A Comparative Study of List Sphere Decoders for MIMO Systems

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ABSTRACT

In this paper, we investigated the list sphere decoders (LSD) for multiple-input multiple-output (MIMO) systems. We showed that the ordering procedures play an important role in LSD in order to achieve the low complexity without degrading the bit-error-rate (BER) performance. Then, we proposed a novel ordering algorithm for the LSD which uses a look-up table and simply comparative operations. Comparative results in terms of BER performance and computational complexity are provided through computer simulations.

Keywords

Wireless Communication, Maximum-likelihood Detection, List Sphere Decoder, Multiple-input multiple-output System

Ⅰ. Introduction

Recently, to cope with the steeply increasing demand of broadband services such as video, high speed data down/up loadings, much attention has been paid to the application of multiple transmit and receive antennas, i.e., the so-called multiple-input multiple-output (MIMO) systems, to wireless communication systems. This is mainly because the MIMO systems are theoretically shown to significantly improve the spectral efficiencies [1]. To achieve high spectral efficiencies, a typical MIMO technique, known as the Vertical Bell Laboratories Layered Space Time (V-BLAST) architecture [2], have been reported. In order to achieve an optimal system performance, the maximum likelihood (ML) decoder has been required for the V-BLAST. Unfortunately, brute-force maximum likelihood (BFML) detection, so-called MLD, is an impractical detection due to its complexity, which increases exponentially with the number of the transmit antennas and the level of modulation.

For achieving ML-like performance even at low complexity level, a class of detection techniques, collectively referred to as sphere decoders (SDs), have been developed [3]-[4]. Both computer simulation and theoretical analysis have shown that the average complexities of SDs were remarkably smaller than that of the MLD in many practical scenarios. Mostly, the principles of SDs are based on integer lattice theory. The SDs can be regarded as depth-first or breadth-first tree search decoders with pruning within a hype-sphere constraint. So far, many efforts have been made to develop even superior SDs for both real-valued and complex-valued systems.

However, the SDs can not be directly used in the integrated detector/decoder architecture to exploit the coding gain of the outer channel code. This is mainly because the SDs are targeted to get only the tentative ML solution, the so-called hard decision, which can not generate an extrinsic information for the soft-decoding of the outer decoder. Therefore, a modified SD, the so-called list sphere decoder (LSD) [6], has been introduced. In LSDs, instead of finding a single tentative ML solution, they find a list of most likely symbols within a hyper-sphere. Then, from the list, the soft information is generated to be used for outer decoder.

Since the LSDs extend the list of candidates, they become normally ten times slower than the SDs. In addition, the computational complexities of the LSDs depend largely on the size of the list. In order to reduce the computational complexities of the LSDs, some techniques such as radius reduction, Schnorr-Euchner enumeration, storage pruning, and ordered-list have been used [3],[4]. While searching for the most appropriate candidate, the LSDs will continue to update the list whenever the leaf nodes are reached in depth-first search strategy, or when a new possible group of nodes may be found in the breadth-first search strategy. Since a searching process can be a bottleneck of the LSDs as well as SDs, the ordering procedures can play a crucial role in facilitating the LSDs to lower their complexities.

In this work, we present a comparative study on the LSDs with various optimal ordering approaches to further reduce the complexity. The remaining part of this paper is organized as follows. Section II presents the system model that is used for our work. The overviews on SDs, LSDs and LSDs with various optimal order procedures are described in Section III. To provide a comparative view, computer simulations have been implemented. The simulation results are given in Section IV accompanied by some discussion. Section V concludes this work.

II. System Model

We consider a V-BLAST configuration with n_T transmit and n_R receive antennas, denoted as (n_T, n_R) system.

At the transmitter, the input bit sequence is first encoded by an outer coder scheme resulting an encoded bit sequence $\{x^{(i)}\}$. The encoded bit sequence is then partitioned into n_T sub-streams (layers), each of which is then modulated by an M-ary modulation scheme, for example, M-ary Phase Shift Keying (M-PSK), leading to modulated symbols s_i and transmitted from each different transmit antenna. For the sake of simplicity, we investigate one-time-slot complex baseband signal model, where each symbol period a $n_T \times 1$ transmit signal vector $\boldsymbol{s}$ consisting of n_T symbols, s_i, $i = 1, \ldots, n_T$, is sent through n_T transmit antennas. Under the assumptions that the signals are narrow-band and the channel is quasi-static, i.e., it remains constant during some block of arbitrary length and independently changes from one block to another, the relationship between transmitted and received signals can be expressed in the following form:

$$\boldsymbol{r} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{w} \quad (1)$$

where $\boldsymbol{r} = [r_1 \ldots r_{n_T}]^T$ is the $n_R \times 1$ received signal vector, $(.)^T$ denotes the transpose operator, $\boldsymbol{w} = [w_1 \ldots w_{n_R}]^T$ represents the noise samples at n_R receive antennas, which are modelled as independent samples of a zero-mean complex Gaussian random variable with noise variance σ_n^2, $\boldsymbol{H}$ is the $n_R \times n_T$ channel matrix, whose entries are the path gains between transmit and receive antennas modelled as the samples of zero-mean complex Gaussian random variables with equal variance of 0.5 per real dimension. In addition, we assume that the signals transmitted from individual antenna has equal powers of P/n_T, i.e., $E\{\boldsymbol{s}\boldsymbol{s}^H\} = P/n_T \boldsymbol{I}_{n_T}$, where $(.)^H$ denotes the Hermitian transpose operator, $\boldsymbol{I}_{n_T}$ indicates the $n_T \times n_T$ identity matrix, and $E\{.\}$ denotes the expectation operator.

Under the assumption that $\boldsymbol{H}$ is perfectly known at the receiver, the signal can be detected by using MLD according to:

$$\hat{\boldsymbol{s}} = \arg\min_{\boldsymbol{s} \in \boldsymbol{\Omega}} \| \boldsymbol{r} - \boldsymbol{H}\boldsymbol{s} \|^2 \quad (2)$$

where $\boldsymbol{\Omega}$ is the transmission constellation, $\| \boldsymbol{A} \|$ denotes the Euclidean norm of matrix $\boldsymbol{A}$ defined by $\| \boldsymbol{A} \| = tr(\boldsymbol{A}\boldsymbol{A}^H)$, $tr(\boldsymbol{A})$ denotes the trace of matrix $\boldsymbol{A}$.

From solution of (2), the outer decoder uses $\hat{\boldsymbol{s}}$ to recover the original bits.

III. List Sphere Decoders

In this part, for sake of clarity, before investigating the LSD, we will present the overview on the SD. Then, the LSD with various ordering procedures will be introduced.

A. Sphere Decoder

Instead of using exhausted searching, the SD tries to solve the equation (2) by examining only the candidates within a n_R-dimensional hyper-sphere $S(\boldsymbol{r}, \sqrt{C_0})$. The condition can be expressed as:

$$\| \boldsymbol{r} - \boldsymbol{H}\boldsymbol{s} \|_F^2 \le C_0 \quad (3)$$

where C_0 is the squared radius of the sphere.

If we take QR decomposition (QRD) of the channel matrix $\boldsymbol{H}$, (3) can be re-written as:

$$\| \boldsymbol{r} - \boldsymbol{QRs} \|_F^2 \leq C_0 \quad (4a)$$

$$\| \boldsymbol{Q}^H\boldsymbol{r} - \boldsymbol{Rs} \|_F^2 \leq C_0 \quad (4b)$$

$$\| \boldsymbol{y} - \boldsymbol{Rs} \|_F^2 \leq C_0 \quad (4c)$$

where $\boldsymbol{R}$ is an $n_R \times n_T$ upper triangular matrix with positive diagonal entries, $\boldsymbol{Q}$ is an $n_R \times n_R$ orthogonal matrix, $\boldsymbol{y} = \boldsymbol{Q}^H\boldsymbol{r}$.

Since $\boldsymbol{R}$ is an upper triangular matrix, from (4c) the partial Euclidian distance of $\boldsymbol{s}_{(i)}$ $(i = n_T, \ldots, 1)$ can be found by

$$d(\boldsymbol{s}_{(i)}) = d(\boldsymbol{s}_{(i+1)}) + \left| y_{(i)} - \sum_{j=i}^{n_T} R_{i,j} s_j \right|^2 \quad (5)$$

The next potential candidate for evaluating (5) can be determined by computing all possible signal constellations and then choosing the lowest one from a certain enumeration such as Schnorr-Euchner enumeration, ordered list.

B. List Sphere Decoder

The LSDs, by exploiting (4c) and (5), find a list of size $\wp$ $(1 \leq \wp \leq 2^{\Omega n_T})$ of potential candidate symbols vectors. It is obvious that the larger the list size $\wp$ is, the better the BER performance as well as the higher the complexity will be.

From the list, the log-likelihood ratio (LLR) value of a posteriori probability is computed as follows

$$L_D(x^{(i)}|\boldsymbol{y}) = L_A(x^{(i)}) + L_E(x^{(i)}|\boldsymbol{y}) \quad (6)$$

where $L_A(x^{(i)})$ is a priori LLR value and $L_E(x^{(i)}|\boldsymbol{y})$ is an extrinsic LLR value.

B.1. Schnorr-Euchnerr Enumeration based LSD (SE-LSD)

The SE-LSD, a modification of SE-SD for LSD, is a depth-first search tree algorithm. The SE [3] can be seen as a combination of Pohst enumeration and Viterbo-Boutros SD, in which the admissible nodes of each layer are spanned in a zig-zag order starting at the closest middle point.

B.2. Comparative Ordering LSD (CO-LSD)

Instead of using SE, we apply the optimal ordering procedures in [4] to determined the admissible nodes. In this method, the order of candidates are determined by comparing the ratio of real and imaginary parts of y_{i+1} with pre-determined slopes of $\{s_i\}$. The illustration of the process for M-PSK signal is presented in Fig.1.

B.3. Rotated Comparative LSD (RC-LSD).

Similar to CO-LSD, RC-LSD also uses the optimal ordering procedure which is called direct enumeration [6]. In general, the preferred child $s_i^{(0)}$ in the forward pass is given by the Babai point. All constellation points lie inside a circle around the origin and $R_{i,j}$ can be chosen to be positive real without any loss of generality. Then, one can easily show that the preferred child can also obtained from

$$s_i^{(0)} = \arg\min_{s_i \in \Omega} |\mathrm{arc}(y_{i+1}) - \mathrm{arc}(s_i)| \quad (7)$$

where $\mathrm{arc}(.)$ denotes the phase of a complex number. The ordering procedure starts by finding phases of y_{i+1} and two neighbors of $s_i^{(0)}$, then the evaluation is processed in zig-zag fashion along the circle until the condition of radius is violated. The illustration of the process is depicted in Fig.2.

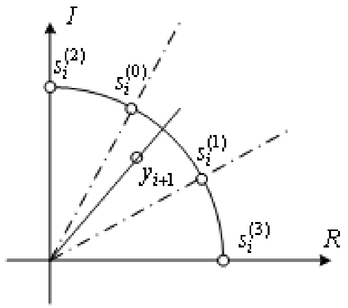

Fig. 1 : Illustration of CO processing for M-PSK signal.

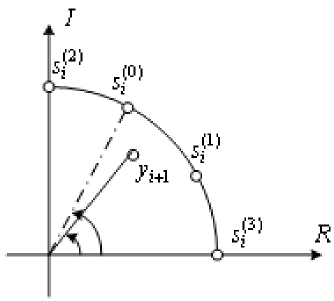

Fig. 2 : Illustration of RC processing for M-PSK signal.

Ⅳ. Computer simulation results and Discussion

In order to provide a comparative view of the algorithms performance, we implement the computer simulation. In the simulation, we apply the SE-LSD, CO-LSD and RC-LSD decoders into V-BLAST system of (4,4) employing 64-QAM with outer code of LDPC. We set the list size for all decoders to 16. The complexities of decoders are evaluated by examining the average number of additions, multiplication and comparison operations required by the inner list decoder only. For clearly showing the benchmark of the decoders, we also compare their performance to that of MLD. Due to the exponential complexity of MLD, we do not show its complexity in our complexity comparison.

Fig.3 shows the comparison of the BER performances of the decoder. As shown in the figure, CO-LSD, RC-LSD and SE-LSD can achieve almost the same bit-error-rate (BER) performances. However, due to the limitation of list size, the CO-LSD, RC-LSD and SE-LSD all suffer from the BER degradation as compared to the MLD.

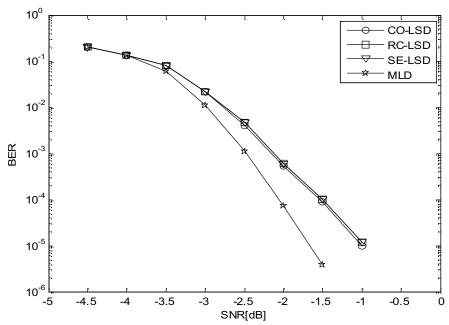

Fig. 3 : Comparison of the BER performance of the CO-LSD, RC-LSD, SE-LSD and MLD in V-BLAST system of (4,4) employing 64-QAM with LDPC outer coding.

The comparisons of the average complexities of the CO-LSD, RC-LSD and SE-LSD are illustrated in Fig.4. It is shown clearly that the CO-LSD results in the lowest complexity while SE-LSD results in the highest complexity. This is believed due to the fact that in the CO-LSD, the simple technique used for preparing candidates implementing only the real-value comparison has been exploited. In RC-LSD, although the similar technique is also used, it needs an additional computation for evaluating the arc() of a complex value.

Ⅴ. Conclusion

In this paper, a comparative view on the list sphere decoders, modified versions of SDs, has

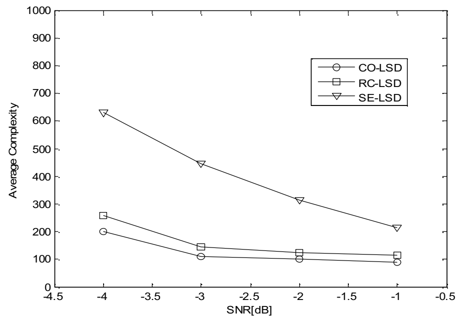

Fig. 4 : Average complexity performance of the inner list decoder of CO-LSD, RC-LSD, and SE-LSD.

been presented. By applying a simple optimal ordering procedure, the list decoders could get not only the comparable low BER but also significantly low complexity.

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