An Analysis Method for Dynamical System

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Abstract

This paper provides a method to analyze the dynamical system. It considers the fact of realistic delay in dynamical system analysis for the first time. The method uses timeline and state space to emulate the inhibitive coupling nodes evolving procedure in transmission delayed environment. The resultant finite state machine shows the system predictability and hardware implementation feasibility.

1. Introduction

The dynamical system has some fixed 'rule' which "describes the time dependence of a point's position in its ambient space" [1]. Following the 'rule', this paper uses timeline, state space and finite state machine to analyze a realistic delay considered dynamical system.

2. Methodology

2.1 Model choosing

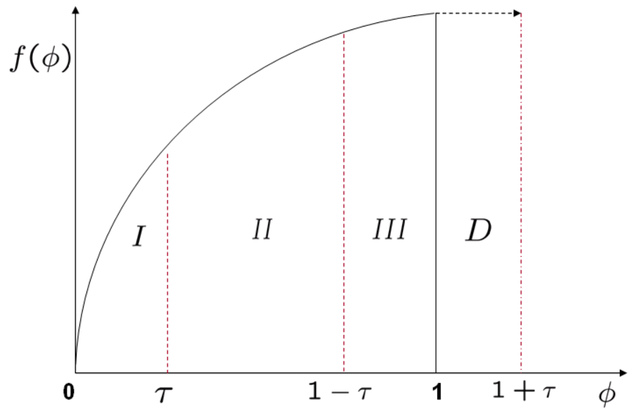
To analyze and predict a dynamical system's convergence and activities, the basic node model choosing is critical, because it affects the success and difficulty of the whole work.

Here Integrate-and-Firing (IF) model [2] is chosen to emulate the interactive nodes. Every node is regarded as an active oscillator and its potential is accumulated by integrating until up to the threshold, then subsequently it fires and emits a pulse signal. The potential falls to zero. And the integrating cycle is repeated. The function $f(\phi) = \frac{1}{b}\ln[(e^{b}-1)\phi+1]$ describes the integrating procedure: b is a set constant $\phi \in [0,1]$ $f(\phi) \in [0,1]$ and period T=1. To guarantee the convergence, the function should be monotonically increasing and concave down ($f' > 0, f'' < 0$). The coupling is triggered by the firing pulses sent out by oscillators. When an oscillator receives the pulse, it will adjust its state $f(\phi)$ with some amount ε (coupling strength). When the state jumps down, the coupling is called inhibitive. No matter how much the state jumps, the state value still should be bounded in [0, 1]. Here use F_{-} to denote the phase after coupling:

$$F_{-}(\phi) = MAX(0, f^{-1}(f(\phi)-\varepsilon))$$

2.2 Environment setting up

Consider the transmission delay everywhere in the realistic network, the original IF curve extends τ length after firing. The reaction does not happen simultaneously with the firing at *0* or *1* point as ideally assumed, but τ time later. In the procedure of mutual coupling and iteration, the τ length before and after firing point become critical districts. Assume the delay time is less than half of the period $\tau < 1/2T$, so $\tau < 1-\tau$. The critical districts divide the period T into three regions $R_1 : (1,\tau)$ $R_2 : (\tau, 1-\tau)$ $R_3 : (1-\tau, 1)$. Fig.1 shows the district division and the delay region.



(Fig.1.)Region division in one oscillating period

2.3 Timing analysis

After modeling the basic node and environment, the first step is to observe the nodes' phase changes and the coupling events at discrete time points. This shows the evolving mechanism of the system.

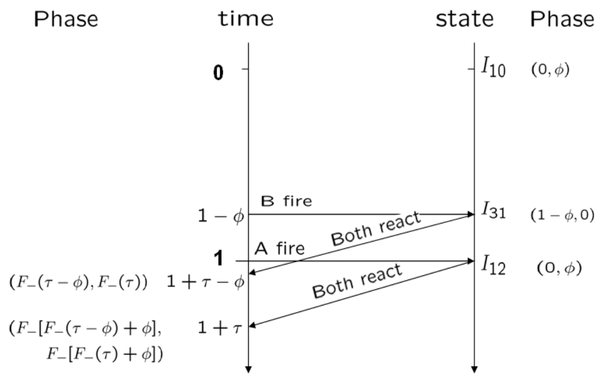
Subsequently two inhibitive coupling nodes converging procedure is used as an example to show how the method steps work.

Given two nodes A, B with their initial phases $\phi_A = 0$ $\phi_B = \phi$; at time $1-\phi$, node B fires and resets to 0; A also moves to phase $1-\phi$. Next: at time 1, A will fire; B's pulse will be reacted at time $1-\phi+\tau$ (Fig.2). Now: the values of ϕ and τ are compared to decide which event occurs first. The subsequently two nodes' action and succeeding system events are dependent on this comparison; although the relationship of ϕ and τ is decided by ϕ_B's initial region.

2.4 State space and state transfer

The system changes are based upon the iteration and coupling of nodes. Determine the state space and state transfer condition allows logical abstraction of the analysis of dynamical system.

Here I_{ij} denotes the system state with i representing which district one node's phase located in when the other one's phase is zero; j means how many firing signals are floating in air waiting for reaction. In that two nodes system, at the beginning, ϕ can be in any regions and there is no floating signal, so the system starting state is I_{i0}. After node B fires but before the pulse is reacted, there is one floating signal, so based on the region that $1-\phi$ is located in, the system state transfers to I_{k1} (k is a function of i). Next as analyzed in 2.3, different initial phases cause different event happening orders and cause the system to transfer to different further system states. The state space of two nodes system is $(I_{i0}, I_{i1}, I_{12} \mid i \in \{1,2,3\})$.



(Fig.2.) Timeline and the state changing by starting from region R_1

2.5 Tabular expression

Finally, the previous two steps are combined in tabular form: specifically all the nodes' phase in corresponding states at discrete time point [3]. The different values of coupling strength will cause multiple cases. And for different cases, checking the change of phase difference $\Delta\phi$ after several cycles can predict the system's converging direction and final ending state.

Table 1 shows the phase and state changing situation of the two nodes system; which starts from state I_{10}, case 1.

<Table 1> two nodes system starting from I_{10}, case 1

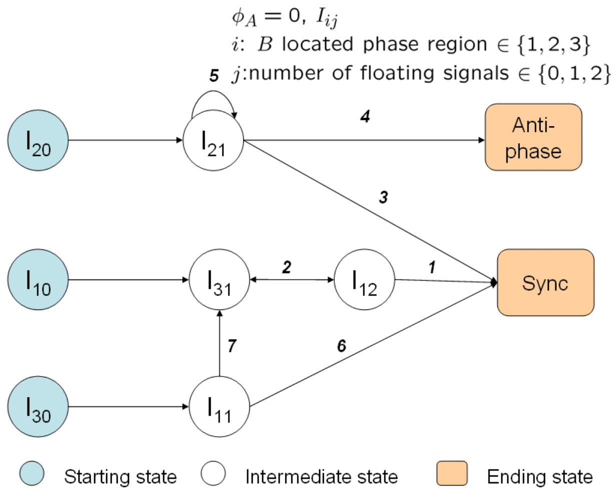
In1.1: $\epsilon < f(\tau-\phi)$, $\phi \in I_{10}$

time	ϕ_A	ϕ_B	signal	*state*
0	0	ϕ	0	I_{10}
$1-\phi$	$1-\phi$	$1\to 0$	1	I_{31}
1	$1\to 0$	ϕ	2	I_{12}
$1+$ $\tau-\phi$	$a_0=\tau-\phi\to$ $a_1=F_-(a_0)>0$	$b_0=\tau\to$ $b_1=F_-(b_0)$	1	
$1+\tau$	$a_1+\phi\to$ $a_2=$ $F_-[a_1+\phi]$	$b_1+\phi\to$ $b_2=$ $F_-[b_1+\phi]$	0	

2.6 Finite state machine

After analyzing all the initial states and cases, the above information can be reinterpreted as a finite state machine.

Fig.3 shows the finite state machine of two nodes with inhibitive coupling. There are three starting states, several intermediate states and two ending stable states: ***Synchronization*** and ***Anti-phase***. The figure shows that the ***Synchrnization*** ending state can be arrived at from every starting state, either directly or after finite round of intermediate state transfer. However, the ***Anti-Phase*** state can only be reached by starting from I_{20}.



(Fig.3.) Finite machine of two nodes inhibitive coupling

3. Conclusion

The method analyzes the dynamical system in delayed environment. The whole analyzing procedure provides a prototype to program and analyze the numerous nodes system on computer in our future work.

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