H \qquad [1]$$

The estimated received $_{zf}$ =**Wy**
can simply be written as follows

$$\tilde{x}=D(Gy)=\arg\{_x s1^*\cdots{}^*sM \min||Gy-x||\}$$

where D( ) is the decision function which finds the nearest point in the constellation to the received signal point and **G=W**

2. Minimum Mean Square Error

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient W which minimizes the criterion,

$$E\{[Wy-x][Wy-x]^H\}.$$

Solving,

$$W=(H^H{}_*H+N_0I)^{-1}H^H$$

3. Maximum Likelihood

The Maximum Likelihood receiver tries to find out which minimizes,

$$J=|y-H\hat{x}|^2$$

Since the modulation is BPSK, the possible values of x_i are **+1** or **-1**. Hence, to find the Maximum Likelihood solution, we need to find the minimum from the all four combinations.

$$J+1,-1=\left[\begin{bmatrix}y_1\\y_2\end{bmatrix}-\begin{bmatrix}h_{11}\,h_{12}\\h_{21}\,h_{22}\end{bmatrix}\begin{bmatrix}+1\\-1\end{bmatrix}\right]^2$$
$$J-1,+1=\left[\begin{bmatrix}y_1\\y_2\end{bmatrix}-\begin{bmatrix}h_{11}\,h_{12}\\h_{21}\,h_{22}\end{bmatrix}\begin{bmatrix}-1\\+1\end{bmatrix}\right]^2$$
$$J+1,+1=\left[\begin{bmatrix}y_1\\y_2\end{bmatrix}-\begin{bmatrix}h_{11}\,h_{12}\\h_{21}\,h_{22}\end{bmatrix}\begin{bmatrix}+1\\-1\end{bmatrix}\right]^2$$
$$J-1,-1=\left[\begin{bmatrix}y_1\\y_2\end{bmatrix}-\begin{bmatrix}h_{11}\,h_{12}\\h_{21}\,h_{22}\end{bmatrix}\begin{bmatrix}-1\\-1\end{bmatrix}\right]^2$$

The estimate of the transmit symbol is chosen based on the minimum value from the above four values i.e

if the minimum is J+1,+1 → [1 1],
if the minimum is J+1,-1 → [1 0],
if the minimum is J-1,+1 → [0 1]and
if the minimum is J-1,-1 → [0 0].

Simply ML can be written like

$$\tilde{x} = \arg\{xs_1^*\cdots{}^*sM\min||y-Hx||\}$$

Where ||.|| is the **Frobenius-norm**, arg{min()}is the parameter to enable the expression minimums, and "*****" represents the Descartes product of sets.

III. Reliability condition

First verification

$$K=||y-H\hat{x}||^2/\sigma^2$$
(||.|| : Frobenius norm[4]),

$$K=||\ H(x-\hat{x})+n)||^2/\sigma^2$$

if $\hat{x}$ is the right estimate of transmit symbols then **x =** $\hat{x}$ therefore

$$K=\sum_{i=1}^{nr}\frac{(n_i)^2}{\sigma^2}$$

and if we get $k< n_r{}^*\sigma^2$ with $\mathbf{n_r}$ the degree of freedom of the receiver antennas then we take

$\hat{x}$ **as the final result Second verification** if is the wrong estimate of transmitted symbols $\mathbf{x} \neq \hat{x}$. Then calculate **di** = $|| y - H x_i ||^2$ with $\mathbf{x_i}$ as possible transmit symbol vector and **di** the squared distance.

If di < $\mathbf{n_r} * \sigma_z^2$ take xi as the final result, and if not, choose the lowest squared distance and output its corresponding x as the result.

Ⅳ. Simulation

The simulation is made under Rayleigh fading channel environment, 2x2 MIMO with BPSK as modulation, lets compare the BER performance between ZF, MMSE, ML and ZF_ML. the figure 3 shows the BER performance of different above detection techniques, assuming the channel to be known at the receiver.

From the figure 3, MMSE_ML, ZF_ML and ML perform similarly, precisely in the low SNR the performance is the same.

This is the result of the reliable estimation that has been done by using linear detectors combined with conditional ML, this considerably reduces the exhaustive search absolutely found in ML. Moreover, ZF and MMSE give a reliable estimate of the received signal, because of the condition of reliability that must be fulfilled ,it is a kind of threshold in order to get the relevant estimate. However ,the combination is performing quite well in terms of BER as ML does, because in all cases, the detector search always the accurate estimation according to the condition. therefore, this adds up something about the efficiency of the detection hence optimal estimate. As the figure shows, we have ZF-ML, MMSE-ML and ML performing almost in the same BER and really in the a better way comparing to linear detectors such as ZF and MMSE.

Figure 4 presents the search complexity of various detectors, during search process. As we know, linear detectors ZF and MMSE are low complex to implement, since combined with advanced detector ML as well as the estimation reliability condition, they become less complex comparing to ML. Like the figure 4 is showing, the tree search is significantly reduced comparing to ML when it is used as the only detection. As it is shown, ML detector does a long search comparing to the combined method of linear detectors with conditional ML. Due to this, we realized that the computational complexity of linear detectors (ZF and MMSE) combined with conditional ML is minimal comparing to ML detector.

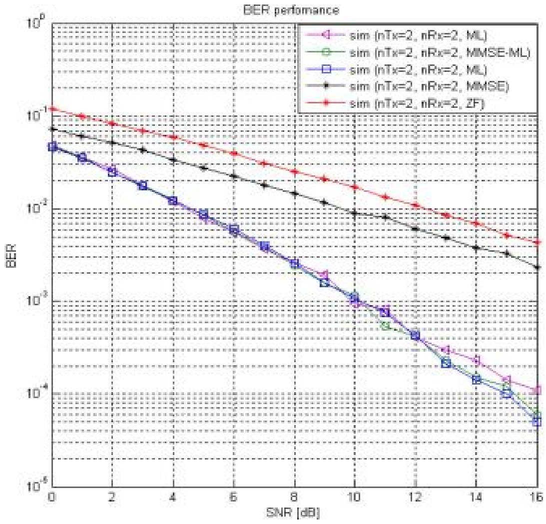

Figure 3. BER performance for different detectors.

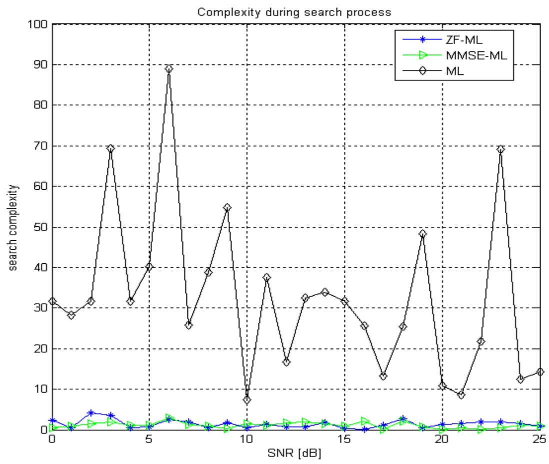

Figure 4. Complexity during search process.

Ⅴ. Conclusion

After simulating all the above detectors, we realized that linear detectors combined with ML achieve quite the same BER performance as ML at reduced complexity. Moreover in low SNR these combinations are more efficient.

Therefore we recommend this combination

for a tradeoff between BER and computational complexity. Further reducing about reducing much more the complexity of the detection with much high BER performance by simplifying the search tree algorithm.

Reference

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