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EVALUATION OF MINIMUM REVENUE GUARANTEE(MRG) IN BOT PROJECT FINANCE WITH OPTION PRICING THEORY

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ABSTRACT: The limited public funds available for infrastructure projects have led governments to consider private entities' participation in long-term contracts for finance, construction, and operation of these projects to share risks and rewards between the public and the private. Because these projects have complicated risk evolutions, diverse contractual forms for each project member to hedge risks involved in a project are necessary. In light of this, Build-Operate-Transfer(BOT) model is considered as effective to accomplish Public Private Partnerships(PPPs) with a characteristic of an ownership-reversion. In BOT projects, the government has used such an incentive system as minimum revenue guarantee(MRG) agreement to attract the private's participation. Although this agreement turns out critical in success of BOT project, there still exist problematic issues in a financial feasibility analysis since the traditional capital budgeting theory, Net Present Value(NPV) analysis, has failed to evaluate the contingent characteristic of MRG agreement. The purpose of this research is to develop real option model based on option pricing theory so as to provide a theoretical framework in valuing MRG agreement in BOT projects. To understand the applicability of the model, the model is applied to the example of the BOT toll road project and the results are compared with that by NPV analysis. Finally, we found that the impact of the MRG agreement is significant on the project value. Hence, the real option model can help the government establish better BOT policies and the developer make appropriate bidding strategies.

Keywords: NPV Analysis; Real Option Analysis; Minimum Revenue Guarantee(MRG); Build-Operate-Transfer(BOT) Project Finance

1. INTRODUCTION

In many countries, limitations on the public funds available for infrastructure have led governments to attract the private to take part in long-term contractual agreements for financing, constructing and operating huge infrastructure projects. In these projects, shareholders often try to follow specific scheme, which is called project finance, where they just can put little equity and the project's continuation solely relies on direct cash flows coming from the project itself to cover operating and financing costs. Since risk evolution in these projects is generally complicated so that the investor conducts proper risk analysis, the public and the private may incorporate diverse features to hedge risks they may face. Among the various ways to accomplish these PPP projects, Build-Operate-Transfer(BOT) is one of the most frequently used types due to the characteristics of a shared-ownership between the public and private.

Basically, BOT scheme is implemented following risk and return negotiations among the government, project company, and lender. Through the bidding stage, those members negotiate one another to develop a mutually satisfactory project financing structure. Therefore, it is no wonder that the effective risk and return sharing scheme depends on the financial soundness of the BOT proposal since better financial planning provides a higher probability that the BOT project is successful.

In a BOT project, due to its unique characteristics coming from the uncertainties of huge project size, long concession period, and contractual complexity, there should exist asymmetric payoffs, which can not be easily assessed based on the traditional capital budgeting theory such as Net Present Value(NPV) analysis. Among diverse factors which cause asymmetric payoffs in BOT projects, such an incentive system as a minimum revenue guarantee(MRG) agreement is used to address the concerns of the private sector and attract investor participation in financing the project(Zhang, 2005). Despite the MRG agreement often found in BOT projects, the cost to the government of this financial incentive and its positive value to the private are not well understood(Mason and Baldwin, 1988). Furthermore, it is controversial to estimate the exact MRG value that the government has to support in order for the developers to decide to undertake the project. More seriously, unsolicited proposals of the private sector's participation in projects are not uncommon and these factors can cause government to be exploited by the BOT developer's proposing an unfair deal. This situation can become worse when it comes to the regulatory frameworks of the weak host country(Hodges, 2003). The MRG agreement adds direct value to the transaction. Developers failing to consider the value of guarantee will underestimate the investment value, and, if the guarantee value is too large, the government over-subsidizes the BOT firm. However, because traditional capital budgeting theory, NPV analysis, can not price the value of the MRG that creates an asymmetric payoff, studies on the valuation of infrastructure projects based on managerial flexibilities have been limited.

Fortunately, modern financial theory suggests that option pricing theory can be applied in the valuation to consider various complicated asset features of financing schemes and managerial flexibilities. Analogies in a process between the BOT financial feasibility evaluation and option pricing can help assess the asymmetric payoff condition coming from the contingency of the MRG agreement. Therefore, the purpose of this research is develop real option model based on discrete-time approach in option pricing theory so as to provide a theoretical framework in valuing MRG agreement in BOT project. To understand the applicability of the model, the model is applied to BOT toll road project and the results are compared with those by traditional capital budgeting theories.

2. THEORIES

2.1 Traditional Capital Budgeting Theory-NPV Analysis

One of the traditional capital budgeting theories, NPV analysis, works well while the risks of an asset remain stable as the project goes forward. This valuation method is appropriate for investment decisions as to the assets-in-place if operations guarantee relatively stable cash flows(Luehrman, 1998; Myers, 1984).

Projects often create contingencies such as delaying, abandoning or expanding the projects by the management decision changes. And, future cash flows may change as development proceeds or new information is received. In this case, NPV analysis is likely to either underestimate or ignore the value of this managerial flexibilities(Amram and Kulatilaka, 1999; Trigeorgis, 1999; Dixit and Pindyck, 1994; Myers, 1984). Once risk is recognized in investment based on the NPV analysis, this analysis reflects risk through a risk-adjusted discount rate to discount the expected cash flows. In a real world, many firms classify different risk categories of projects and assign each category different rates to reflect the risk involved(Trigeorgis, 1999) or use different discount rates in different periods to reflect the change of nominal rates of interest(Aggarwal, 1993). Even as the NPV analysis has been widely used in various industries as an effective valuation method, there are also some criticisms that can be leveled against it.

First, the NPV analysis assumes that the cash outflow is stable. Even when there are cash outflows in different time periods other than time 0, they are assumed to have the same risk characteristic as the cash inflows. But, in real projects such as large construction projects although the future cash inflows are assumed to be certain based on the contract, the uncertainty mainly comes from the cash outflows. Second, when NPV is applied to construction projects, it can not properly evaluate managerial flexibility to adjust later decision when, as uncertainty is resolved, future events turn out differently from what management expected at the beginning of the project(Copeland and Antikarov, 2001; Dixit and Pindyck, 1995; Trigeorgis, 1999). When a project is associated with high uncertainty, if an investment requires sequential decision-making and if early investment can reveal information about the future profitability of the project, it deserves to invest even when NPV is negative. When mistakenly ignoring the operating and managerial flexibilities involved in a project can cause a significant underestimation of its value(Mason and Merton, 1985). For the evaluation of long-term projects where future profitability is uncertain, it is critical to consider the associated managerial or strategic options.

For the reasons above, the real options analysis is suggested by many researchers as an effective method to incorporate the management flexibilities into the project value. In infrastructure projects, design flexibility may let the projects be flexible to varying conditions if demand risk is involved. Moreover, staged infrastructure projects give management the chance to receive new information as market become more certain. Even if flexibility adds value, because it requires efforts with regard to time, money, or complexity, it needs to be appropriately assessed. However, the NPV analysis has its limitation to evaluate this added value.

Equation (1) is the basic form of the NPV analysis with the concept of discounting the future cash flows at a required rate of return(Brigham and Houston, 2004):

$$NPV = -I_0 + \sum_{i=1}^{t} \frac{FCF_i}{(1+WACC)^i} \tag{1}$$

where, I_0 is the initial investment, FCF_i is the future net cash flow after tax at time i, WACC(Weighted Average Cost of Capital) is the required rate of return used to discount the future cash flow FCF_i, and i is the time increment. WACC of the firm or project as defined by Equation (2).

$$WACC = R_e \cdot \frac{E}{A} + R_d \cdot \frac{D}{A} \cdot (1-T) \tag{2}$$

where E is the equity, R_e is the cost of equity, A is total invested capital, R_d is the cost of debt, D is the debt, and T is the corporate tax. As for the infrastructure projects, WACC can be determined based on Equation (2) and the obtained WACC is used in Equation (1) to find Net present value of the project. WACC stands for a company's weighted average cost of capital reflecting cost of debt and cost of equity, and it is employed to evaluate projects matching a firm's existing operating assets and associated risks. Thus, determining R_d, T, D, E and A is not difficult, and the last variable, cost of equity, R_e, is often estimated by Capital Asset Pricing Model(CAPM). R_e is a measurement of the appropriate required return that equity investors should expect on equity investments, given the level of risk of such investments. Equation (3) used to estimate R_e is based on the CAPM developed by Sharpe(1964), which is expressed as follows.

$$R_e = R_f + \beta_e (R_m - R_f) \tag{3}$$

When it comes to the infrastructure projects, since some risk premiums coming from the uncertainties involved in the projects such as country or sector risk should be added to the cost of equity, actual risk-adjusted discount rate used in investment analysis can be greater than R_e.

2.2 Option Pricing Theory

The option pricing theory developed by Black, Scholes(1973), and Merton(1973), for pricing financial assets is the building block of this paper. The concept of option pricing theory was imported to seek to value options on real assets. This theory is based on the assumption that stock price follows a log-normal distribution, which is called a 'Geometric Brownian motion process' and has been proven to be appropriate for modeling the price of an asymmetric payoff of securities (Luenberger, 1998). The uncertainty of the value of real asset is reasonably reflected through this diffusion process(Brennan and Schwartz, 1984; Dixit and Pindyck, 1994). Equation (4) describes the diffusion process based on Geometric Brownian motion process:

$$\frac{dS}{S} = \mu\, dt + \sigma\, dz \tag{4}$$

where S is the stock price, μ is the instantaneous rate of return, σ^2 is the instantaneous variance of the rate of return, and dz is a random increment to a standard Wiener process. The value of a European call option can be obtained by solving the partial differential equation derived by Black and Scholes(1973), subject to one terminal and two boundary conditions. This Black-Scholes equation is the mathematical treatment of the option pricing framework but it is analytically limited because of its mathematical complexity of modeling and calculating processes. For this reason, it needs to take into account the numerical method such as in this paper. Recently, there have been some efforts to evaluate real assets based on the option pricing theory. The analyzing methodology used in this paper also can be regarded as a real option approach based on the option pricing theory.

2.3 Management Flexibility: Asymmetric Payoff caused by the MRG Agreement

Even if the NPV analysis is considered effective because of its consistency with the firm's objective of maximizing the shareholders' utilities, when the uncertainty is involved in an investment analysis, the discount rate used in this method should be appropriately adjusted for related risks based on the CAPM(Copeland and Weston, 1988). However, there exits argument that the NPV analysis is not appropriate to capture the characteristics of the managerial flexibilities in case of changing later decisions when the future seems different from what management expected. The managerial flexibilities provide specific kinds of asymmetric payoff relationships identical to the payoff forms of the stock options and, therefore, the option pricing theory can be effective to price such complicated dynamics of the contingencies. Followings are the examples of the asymmetric payoffs in financial call and put options.

$$F_{call}(S_t, t) = Max[0, S_t - X] \tag{5}$$

$$F_{put}(S_t, t) = Max[0, X - S_t] \tag{6}$$

where F_{call} and F_{put} are the option values of call and put options respectively, S_t is the stock price at time t, and X is the exercise price.

When it comes to the MRG agreement, it can be formulated based on the concept of financial put option described in Equation (6). The basic idea of the MRG agreement is that during concessionaire period if the projected cash flow in each year i satisfies the guaranteed cash flow level, which is already signed in the BOT contract based on the expected cash flow agreed by both of the public and the private, the government does not have to pay any MRG to the BOT developer. Otherwise, the government should compensate for the shortfall in revenue by paying the BOT developer. As an MRG value, the government's obligation to pay in each year, SF_i, would depend on the relative value between guaranteed cash flow at year i, CF_{ig}, and projected cash flow at year i, CF_{ip}, as shown in Equation (7)(Cheah and Liu, 2006). When the guaranteed cash flow is greater than the projected cash flow, the government pays the difference between the guaranteed cash flow and projected cash flow to the BOT developer as the MRG. Therefore, the government's MRG payment at year i, SF_i, can be estimated based on the asymmetric payoff of Equation (7).

$$\begin{aligned} SF_i &= Max\,[\,Guaranteed\ \ FCF_e\ \ at\ \ Year\ \ i \\ &\qquad - Projected\ \ FCF_e\ \ at\ Year\ \ i,\ 0] \\ &= Max\,[\,CF_{ig} - CF_{ip},\ 0] \end{aligned} \tag{7}$$

where FCF_e is free cash flow on equity at year i and SF_i is free cash flow difference on equity at year i between the guaranteed cash flow and projected cash flow in the MRG agreement. Finally, we can find the MRG value as follows:

$$MRG = \sum_{i=1}^{n} \frac{SF_i}{(1+r)^i} \tag{8}$$

where MRG is the present value of the total MRG value during concession period at time "0", r is the risk-free rate, and n is the years of the BOT concession period.

3. METHODS

Based on the option pricing theories, this paper develops a real option model to evaluate the MRG agreement in a BOT project. This paper considers the financial feasibility of the BOT project at the equity level reflecting the dynamics of the project value from the developer and the government's points of views. The real option model is tested by the case example of the real BOT toll road to see its applicability. The results show that the real option model is capable of better representing the BOT project situations. Following are the processes to construct the real option model and an illustrative example in the next section.

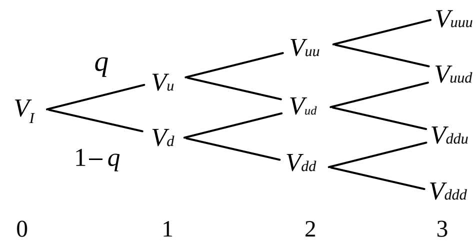
Sept 1. Selection of the Underlying Risky Asset and Determination of Its Dynamics

The first step is to choose the underlying asset and determine its dynamics. The change of the underlying asset value, project value, has an impact on the option value since the option value is contingent on the underlying asset. When it come to the project finance, as the lenders look especially to the forecasted cash flow rather than to project assets as collateral for the loan, the forecasted cash flow is the main credit support of the capital needed(Beidleman et al., 1990). Therefore, this paper determines the value of the BOT project based on its forecasted cash flow rather than the value of the physical asset. The uncertainty of the operating profit is the main risk during the operation in a BOT project because, in some projects exposed to market competition, operating and economic risks could be large(Finnerty, 1996). The BOT project value is defined as the summation of the expected cash flow discounted at an appropriated risk-adjusted discount rate for time '0' during the operation. As a result, this value is based on the future cash flow of the entire concession period. This research concerns the stochastic nature of the project value on equity, which is subject to change or fluctuations due to various market conditions during the operation period. To model the dynamics of the project value during the operation period, the project value V is defined as an underlying asset being assumed to follow a geometric Brownian motion process as follows(Dixit and Pindyck, 1994):

$$\frac{dV}{V} = \mu \ dt + \sigma \ dz \qquad (9)$$

where V represents the market value of a completed project, μ is the market required rate of return from the project, σ describes the volatility of the rate of return in the project value, and dz is an increment to a standard Wiener process. This step helps assume a structure for the dynamics and uncertainties of the underlying risky asset 'project value.'

Figure 1 Binomial Tree of Underlying Asset, V



Step 2. Finding the Initial Project Value "V_I"

The initial project value is the present value of the expected cash flows which consist of all the revenues and expenditures generated from the investment excluding the initial investment cost in the project. Then, by discounting these future cash flows at a proper discount rate to the present, the initial project value can be estimated. Because this research focuses on the BOT developer and the government's points of views, the dynamics of the project value on equity should be captured as follows.

$$V_I = \sum_{i=1}^{n} \frac{FCFe_i}{(1 + R_e)^i} \qquad (10)$$

where $FCFe_i$ is the free cash flow on equity at year i and R_e is the cost of equity. $FCFe_i$ is obtained by deducting the annual debt service from the annual free cash flows.

Step 3. Selection of Volatility "σ"

Volatility σ, which is defined as a standard deviation of rate of return in cash flow returns, can be obtained from the logarithmic value of the cash flow returns. Because this value is calculated with historic or future estimates of cash flow returns agreed between the public and the private, this approach is easy to be simply applied in a financial feasibility process and has been widely used in estimating the volatility of real assets in many industries. In the case example of this paper, it is assumed that the volatility is given for the convenience of the calculation.

Step 4. Up and Down Movements, "*u*" and "*d*" and Risk Neutral Probability, "q" and "$1-q$"

The next is to calculate the value of the up and down movement '*u*' and '*d*' which will be multiplied with the initial project value V_I to reflect the dynamics of the project value V. Under the binomial tree framework, *u*, *d*, and R equal to $e^{r\Delta t}$ are needed in order to compute the risk neutral probabilities q and $1-q$. By imposing $u = 1/d$, for convenience, the up and down movements and risk neutral probabilities can be obtained from Equation (11) to (14)(Cox et al., 1979):

$$u = e^{\sigma\sqrt{\Delta t}} \quad (11) \qquad \text{and} \qquad d = e^{-\sigma\sqrt{\Delta t}} \quad (12)$$

$$q = \frac{R - d}{u - d} \quad (13) \qquad \text{and} \qquad 1 - q = \frac{u - R}{u - d} \quad (14)$$

where Δt is time interval. We alternatively obtain u and d by imposing a fixed pseudo probability, $q = 1-q = 0.5$, for the convenience of the calculation. This helps replace Equation (13) and (14) with Equation (15) and (16), thereby keeping the risk neutral probabilities remain constant regardless of the value of σ or the number of time step Δt (Hull, 1997):

$$u = e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}} \quad (15) \quad \text{and} \quad d = e^{\left(r-\frac{1}{2}\sigma^2\right)\Delta t - \sigma\sqrt{\Delta t}} \quad (16)$$

Step 5. Construct a Reverse Binomial Tree with an Underlying Asset "V_I"

It is time to construct the binomial tree with V_I, σ, u, and d taken from above steps. As shown in Figure 1, since the binomial tree involves all the likely project values considering the uncertainties over time, project values in the binomial tree reflect the projected project values. The change of this project value over time is as follows:

At $t = 1$

$$V_u = u \cdot V_I = e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot V_I \quad (17)$$

$$V_d = d \cdot V_I = e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \quad (18)$$

At $t = 2$

$$V_{uu} = u^2 \cdot V_I = e^{2\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot V_I \quad (19)$$

$$V_{ud} = u \cdot d \cdot V_I = e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \quad (20)$$

$$V_{dd} = d^2 \cdot V_I = e^{2\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \quad (21)$$

At $t = 3$

$$V_{uuu} = u^3 \cdot V_I = e^{3\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot V_I \quad (22)$$

$$V_{uud} = u^2 \cdot d \cdot V_I = e^{2\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \quad (23)$$

$$V_{ddu} = d^2 \cdot u \cdot V_I = e^{2\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot e^{1\cdot\left(\left(r-\frac{1}{2}\right)\Delta t+\sigma\sqrt{\Delta t}\right)} \cdot V_I \quad (24)$$

$$V_{ddd} = d^3 \cdot V_I = e^{3\cdot\left(\left(r-\frac{1}{2}\right)\Delta t-\sigma\sqrt{\Delta t}\right)} \cdot V_I \quad (25)$$

Step 6. Formulation of the MRG Agreement as a Put Option

The MRG agreement can be formulated as a put option based on the asymmetric payoff condition in Equation (7). However, this paper takes into account the real option model at the level of project value as shown in Equation (26). If the projected project value at each time step is higher than the guaranteed project value, there is not any reason for the government to pay the MRG to developer. On the other hand, if the projected project value is less than the guaranteed project value, there should be an MRG for the developers to quit an averse condition where they can not obtain the minimum revenue to cover the cost, expense, or debt.

$$MRG_i = Max[Guaranteed\ Project Value\ on\ Equity\ at\ Year\ i - Projected\ Project\ Value\ on\ Equity\ at\ Year\ i,\ 0] \quad (26)$$

The project value follows a geometric Brownian motion process which has two major factors in its value change: the term of the fixed rate of return and that of the uncertain rate of return, which is randomly selected at every time step over time. When the initial project value increases at a fixed rate of return over time, this represents the dynamics of the project value without uncertainty. Therefore, this varying project value is assumed as the exercise price. The exercise price is as following:

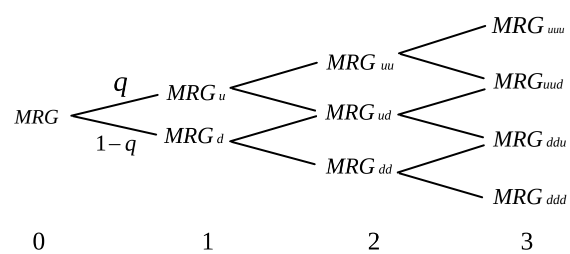
$$\text{At } t = n \qquad X_n = e^{n\cdot\left(\left(r-\frac{1}{2}\right)\Delta t\right)} \cdot V_I \quad (27)$$

Where, X_n is the exercise price at time n.

Step 7. Asymmetric Payoff Condition at Each Node in the Binomial Tree

Throughout the above steps, we finally can construct the asymmetric payoff conditions for each time step as shown in Figure 2 and Equation (28) to (36).

Figure 2 MRG Option Value and Asymmetric Payoff Condition in Binomial Tree



where,

$$\text{At } t = 1 \quad MRG_u = Max\left[X_1 - V_u,\ 0\right] \quad (28)$$

$$MRG_d = Max\left[X_1 - V_d,\ 0\right] \quad (29)$$

$$\text{At } t = 2 \quad MRG_{uu} = Max\left[X_2 - V_{uu},\ 0\right] \quad (30)$$

$$MRG_{ud} = Max\left[X_2 - V_{ud},\ 0\right] \quad (31)$$

$$MRG_{dd} = Max\left[X_2 - V_{dd},\ 0\right] \quad (32)$$

$$\text{At } t = 3 \quad MRG_{uuu} = Max\left[X_3 - V_{uuu},\ 0\right] \quad (33)$$

$$MRG_{uud} = Max\left[X_3 - V_{uud},\ 0\right] \quad (34)$$

$$MRG_{ddu} = Max\left[X_3 - V_{ddu},\ 0\right] \quad (35)$$

$$MRG_{ddd} = Max\left[X_3 - V_{ddd},\ 0\right] \quad (36)$$

Step 8. Implementing the Calculation Backward Recursively

To find the MRG value, calculation is implemented backward recursively from the end of the binomial tree in Figure 2. The selected option value based on the asymmetric payoff condition at each node is calculated by q, $1-q$, and R. For instance, while finding the option value of MRG_{uu} at time 2 in Figure 2, it needs to consider whether an MRG option is exercised or not. Because the real option analysis chooses the maximized value in each time step, we have to have the larger value between when the option is exercised and not exercised. This represents the only one is chosen between two option values whichever is larger and this process is iterated in every time step from the end of the binomial tree to present. When the MRG agreement is not exercised, MRG_{uu} is described in Equation (37):

$$MRG_{uu\,(not\ exercised)} = \frac{[\,q \cdot MAX\,(X_3 - V_{uuu},\,0) + (1-q)\cdot MAX\,(X_3 - V_{uud},\,0)\,]}{e^{r\Delta t}} \quad (37)$$

And, when being exercised:

$$MRG_{uu\,(exercised)} = MAX\,[\,X_2 - V_{uu}\,,\,0\,] \quad (38)$$

Finally, since the either one needs to be chosen at time 2 whichever is larger, its value will be as follows.

$$MRG_{uu} = Max\left[\begin{matrix}\dfrac{q\,Max\,[X_3 - V_{uuu},0] + (1-q)\,Max\,[X_3 - V_{uud},0]}{e^{r\Delta t}} \\ ,\ Max\,[X_2 - V_{uu},0]\end{matrix}\right] \quad (39)$$

Through the iterations of this process at every node for every time step, the MRG value at time 0 can be calculated. Followings show all asymmetric conditions and MRG option values at each node for the binomial tree.

At *t=2*

$$MRG_{uu} = Max\left[\frac{q\,MRG_{uuu} + (1-q)\,MRG_{uud}}{e^{r\Delta t}},\ X_2 - V_{uu}\right] \quad (40)$$

$$MRG_{ud} = Max\left[\frac{q\,MRG_{udu} + (1-q)\,MRG_{udd}}{e^{r\Delta t}},\ X_2 - V_{ud}\right] \quad (41)$$

$$MRG_{dd} = Max\left[\frac{q\,MRG_{ddu} + (1-q)\,MRG_{ddd}}{e^{r\Delta t}},\ X_2 - V_{dd}\right] \quad (42)$$

At *t=1*

$$MRG_u = Max\left[\frac{q\,MRG_{uu} + (1-q)\,MRG_{ud}}{e^{r\Delta t}},\ X_1 - V_u\right]$$

$$= Max\left[\frac{qMax[X_2 - V_{uu},0] + (1-q)Max[X_2 - V_{ud},0]}{e^{r\Delta t}},\ X_1 - V_u\right] \quad (43)$$

$$MRG_d = Max\left[\frac{qMRG_{du} + (1-q)MRG_{dd}}{e^{r\Delta t}},\ X_1 - V_d\right]$$

$$= Max\left[\frac{qMax[X_2 - V_{du},0] + (1-q)Max[X_2 - V_{dd},0]}{e^{r\Delta t}},\ X_1 - V_d\right] \quad (44)$$

At *t=0*

$$MRG = \frac{q\,MRG_u + (1-q)\,MRG_d}{e^{r\Delta t}}$$

$$= \frac{\left[\begin{matrix} qMax\left[\dfrac{q\,MRG_{uu} + (1-q)\,MRG_{ud}}{e^{r\Delta t}},\ X_1 - V_u\right] \\ +(1-q)Max\left[\dfrac{qMRG_{du} + (1-q)\,MRG_{dd}}{e^{r\Delta t}},\ X_1 - V_d\right]\end{matrix}\right]}{e^{r\Delta t}} \quad (45)$$

4. ILLUSTRATIVE EXAMPLE

Table 1 describes an illustrative example of the BOT toll road system in order to show whether the developed real option model can be applicable. In this example, capital expenditure, operating expenditure, and average toll rate are assumed to annually increase at 3%.

4.1 NPV Analysis

Market risk premium, *MRP* in Equation (46), is the difference between the return of the market and the risk-free rate. So, *MRP* is 5.1%. β, which is a measurement of risk for the BOT developer, is 1.335.

$$R_e = R_f + (MRP \times \beta) = 5.3\% + (5.1 \times 1.335) = 12.11\,\% \quad (46)$$

Table 1. BOT Toll Road Case Example

Capital Structure	
Project Construction Cost	\$316 M(4 years)
Debt : Equity = 81.3:18.7	\$257 M : \$ 59 M
Debt	Senior: 15 years(8.11%) Sub: 20 years(20%)
Capital Expenditure	\$3.66 M(Every 5 years)
Operating Expenditure	\$2.53 M(Every year)
Corporate Tax Rate	27.5%
Initial Traffic Volume	8.61 M(Year)
Traffic Volume Growth Rate(Standard Deviation)	2.3(0.989)%
Average Toll Rate	\$ 3
Concession Period	30 Years(From 2008)
MRG Agreement	80% of Expected Revenue(30 years)
Market Rate of Return	10.4%
Risk Free Rate	5.3%
Cost of Equity	12.11%
Volatility	0.095

Based on the Equation (1) with the information in Table 1, we can build the cash flow model of BOT case example as shown in Figure 3 and have the NPV on equity of \$6.11 million without considering the MRG agreement as follows:

$$\mathrm{NPV}_e = \frac{-59.00}{(1+0.12.11)^0} + \frac{0}{(1+0.1211)^1} + \frac{0}{(1+0.1211)^2} + \ldots\ldots + \frac{69.860}{(1+0.1211)^{32}} + \frac{67.070}{(1+0.1211)^{33}} = \$\ 6.11\ \mathit{Million} \qquad (47)$$

4.2 Real Option Analysis

Through the Equation (10), we can calculate the initial project value V_I used to reflect the dynamics of the underling asset in Equation (48).

$$V_I = \sum_{i=1}^{n} \frac{FCFe_i}{(1+R_e)^i} = \frac{-0.70}{(1.121)^1} + \frac{0.61}{(1.121)^2} + \ldots + \frac{69.86}{(1.121)^{29}} + \frac{67.07}{(1.121)^{30}} = \$\ 91.82\ \mathit{Million} \qquad (48)$$

With the given volatility of 0.095 in Table 1, we have the up and down movements based on Equation (15) and (16):

$$u = e^{\left(\left(r-\frac{1}{2}\sigma^2\right)\Delta t+\sigma\sqrt{\Delta t}\right)} = e^{\left(\left(0.053-\frac{1}{2}(0.0948)^2\right)\cdot 1 + 0.0948\cdot\sqrt{1}\right)} = 1.154 \qquad (49)$$

$$d = e^{\left(\left(r-\frac{1}{2}\sigma^2\right)\Delta t-\sigma\sqrt{\Delta t}\right)} = e^{\left(\left(0.053-\frac{1}{2}(0.0948)^2\right)\cdot 1 - 0.0948\cdot\sqrt{1}\right)} = 0.955 \qquad (50)$$

Now, because we know the values of V_I, u, d, q, and $1-q$, we can construct a binomial tree which reflects all possibilities that the project value can have during the concession period of 30 years. Table 2 describes the parameters needed to build the binomial tree.

When it comes to the option formulation, the guaranteed project value used as the exercise price in a put option is defined as a guaranteed project value in Equation (27). Since the guaranteed project value at time '0' is the same as the initial project value multiplied by 0.8(the MRG agreement says that 80 % of the expected cash flow will be guaranteed as minimum revenue), we have the exercise price of $0.8 \times 91.82 = 73.50$ at the first year of the operation and this will increase at the annual rate of $r-(1/2)\sigma^2 = 0.053-(1/2)\cdot 0.0948^2 = 0.049$ over

Table 2. Calculated Parameters

V_I (Million, \$)	91.82	Volatility	0.0948
u	1.154	Concession Period (Year)	30
d	0.955	Δt	1
r	0.053	$q = 1-q$	0.5

time during the concession period. The MRG option will be exercised whenever the condition to exercise the option is met. The last step is to calculate the MRG value. The selected option value at each node in the binomial tree is calculated backwards recursively using the risk neutral probabilities 0.5 and a risk-free rate of 5.3%. In Figure 4, we have the MRG value in 2007, which is \$5.948 million and, by discounting this value to 2004 with a risk-free rate, we obtain the MRG of \$5.075 million. As results of the analysis, we have the NPV on equity of \$6.11 million based on the traditional NPV analysis without considering the MRG agreement and, by the real option model, the MRG agreement value, \$5.075 million which accounts for 83.06% of NPV on equity and 8.60% of initial equity investment respectively.

5. CONCLUSIONS

The MRG agreement is an important concern for both of the BOT developer and the government. Through this paper, a binomial real option model is developed to seek to evaluate this agreement. The conclusions drawn from this paper are as follows. First, the MRG value obtained through the real option approach has significant impact on the static NPV on equity in BOT case example. Therefore, the BOT developer does not fully assume the operational risk should things go wrong. Second, the negotiation process associated with the MRG agreement is critical because it directly affects the MRG value. While most of the input variables are deterministic, the exercise price that the government and the BOT developer can decide through the negotiation process is the only controllable factor. In addition, because the exercise price determined by a certain percentage of the expected project revenue depends on the MRG agreement, the negotiation process for the MRG agreement should be carefully taken into account. Finally, the developed model seems to be relatively easier for the management to use in the practical world as opposed to the Black-Scholes equation because it is derived from simple numerical framework and not from a set of complicated mathematics. It may be available for BOT project members who are already familiar with the algebra level of the NPV analysis. Moreover, due to the simplicity of

Figure 3. BOT Toll Road Cash Flow Model

Year	2004	2005	2006	2007	2008	2009	2010	…	…	2036	2037
Traffic Volume (M)					8.61	8.97	9.32	…	…	16.39	16.62
Toll Rate (\$)					3.00	3.09	3.18	…	…	6.86	7.07
Gross Revenue (M, \$)					26	28	30	…	…	112	117
CAPEX (M, \$)	59							…	…		8.67
OPEX (M, \$)					2.53	2.61	2.68	…	…	5.79	5.96
EBIT (M, \$)	-59				23.30	25.11	26.98	…	…	106.71	102.86
Senior Debt Service (M, \$)					24.27	24.27	24.27	…			
Sub Debt Service (M, \$)								…	…	10.35	10.35
Taxes (M, \$)					-0.27	0.23	0.75	…	…	26.50	25.44
FCF on Equity (M, \$)	-59				-0.70	0.61	1.97	…	…	69.86	67.07

Figure 4. MRG Option Value of the BOT Toll Road

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
5.948	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	8	6	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		11	9	6	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
			14	11	9	7	5	3	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				18	15	12	9	7	5	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					23	20	16	13	10	7	5	3	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						30	26	21	17	14	10	7	4	3	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							37	32	28	23	19	14	10	7	4	2	1	0	0	0	0	0	0	0	0	0	0	0	0	0
								45	40	36	30	25	20	15	10	6	4	2	1	0	0	0	0	0	0	0	0	0	0	0
									53	49	45	39	33	27	21	15	10	6	3	1	0	0	0	0	0	0	0	0	0	0
										62	59	54	49	43	36	29	22	15	10	5	2	0	0	0	0	0	0	0	0	0
											70	68	65	60	54	47	39	31	23	15	9	4	1	0	0	0	0	0	0	0
												79	77	75	71	66	60	51	42	33	23	14	7	2	0	0	0	0	0	0
													88	87	85	82	78	73	66	57	46	35	23	13	4	0	0	0	0	0
														97	97	96	94	91	86	80	72	62	50	37	22	9	0	0	0	0
															106	107	106	105	103	100	95	89	80	69	55	38	20	0	0	0
																116	117	118	117	116	114	110	105	98	88	76	60	41	0	0
																	126	128	129	130	129	128	125	121	115	107	97	83	0	0
																		136	139	141	142	143	142	141	138	134	127	118	0	0
																			147	150	153	155	157	157	157	155	152	147	0	0
																				157	161	165	168	171	173	173	173	171	0	0
																					169	173	178	182	185	188	190	191	0	0
																						180	186	191	196	200	204	207	0	0
																							192	199	205	211	216	221	2	0
																								205	212	219	226	232	15	0
																									218	226	234	241	26	0
																										232	240	249	35	0
																											246	255	42	0
																												261	48	0
																													53	0
																														0

the binomial model in formulating the complex management flexibilities, to model the MRG agreement as an option may be easily applied to other management flexibilities in BOT contracts through proper modifications. Even if there have been some research as to the issue of the BOT project valuation by the real option approach, we still have some open issues which mainly arise from the characteristics of the BOT project itself. However, in a real world, since there may be more diverse and complex managerial flexibilities which are likely to be formed as asymmetric payoff conditions, efforts to identify, formulate, and evaluate these possible contingencies are necessary.

REFERENCES

[1] Amram, M. and Kulatilaka, N., *Real Options: Managing Strategic Investment in an Uncertain World*, Cambridge, MA: Harvard Business School Press, 1999.

[2] Beidleman, C.R., Fletcher, D. and Veshosdy , D., "On Allocating Risk: The Essence of Project Finance", *Sloan Management Review*, 31, pp. 47-55, 1990.

[3] Black, F. and Scholes, M., "The Pricing of Options and Corporate Liabilities", *Journal of Political Economy*, 81, pp. 637–54, 1973.

[4] Brennan, M.J. and Schwartz, E.S., "Optimal Financial Policy and Firm Valuation", *Journal of Finance*, 39, pp. 593–607, 1984.

[5] Brigham, E.F. and Houston, J.F., *Fundamentals of Financial Management* (10th ed.). Mason, OH: Thompson South-Western, 2004.

[6] Cheah, Y.J. and Liu, J., "Valuing Governmental Support in Infrastructure Projects as Real Options Using Monte Carlo Simulation", *Construction Management and Economics*, 24, pp. 545–554, 2006.

[7] Copeland, T.E. and Antikarov, V., *Real Options: A Practitioner's Guide*, New York: Texere LLC, 2001.

[8] Copeland, T.E. and Weston, J.F., *Financial Theory and Corporate Policy*, New York: Addison-Wesley, 1988.

[9] Cox, J.C., Ross, S.A. and Rubinstein, M., "Option Pricing: A Simplified Approach", *Journal of Financial Economics*, 7, pp. 229–63, 1979.

[10] Dixit, A.K. and Pindyck, R.S., *Investment under Uncertainty*, Princeton, NJ: Princeton University Press, 1994.

[11] Dixit, A.K. and Pindyck, R.S., "The Option Approach to Capital Investment", *Harvard Business Review*, May–June, pp. 105–115, 1995.

[12] Finnerty, J.D., *Project Financing: Asset-Based Financial Engineering*, New York: John Wiley & Sons, Inc, 1996.

[13] Hodges, J., *Unsolicited Proposals Private Sector and Infrastructure Network*, Note No. 257, World Bank, Washington, DC, 2003.

[14] Hull, J., *Options, Futures, and Other Derivatives*. (3rd ed.)., Englewood Cliffs, NJ: Prentice Hall, 1997.

[15] Luehrman, T.A., "Investment Opportunities as Real Options: Getting Started on the Numbers", *Harvard Business Review*, July–August, 1998.

[16] Mason, S.P. and Baldwin, C.Y., "Evaluation of Government Subsidies to Large-Scale Energy Projects", *Advances in Futures and Options Research*, 3, pp. 169–81, 1988.

[17] Mason, S.P. and Merton, R.C., *The Role of Contingent Claims Analysis in Corporate Finance. Recent Advances in Corporate Finance,* edited by Altman, E.I. and M.G. Subrahmanyam, Homewood, IL: Richard D. Irwin Inc, 1985.

[18] Merton, R.C., "Theory of Rational Option Pricing", *Bell Journal of Economics and Management Science*, 4, pp. 141–83, 1973.

[19] Myers, S.C., "The Capital Structure Puzzle", *Journal of Finance*, 39, pp. 575–592, 1984.

[20] Sharpe, W. F., "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk", *Journal of Finance*, Vol. 19(3), pp. 425–442, 1964.

[21] Trigeorgis, L., *Real Options: Managerial Flexibility and Strategy in Resource Allocation*, Cambridge, MA: The MIT Press, 1999.

[22] Zhang, X., "Critical Success Factors for Public–private Partnerships in Infrastructure Development", *Journal of Construction Engineering and Management*, Vol. 131(1), pp. 3–14, 2005.