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다중의 불완전성을 갖는 반구형 셸의 진동

Vibration of Hemispherical Shell with Multiple Imperfections

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1. Introduction

The shell structures have been studied steadily in aerospace engineering fields. Especially, as a high precision gyroscope, resonating gyros became one of the intriguing subjects. Fox[1] investigated imperfection as an unbalancing factor on a circular ring. Also, Hwang[2] presented experimental results of vibrations on hemi-spherical shell and compared to analytical data. Zhuravlev[3] studied imperfect shell of which density varies with periodic manners thorough circumferential angle. In this study, mathematical model of imperfect hemispherical shell by using point mass elements is presented by using energy relations of inextensional vibration of the shell.

2. Formulations

2.1 Imperfections due to point masses

In this section, mathematical model of effects of point masses on the shell is formulated. The natural frequency of the system is determined by using Rayleigh's energy method for bending mode of vibration.

The maximum kinetic and strain energy for the system can be expressed as

$$K_T = K_0 + K_M \qquad U_T = U_0 \tag{1}$$

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where K and U denote kinetic and strain energy respectively. Also, subscripts such as T, 0 and M represent terms with respect to total system, perfect shell, and point mass, respectively.
Applying Rayleigh's procedure onto Eq. (1), the natural frequency of the system can be deter-mined as[2]

$$\omega_n^2 = \omega_0^2 \frac{1}{(1+\varepsilon_K)} \quad \text{where} \quad \varepsilon_K = \frac{K_{M_{\max}}}{K_{0_{\max}}} \tag{2}$$

where ω_0 and ω_n denote the natural frequency of symmetric shell, and of imperfect shell, respect-tively. The natural frequency of symmetric shell can be determined by using Rayleigh's energy method. Further, assuming that the shell has bending mode, and with free boundary conditions, the kinetic energy of point masses can be expressed as

$$K_{M_{\max}} = \frac{1}{2} D^2 m_i \omega_n^2 \left[\tan^{2n} \frac{\phi_i}{2} \left\{ \begin{matrix} (n+\cos\phi_i)^2 \\ \sin^2(n(\theta_i - \zeta)) + \sin^2\phi_i \end{matrix} \right\} \right] \tag{3}$$

where D, m, a, ρ, and h denote magnitude of local displacement, point mass, radius of the shell and thickness of the shell, respectively. The subscript i denotes the number of point masses. Then, substituting Eq. (3) into Eq. (2) results the natural frequency as Eq. (4) The shift angle of mode orientation can be determined by the knowledge that the natural frequencies will be stationary for ζ, and be expressed as[2] in Eq. (5)

2.2 Single point mass case

In this section, the imperfect shell model with single mass case is investigated. The two roots of shift angle of mode orientation can be determined for single point mass, such as Eq. (6)

H and L denote split of the natural frequen-

$$\omega_n^2 = \omega_0^2 \left[1 + \frac{\frac{1}{2}\sum_i m_i \left[\tan^{2n}\frac{\phi_i}{2} \begin{Bmatrix} (n+\cos\phi_i)^2 \\ \sin^2\left(n\left(\theta_i-\zeta_j\right)\right) \\ +\sin^2\phi_i \end{Bmatrix} \right]}{\frac{1}{2} a\pi\rho h \int_0^{\frac{\pi}{2}} \tan^{2n}\frac{\phi}{2} \begin{Bmatrix} (n+\cos\phi)^2 \\ +2\sin^2\phi \end{Bmatrix} \sin\phi d\phi} \right]^{-1} \quad (4)$$

$(j = H, L)$

$$\frac{\partial \varepsilon_K}{\partial \zeta} = \frac{\partial K_{M_{\max}}}{\partial \zeta} = 0 \quad (5)$$

$$\zeta_{H,L} = \theta_1 + \frac{\pi}{2n},\ \theta_1 \quad (6)$$

cies that the shape of H-mode has its node on the point mass and of L-mode has its anti-node on the point mass. Additionally, normalized frequency can be defined by following relation.

$$\omega_{norm} = \frac{\omega_{H,L} - \omega_0}{\omega_0} = \frac{\Delta\omega_n}{\omega_0} \quad (7)$$

Also, the normalized mass can be defined by

$$m_{norm} = \frac{m_{point}}{m_{shell}} \quad (8)$$

3. Results and discussions

The results for single point mass model are presented in this section. The point mass is located on the 0 deg. of circumferential angle, and 90 deg. of meridian angle. Fig. 1 and Fig. 2 show the normalized natural frequencies for each H- and L-mode. As shown in figures, changing mode number and weight of point mass affect directly to the split of natural frequencies. The split amount for L-mode is relatively larger than for H-mode, and as mode number increases, the split of H-mode decreases while for L-mode increases. This means the split frequencies for H-mode is very close to the natural frequencies of symmetric shell.

4. Conclusions

The mathematical model of imperfect hemi-spherical shell is presented by using Rayleigh' s energy method and using energy relations.

Through this study, the unbalanced vibration of manufactured gyroscope can be expressed analy-tically. The imperfect vibration of the shell can be mathematically modeled by using finite number of point masses, and further study will consider tri-mming mechanism.

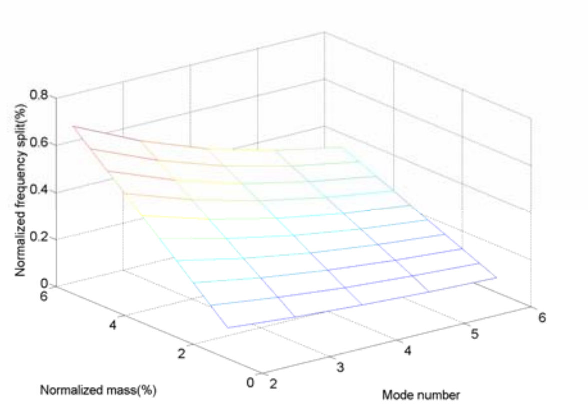

Fig. 1 Normalized frequencies for H-mode

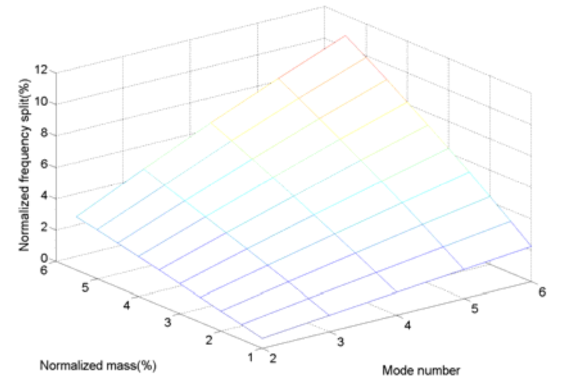

Fig. 2 Normalized frequencies for L-mode

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References

[1] C. H. J. Fox, " A simple theory for the analysis and correction of frequency splitting in slightly imperfect rings, Journal of Sound and Vibration, Vol. 142(2), pp. 227 – 243, 1990.

[2] C. Hwang, " Some experiment on the vibration of a hemi-spherical shell, Journal of Applied Mechanics, Transactions of the ASME, pp.817-824, 1966

[3] V. Ph. Zhuravlev, " Drift of an imperfect hemispherical resonator gyro, Mechanics of Solids, Vol. 39(4), pp. 15 – 18, 2004