

## EFFECT OF CONCRETE STRENGTH ON FLEXURAL DEFLECTION OF HIGH-STRENGTH REINFORCED CONCRETE BEAMS

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**ABSTRACT:** Deflections of Reinforced concrete structures must satisfy the permissible values and it is hard to predict the due to uncertainty of deflection of the reinforced concrete structures. Thus, many researchers have suggested a number of experimental equation of deflection against the uncertainty. In a specification, a procedure to evaluate flexure deflection using effective moment of inertia and moment-curvature relation is suggested. ACI offers a method using effective moment of inertia, which has been developed by Branson. Eurocode 2(EC2) suggests a procedure to evaluate deflection of reinforced concrete structure using moment-curvature relation. In this paper, a series of experiments were conducted on the singly reinforced concrete beams which have the same reinforcement ratio and different concrete strength. Therefore, the effect of the concrete strength on the deflection of the beams was analysed. The deflections obtained from the experiment were compared with the deflections calculated with ACI code and EC2.

**Keywords:** flexural deflection, reinforced concrete, concrete strength, ACI, EC2

### 1. INTRODUCTION

Deflection of reinforced concrete member must satisfy the permissible values. It is hard to predict deflection of reinforced concrete member due to uncertainty of deflection. When the load is acting on the reinforced concrete, crack will occur in the uncertain. The difficulty of prediction the deflection is the uncertain occurrence of cracks. Thus, many researchers have suggested a number of experimental equation of deflection against the uncertainty. In a specification, a procedure to evaluate flexural deflection using effective moment of inertia and moment-curvature is suggested. ACI offers a method to evaluate deflection of reinforced member using effective moment of inertia, which has been developed by Branson. Eurocode 2 suggests a procedure to evaluate deflection of reinforced concrete members using moment-curvature relation.

In this study, experimental research has been carried out for 4 simply supported reinforced concrete beams under two point loadings. Parameters of test include concrete strength. This study investigated effect of concrete strength on deflection of reinforced beams. The deflections obtained from the experiment were compared with the deflections calculated with ACI code and Eurocode 2.

### 2. MODEL OF EVALUATE TO DEFLECTION OF REINFORCED CONCRETE MEMBERS

#### (1) ACI318-05

ACI controls deflection of reinforced concrete structure subjected to flexure by using two kinds of method. The first is method to limit minimum thickness of structure. The second is to calculate deflection of reinforced concrete structure by directly. Unless stiffness values are obtained by more comprehensive analysis, immediate deflection shall be computed with the modulus of elasticity for concrete and with the effective moment of inertia as follows, but not greater than  $I_g$ .

$$I_e = \left( \frac{M_{cr}}{M_a} \right)^3 I_g + \left[ 1 - \left( \frac{M_{cr}}{M_a} \right)^3 \right] I_{cr} \leq I_g$$

where  $I_e$  = effective moment of inertia,  $I_g$  = moment of inertia of gross concrete section about centroidal axis, neglecting reinforcement,  $I_{cr}$  = moment of inertia of cracked section transformed to concrete,  $M_{cr}$  = cracking moment,  $M_{cr} = \frac{f_r I_g}{y_t}$ ,  $M_a$  = Maximum unfactored

moment in member at stage deflection is computed.

effective flexural stiffness shall be computed with the moment of inertia and modulus of elasticity for concrete. effective flexural stiffness of reinforced concrete structure

is shown in Fig 1. effective flexural stiffness values are more values when cracking when no cracking as shown in Fig.1. Expression of effective flexural stiffness is induced by experimental study. Thus, effective flexural stiffness is different from actual flexural stiffness. Deflection of reinforced concrete structures is computed by linear elastic analysis. In ACI codes, deflection of reinforced concrete structure is calculated assuming that stiffness is same along span and considering maximum moment at section of members. In fact, however, deflection of reinforced concrete structure is changed when load distribution is changed. Namely, effect of load distribution on deflection of reinforced concrete structure is neglected in ACI codes. JSCE codes propose developed expression of effective moment of inertia to distinguish whether stiffness is function of moment at section.

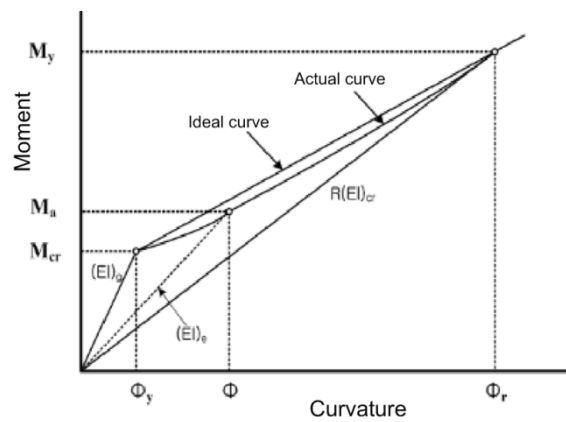


Fig 1. Effective flexural stiffness of reinforced concrete members

(2) Eurocode 2

Eurocode 2 is using method to control deflection of reinforced concrete similar to ACI. Namely, Eurocode 2 control deflection by limiting span/depth ratio of reinforced concrete structure or calculating deflection by directly. The limiting span/depth ratio may be estimated using Eq. 1, 2 and multiplying this by correction factors to allow for the type of reinforcement used and other variables. No allowance has been made for any pre-camber in the derivation of these Expressions.

$$\frac{l}{d} = K \left[ 11 + 1.5\sqrt{f_{ck}} \frac{\rho_0}{\rho} + 3.2\sqrt{f_{ck}} \left( \frac{\rho_0}{\rho} - 1 \right)^{3/2} \right] \text{ if } \rho \leq \rho_0 \quad (1)$$

$$\frac{l}{d} = K \left[ 11 + 1.5\sqrt{f_{ck}} \frac{\rho_0}{\rho - \rho_0} + \frac{1}{12}\sqrt{f_{ck}} \sqrt{\frac{\rho'}{\rho_0}} \right] \text{ if } \rho > \rho_0 \quad (2)$$

where,  $l/d$  = the limit span/depth,  $K$  = the factor to take into account the different structural system,  $\rho_0$  = the reference reinforcement ratio =  $\sqrt{f_{ck}} 10^{-3}$ ,  $\rho$  = the required tension reinforcement ratio at mid-span to resist the moment due to the design loads (at support for cantilevers),  $\rho'$  = the required compression reinforcement ratio at mid-span to resist the moment due to design loads (at support for cantilevers),  $f_{ck}$  = in MPa units.

when deflection of reinforced concrete members shall be calculated, it is calculated by the distinction between cracked section and uncracked section in member. Members which are not expected to crack, but may not be fully cracked, will behave in a manner intermediate between the uncracked and fully cracked conditions and, for members subjected mainly to flexure, an adequate prediction of behaviour is given by Eq. 3.

$$\alpha = \zeta \alpha_{ii} + (1 - \zeta) \alpha_i \quad (3)$$

where,  $\alpha$  = the deformation parameter considered which may be, for example, a strain, a curvature, or a rotation,  $\alpha_i, \alpha_{ii}$  = the values of the parameter calculated for the uncracked and fully cracked conditions respectively,  $\zeta$  = a distribution coefficient (allowing for tensioning stiffening at a section) given by Eq. 4.

$$\zeta = 1 - \beta \left( \frac{\sigma_{sr}}{\sigma_s} \right)^2 \quad (4)$$

where, for uncracked sections,  $\beta$  = a coefficient taking account of the influence of the duration of the loading or of repeated loading on the average strain = 1.0 for a single short-term loading = 0.5 for sustained loads or many cycles of repeated loading,  $\sigma_s$  = the stress in the tension reinforcement calculated on the basis of a cracked section,  $\sigma_{sr}$  = the stress in the tension reinforcement calculated on the basis of a cracked section under loading conditions causing first cracking

$\alpha$  is taken as curvature when deflection of member shall be calculated. Deflection of member is computed to the curvatures along the span and then calculated by integration. When deflection shall be calculated, Eurocode 2 is more complex than ACI because of integration progress.

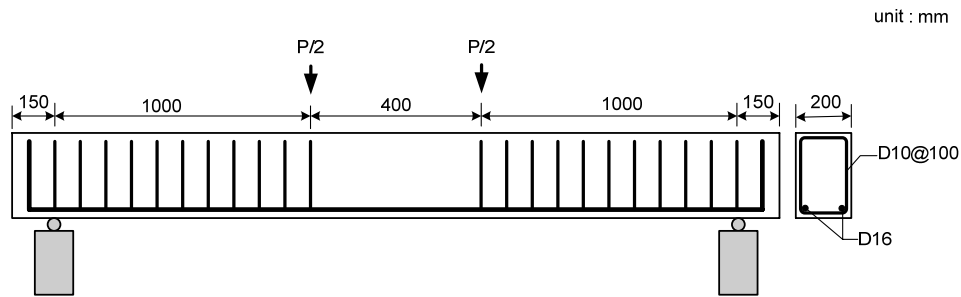


Fig 2. Properties of test beams

Table 1. Properties of steel used in test beams

Type	Diameter of steel (cm)	Area of steel (cm <sup>2</sup> )	Yield strength (MPa)	Modulus of elasticity (MPa)	
D10	0.953	0.713	400	200,000	stirrups
D16	1.59	1.986	400	200,000	Tensile stell

Table 2. Name of test beams &amp; Strength of concrete

Name of test beams	S1	S2	S3	S4
Strength of concrete (MPa)	71.5	76.3	82.6	86.8

### 3. TEST PROGRAM

Experimental research has been carried out for 4 simply supported reinforced concrete beams under two point loadings as shown in Fig 2. Dimension of all test beams have the same 200×300 mm rectangular section, with a simply supported clear span of 2,400 mm and area of steel of 397.2 mm<sup>2</sup>. Stirrups were eliminated in pure bending region and installed at intervals of 10 cm in order to induce flexural failure. The distance between the loading points is 40 cm and loading speed is 1mm/min in pure bending. Three cylinder were also cast from each batch, cured under conditions similar to the corresponding beams, and tested at the age of 28 days.

Properties of steel used in test beams is shown in Table 1. Properties of concrete strength of test beams are shown in Table 2..

### 4. RESULT OF EXPERIMENTAL STUDY

As a result of the experiment, load acting on beams and deflection at mid-span relationship was illustrated. Also,

deflections obtained from the experiment were compared with the deflections calculated with ACI code and Eurocode 2. Modulus of elasticity and tensile strength of concrete was calculated by each specifications. Load acting on test beams and deflection at mid-span relations is shown in Fig 3. Deflection of test beams computed by ACI and Eurocode 2 is shown in Fig 4. Comparison with experimental deflection and deflection computed by specifications is shown in Fig 5~8.

For all test beams, ACI and Eurocode 2 are conservative to predict flexural deflection than experimental deflection of beams. Deflection of reinforced concrete beams in low load stage is different from each reinforced concrete beams, but load acting on test beams is increasing, deflection of beams is similar each others. Eurocode 2 assesses flexural stiffness bigger than Eurocode 2 when crack isn't occurs because of Eurocode 2 assesses modulus of elasticity of concrete bigger than ACI. The higher concrete strength, The more accurate ACI predict deflection of beams than Eurocode 2. For all test beams, Eurocode 2 is more conservative to predict deflection of beams than ACI. Generally, more lower concrete strength, more similarly predict to experimental deflection of beams.

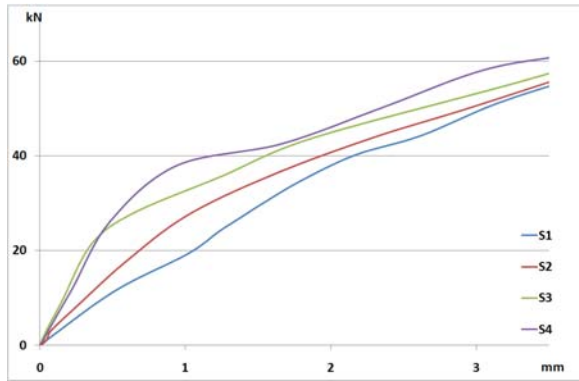


Fig 3. Load-deflection relation of test beams

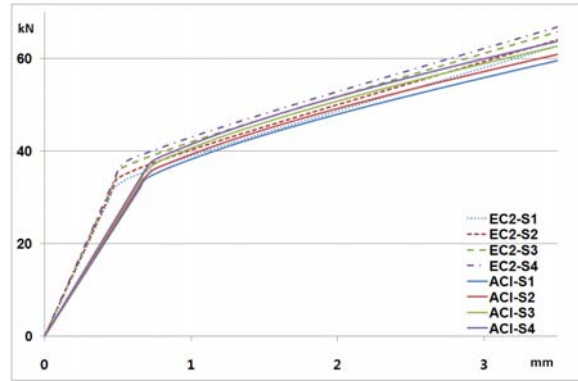


Fig 5. Load-deflection relation comparison with specifications

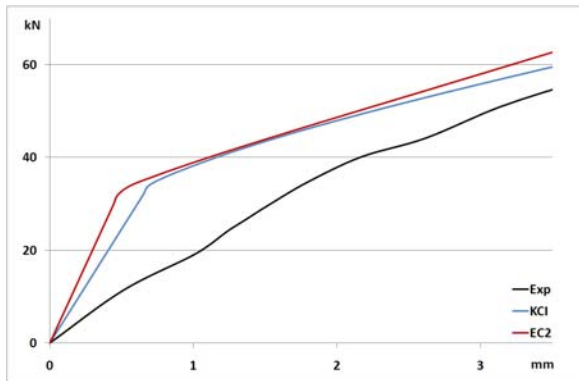


Fig 5. Load-deflection relation of S1 beams

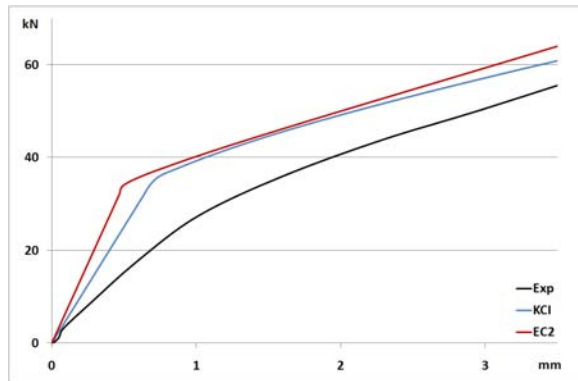


Fig 5. Load-deflection relation of S2 beams

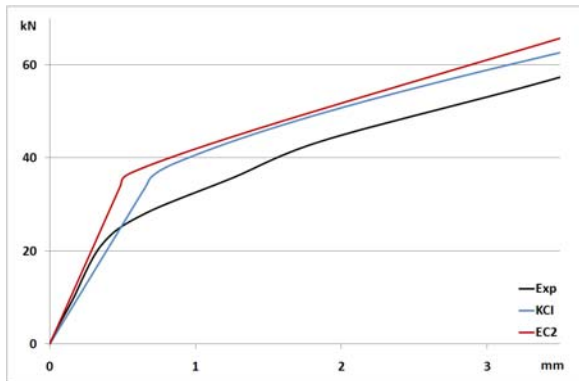


Fig 5. Load-deflection relation of S3 beams

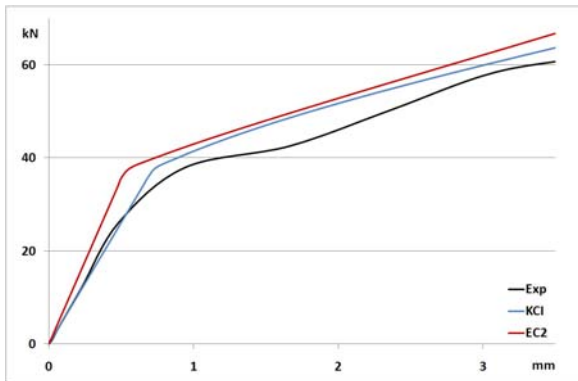


Fig 5. Load-deflection relation of S4 beams

#### 4. CONCLUSIONS

In this study, an experimental research has been carried out for 4 simply supported reinforced beam under 2 point loadings for assess effect of concrete strength on deflection of reinforced concrete. And, it has been concluded as the followings.

① Eurocode 2 assesses flexural stiffness before cracking bigger compared with ACI because of Eurocode 2 assesses modulus of elasticity of concrete bigger than ACI.

② For high-strength reinforced concrete beams, ACI is more accurate than Eurocode 2 to predict actual deflection of reinforced concrete beams.

③ For high-strength reinforced concrete beams, Eurocode 2 is conservative to predict deflection of beams than ACI .

④ Generally, more lower concrete strength, more similarly predict to experimental deflection of beams.

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