Development of Water Supply System under Uncertainty

Chung, Gunhui
Research Professor, Center for Disaster Prevention and Science Technology, Korea University, Seoul, Korea

Kim, Tae-Woong
Assistant Professor, Department of Civil and Environmental System Engineering, Hanyang University, Ansan, Korea

Kim, Joong Hoon
Professor, School of Architecture, Civil and Environmental Engineering, Korea University, Seoul, Korea

ABSTRACT: As urbanization is progressed, the network for distributing water in a basin become complex due to the spatial expansion and parameter uncertainties of water supply systems. When a long range water supply plan is determined, the total construction and operation cost has to be evaluated with the system components and parameter uncertainties as many as possible. In this paper, the robust optimization approach of Bertsimas and Sim is applied in a hypothetical system to find a solution which remains feasible under the possible parameter uncertainties having the correlation effect between the uncertain coefficients. The system components to supply, treatment, and transport water are included in the developed water supply system and construction and expansion of the system is allowed for a long-range period. In this approach, the tradeoff between system robustness and total cost of the system is evaluated in terms of the degree of conservatism which can be converted to the probability of constraint violation. As a result, the degree of conservatism increases, the total cost is increased due to the installation of large capacity of treatment and transportation systems. The applied robust optimization technique can be used to determine a long-range water supply plan with the consideration of system failure.

1 INTRODUCTION

A water supply system is a basic infrastructure in civilized world. In a basin, available water sources such as surface, subsurface, and reclaimed water have to distribute to various users sufficiently over the year. Available water sources are often out of balance between basins. To offset the imbalance and supplement the water shortage from out of the basin, the transportation system inter basins has been constructed. Therefore, water supply system typically includes multiple sources inside and outside of the basin, water users for various purposes such as agricultural, domestic, industrial and commercial, treatment facilities of fresh water and wastewater, and reuse system. The main objective to simulate and optimize the water supply system is to evaluate construction and/or operation cost while meeting water demand of all users.

Future population and climate condition are used to predict water demand and inflow to the basin. Therefore, uncertainty from the prediction has to be implemented when developing a water supply system to prevent system failure and estimate proper net benefit. The complexity of a water supply system and the correlated uncertainties, however, make incorporating uncertainty a challenging exercise. A number of stochastic optimization approaches have been applied to water supply system design and operation. Most works have adopted two-stage, multi-stage linear or nonlinear stochastic programming with recourse or scenario analysis approach (Lund and Israel 1995, Wilchfort and Lund 1997, Jenkins and Lund 2000, Elshorbagy et al. 1997).

System failure, however, has to be evaluated when optimizing the total system cost to consider the future uncertainty. Chance-constrained models limit the probability of not being able to meet a constraint to consider system failure. In the other way to consider the system failure, robust optimization is applied to cope with future uncertainty.

In this paper, water supply system is developed to cope with future uncertainty using robust optimization technique proposed by Bertsimas and Sim (2004). This type of robust optimization was introduced by Soyster (1973) of which solution was too conservative and practically unrealistic. The Soyster's model had been extended by Ben-Tal and Nemirovski (1999) and El-Ghaoui and Lebret (1997) to consider the degree of conservatism. These extensions, however, introduced a higher degree of non-linearity which was not included in the original formulation. This nonlinearity causes the difficulties to solve the real world application which is likely nonlinear. Therefore, Bertsimas and Sim (2004) proposed a new method to control the degree of conservatism for the system reliability without increasing the difficulty in solving the original problem. The hypothetical water is developed and optimized using the robust optimization in this study. The tradeoff between size of system components and total cost is evaluated.

2 Robust Optimization Framework

Known parameters are typically applied in deterministic mathematical programming, however, the measurement and prediction errors are often incorporated in the system parameters. Therefore, robust formulation to cope with parameter uncertainties is proposed for the linear programming model by Soyster (1973). The robust solution obtained from the formulation remains feasible under all possible data uncertainties belong to a convex set:

$$\begin{aligned} &\text{maximize } \mathbf{cx} \\ &\text{subject to } \sum_{j=1}^{n} \widetilde{\mathbf{A}}_{.j} x_j \le \mathbf{b}, \quad \forall \widetilde{\mathbf{A}}_{.j} \in K_j, \quad j = 1,\ldots,n, \qquad (1) \\ &\qquad \mathbf{x} \ge \mathbf{0}, \quad \forall j, \end{aligned}$$

where $\widetilde{\mathbf{A}}_{.j}$ denotes the jth column of the constraint matrix, and the column-wise uncertainty is assumed to belong to a known convex set, K_j. The solution obtained from Eq. (1) tends to sacrifice a significant portion of the optimality of a nominal problem to guarantee robustness.

To control the conservatism and retain the linearity of Soyster's model, Bertsimas and Sim (2004) develop a new approach as the following stochastic optimization problem:

$$\begin{aligned} &\text{maximize} \quad \mathbf{cx} \\ &\text{s.t. } \sum_j \bar{a}_{ij} x_j + \max_{\{S_i \cup \{t_i\} | S_i \subseteq J_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in J_i \setminus S_i\}} \left\{ \sum_{j \in S_i} \hat{a}_{ij} y_j + (\Gamma_i - \lfloor \Gamma_i \rfloor) \hat{a}_{it_i} y_t \right\} \le b_i, \ \forall i \qquad (2) \\ &\qquad -y_j \le x_j \le y_j, \quad \forall j \in J_i, \\ &\qquad \mathbf{l} \le \mathbf{x} \le \mathbf{u}, \\ &\qquad \mathbf{y} \ge \mathbf{0}. \end{aligned}$$

At optimality, $y_j = \left| x_j^* \right|$ for all j.

Uncertainty for the ith constraint, J_i represents the set of indices that correspond to uncertain $\widetilde{a}_{ij}$ which are independent, symmetric and bounded random variables having a half of internal as

$\hat{a}_{ij}$. To control the degree of conservatism, Bertsimas and Sim introduce an additional parameter, Γ_i, that can take any real value within the range of $[0, |J_i|]$, in a manner that the most significant coefficients up to the $\lfloor \Gamma_i \rfloor$th order is fully allowed to vary within their uncertainty intervals and the ($\lfloor \Gamma_i \rfloor$+1)th order significant coefficient, a_{it} is bounded by $(\Gamma_i - \lfloor \Gamma_i \rfloor)\hat{a}_{it}$, while the remaining coefficients are fixed at their nominal values.

The proposed formulation assumes that the uncertain parameters are independent. If coefficient $\tilde{a}_{ij}$ in ith constraint has correlation effects from $|K_i|$ number of uncertainty sources as $\bar{g}_{kj}$. The new robust formulation to consider the parameter correlation is:

$$\begin{aligned}
&\text{maximize} \quad \mathbf{cx} \\
&\text{subject to} \\
&\sum_j \bar{a}_{ij} x_j + \max_{\{S_i \cup \{t_i\} | S_i \subseteq K_i, |S_i| = \lfloor \Gamma_i \rfloor, t_i \in K_i \setminus S_i\}} \left\{ \sum_{k \in S_i} \left| \sum_{j \in J_i} g_{kj} x_j \right| + (\Gamma_i - \lfloor \Gamma_i \rfloor) \left| \sum_{j \in J_i} g_{t_i j} x_j \right| \right\} \le b_i, \ \forall i \quad (3) \\
&\mathbf{l} \le \mathbf{x} \le \mathbf{u}.
\end{aligned}$$

The level of conservatism, Γ_i, is a useful tool to investigate system robustness against failure. If system failure can be presented as a probability, it would give better understanding of system safety. It is possible to relate Γ_i to a probability level and show various probability bounds of constraint violation. Let x^* be an optimal solution to Eq. (3), then the probability that the ith constraint is violated is bounded by:

$$\Pr\left(\sum_j \tilde{a}_{ij} x_j^* > b_i \right) \le B(n, \Gamma_i), \qquad (4)$$

with:

$$B(n, \Gamma_i) \le (1-\mu) C(n, \lfloor \nu \rfloor) + \sum_{l = \lfloor \nu \rfloor + 1}^{n} C(n, l), \qquad (5)$$

where $n = |K_i|$, $\nu = \dfrac{\Gamma_i + n}{2}$, $\mu = \nu - \lfloor \nu \rfloor$ and

$$C(n,l) = \begin{cases} \dfrac{1}{2^n}, & \text{if } l = 0 \text{ or } l = n \\ \dfrac{1}{\sqrt{2\pi}} \sqrt{\dfrac{n}{(n-1)l}} \exp\left(n \log\left(\dfrac{n}{2(n-l)} \right) + l \log\left(\dfrac{n-l}{l} \right) \right), & \text{otherwise.} \end{cases} \qquad (6)$$

3 Application

The robust optimization technique by Bertsimas and Sim (2004) is applied in a hypothetical system for a 15-years-planning period with 2 design periods and 10 operation periods. Groundwater storage of the system at year 1 is lower than the defined minimum required storage, therefore, the aquifer storage has to be recharged in the planning period. The system consists of two demand centers, two treatment facilities, one surface and subsurface source, external water supply, and transportation systems. Decision variables are system capacities and flow allocations in the operation period. The treatment facilities and transportation systems have a capacity variable in terms of treatment volume, pipe diameter, canal depth, and pump size. Flowrates have to be determined in each transportation component every operation period. Therefore, total number of decision variable is 176 including 20 binary variables which ensure the feasibility of constraints.

The mixed-integer nonlinear problem was solved using the GAMS/BARON global optimization solver with the relative termination tolerance of 0.05 (Sahinidis and Tawarmalani, 2005). The parameter uncertainties on future water demand, inflow to the system, and correlation relationship from the inflow to water demand are considered in the system. To demonstrate the effect of robustness on the model results, the system is optimized for violation probabilities ranging from 0.1 (the most conservative) to 1.0 (nominal).

As a result, system reliability is insured by enlarging treatment and transportation components. By doing so, the total cost of construction is increased as the degree of conservatism is raised (Fig. 1). There is an inflection point where the enlargement of system's capacities is substantial. The amount of water purchased from outside of basin is also increased as the robustness requirement increases.

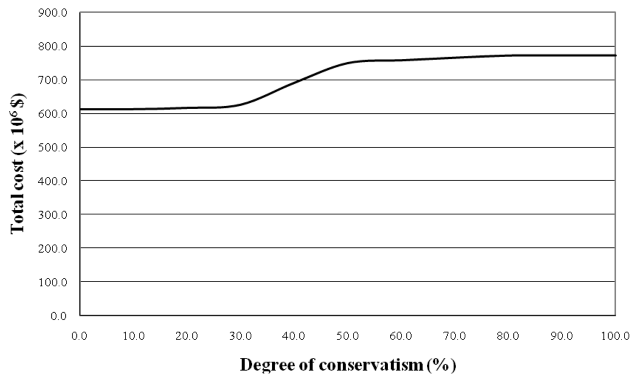

Fig. 1. Total system cost in terms of degree of conservatism of the system

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