FUNDAMENTAL PERFORMANCE OF IMAGE CODING SCHEMES BASED ON MULTIPULSE MODEL

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ABSTRACT

In this paper, we examine the fundamental performance of image coding schemes based on multipulse model. First, we introduce several kinds of pulse search methods (i.e., correlation method, pulse overlap search method and pulse amplitude optimization method) for the model. These pulse search methods are derived from auto-correlation function of impulse responses and cross-correlation function between host signals and impulse responses. Next, we explain the basic procedure of multipulse image coding scheme, which uses the above pulse search methods in order to encode the high frequency component of an original image. Finally, by means of computer simulation for some test images, we examine the PSNR(Peak Signal-to-Noise Ratio) and computational complexity of these methods.

Keywords: Multipulse Model, Correlation Method, Pulse Overlap Search Method, Pulse Amplitude Optimization Method, Cholesky Decomposition, Image Data Compression

1. INTRODUCTION

The multipulse model[1] is a high efficiency speech coding method proposed by B. S. Atal et al. in 1982. Speech coding methods can be classified into two broad categories: The first type of coder is a waveform coder, which attempts to mimic the speech waveform as faithfully as possible. The second type of coder is a vocoder, which synthesizes speech using a parametric model of speech production. The multipulse model is the second type and synthesizes natural-sounding speech at low bit rates by using a parametric model of speech production. It was adopted as the speech coding standard scheme (RPE-LTP) for a digital mobile phone in Europe through various improvements in 1988. The study on applying this model to the time domain of the image signal had been done[2]. However, there was a problem of *saturation effect* of PSNR to the multipulse model when the number of pulses per processing frame increased. This has been thought that the pulse search method is a cause[3].

Because the above issue doesn't have the influence when the compression rate is high (i.e., the number of pulses per processing frame is less than 40), it is not very important. However, it is necessary to evade the issue, because the necessity for increasing the number of pulses per processing frame comes out in case of the application of multipulse image coding model to the high bit rate coding (high-definition image coding) and the image data processing (e.g., feature extraction of person image and the others).

In this paper, we compare the fundamental performance of three kinds of pulse search methods (correlation method by Ozawa et al. [4], pulse overlap search method by Sato [5] and pulse amplitude optimization method by Singhal et al. [3]) for multipulse image coding by means of computer simulation for some test images.

2. MULTIPULSE MODEL

The multipulse model, as shown in Fig. 1, calculates parameters of a synthesizer and a sequence of pulses (or multipulse) from an input signal by means of the multipulse analysis processing. The synthesizer is excited by using the sequence and synthesizes an output signal. Data compression can be done by expressing an input signal with parameters of a synthesizer and a sequence of pulses. Moreover,the input signal is assumed to be generated with the AR(Auto-Regressive) model given by

$$y(n) \quad = \quad \sum_{i=1} \alpha_i y(n \quad i) + e(n), \qquad (1)$$

where $y(n)$ is the synthesizer output, p is AR order, $e(n)$ is the linear prediction error and $\alpha_i(1 \bullet i \bullet p)$ are AR coefficients.

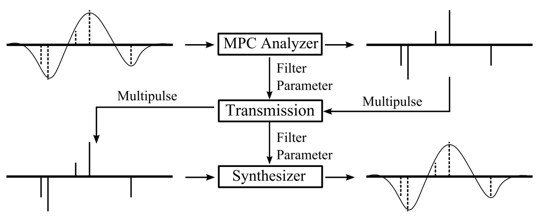

Fig. 1: Overview of multipulse model.

3. OUTLINE OF PULSE SEARCH METHOD

The first pulse search method is A-b-S method (analysis by synthesis) proposed by B. S. Atal et al. in [1]. There is a problem that this method takes considerable time to calculate. Therefore, some efficient pulse search methods had been proposed. In this section, we explain common matters (about the mean square error) in three search methods described in the following section.

First, it is assumed that m pulses exist in the processing frame of N samples. Let $n_1, n_2, n_3, \cdots, n_m$ and $\beta_1, \beta_2, \beta_3, \cdots, \beta_m$ be defined as pulse locations and pulse amplitudes in the frame, respectively. Then the pulse sequence $u(n)$ to excite a synthesizer is called multipulse and given by

$$u(n) \quad = \quad \sum_{k=1}^{m} \beta_k \delta(n \quad n_k). \tag{2}$$

Next, by using the impulse response $h(n)$ of the synthesizer, a synthesized signal $\hat{y}(n)$ is expressed as

$$\hat{y}(n) \quad = \quad \sum_{k=1}^{m} \beta_k h(n \quad n_k). \tag{3}$$

Furthermore, let $w(n)$ be the impulse response of a weighting filter. Then the weighted mean square error E between the original signal $y(n)$ and the synthesized signal $\hat{y}(n)$ is given by

$$E \quad = \quad \sum_{n=0}^{N-1} [y_w(n) \quad \hat{y}_w(n)]^2, \tag{4}$$

$$y_w(n) \quad = \quad y(n) * w(n), \tag{5}$$

$$\hat{y}_w(n) \quad = \quad \hat{y}(n) * w(n), \tag{6}$$

where $*$ stands for convolution operator. From Eqs. (2), (3) and (4), the pulse amplitude $\beta_j (1 \bullet \ j \bullet \ m)$ to minimize E is given by

$$\sum_{k=1}^{m} \beta_k R_{hh}(|n_k \quad n_j|) \quad = \quad R_h \ (n_j), \tag{7}$$

where $R_{hh}(i,j) = R_{hh}(|i \quad j|)$ [4] and

$$R_{hh}(|i \quad j|) \quad = \quad \frac{1}{N} \sum_{n=0}^{N-1} h_w(n) h_w(n \quad |i \quad j|), \tag{8}$$

$$R_h \ (m) \quad = \quad \frac{1}{N} \sum_{n=0}^{N-1} y_w(n) h_w(n \quad m), \tag{9}$$

$$h_w(n) \quad = \quad h(n) * w(n). \tag{10}$$

In general, the above Eq. (7) is called the integral equation of Wiener-Hopf[7]. The difference of three pulse search methods described follows is a difference of how to use Eq. (7).

3.1 Correlation Method

This method is a pulse search method proposed by Ozawa et al. in [4]. From Eq. (7), the equation for calculating the jth pulse amplitude β_j is given by

$$\beta_j \quad = \quad \frac{R_h \ (n_j) \quad \sum_{i=1}^{j-1} \beta_i R_{hh}(|n_i \quad n_j|)}{R_{hh}(0)}. \tag{11}$$

Here, it is assumed that $j \quad 1$ **pulses have already searched**. The jth pulse location n_j is determined by searching the absolute maximum point of β_j in Eq. (11).

In this pulse search method, it is assumed that the each pulse is independent. Therefore, the error is caused in the synthesized signal for closely spaced pulses, because the location and the amplitude of the pulse that has already been determined aren't changed. As a result, the saturation effect of PSNR occurs when the number of pulses per processing frame increases. Therefore, Ozawa et al. proposed some corrective methods in [6]. However, there is no investigation when the number of pulses per processing frame is greater than 50 pulses.

3.1.1 Correlation method using the error evaluation function

Let $R(j,l)$ be the error evaluation function of the correlation method defined by

$$R(j,l) \quad = \quad R(j \quad 1, l) \quad \beta_{j-1} \cdot R_{hh}(|n_{j-1} \quad l|), \tag{12}$$

where $R(1,l) = R_h \ (l)$. Then the jth pulse amplitude β_j and location n_j are given by

$$\beta_j \quad = \quad R(j,l)/R_{hh}(0), \tag{13}$$

$$n_j \quad = \quad l(\neq n_1, n_2, \cdots, n_{j-1}). \tag{14}$$

Moreover, Eq. (13) can be rewritten as follows:

$$R(j+1,l) \quad = \quad R(j,l) \quad \beta_j \cdot R_{hh}(|n_j \quad l|). \tag{15}$$

Thus the pulse amplitude β_j and location n_j can be determined recursively by updating the error evaluation function $R(j,l)$.

3.2 Pulse overlap search

Pulse overlap search method [5] proposed by M. Sato is an improvement of the correlation method. This method permits searching the pulse locations ($n_1 \sim n_k$) that have already been searched in order to correct pulse amplitude. Moreover, it is possible to use the error evaluation function $R(j,l)$.

For example, suppose that the k pulses have already been found and the jth ($j \quad k$) pulse location is n_i ($i \bullet \ k$). Then the pulse amplitude β_i is corrected as follows:

$$\beta_i^{(a+1)} \quad = \quad \beta_i^{(a)} + R(j, n_i)/R_{hh}(0), \tag{16}$$

where a is the number of overlap searches of the pulses. Figure 2 shows the outline of the pulse overlap search method.

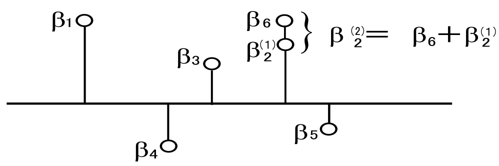

Fig. 2: Outline of pulse overlap search method.

3.3 Pulse amplitude optimization method

This pulse search method was proposed by Singhal et al. [3]. The points different from the above two methods are to rewrite Eq. (7) in the simultaneous Eq. (17) and to optimize all pulse amplitudes of pulse sequence $u(n)$ by using Cholesky decomposition.

$$\boldsymbol{R_{hh}} \boldsymbol{U} = \boldsymbol{R_{hy}}, \tag{17}$$

where $\boldsymbol{R_{hh}} = [R_{hh}(i,j)]$ is the $N \quad N$ matrix, $\boldsymbol{U} = (u(0), u(1), \cdots, u(N \quad 1))^T$ and $\boldsymbol{R_{hy}} = (R_h \ (0), R_h \ (1), \cdots, R_h \ (N \quad 1))^T$. As $\boldsymbol{R_{hh}}$ is a symmetry matrix, there exists a triangular matrix $\boldsymbol{L}$ such that $\boldsymbol{R_{hh}} = \boldsymbol{L}\boldsymbol{L}^t$ and $\boldsymbol{L}$ can be obtained by using Cholesky decomposition of $\boldsymbol{R_{hh}}$. Namely, the element $l(i,j) (i < j)$ of triangular matrix $\boldsymbol{L}$ is given by

$$l(j,j) \quad = \quad \sqrt{R_{hh}(0) \quad \sum_{k=1}^{j-1} l^2(j,k)}, \tag{18}$$

$$l(i,j) \quad = \quad \frac{R_{hh}(|i \quad j|) \quad \sum_{k=1}^{j-1} l(i,k) l(j,k)}{l(j,j)}. \tag{19}$$

Thus Eq. (17) can be written as follows:

$$L^t U = q, \quad (20)$$

$$L q = R_{hy}. \quad (21)$$

The pulse sequence $u(n)$ can be obtained by solving the above simultaneous Eqs. (20) and (21).

4. OVERVIEW OF MULTIPULSE IMAGE CODING

Figure 3 shows the blockdiagram of the multipulse image coding. The multipulse image coding scheme consists of the two parts (i.e., multipulse processing and the generation of the low frequency image). The multipulse processing is applied to the difference signal between the original image and its low frequency image.

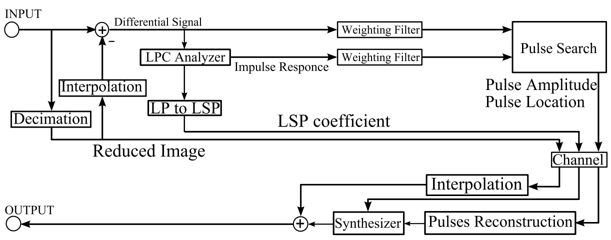

Fig. 3: Blockdiagram of multipulse image coding.

5. COMPUTER SIMULATION

5.1 Conditions for computer simulation

We use the standard image data base SIDBA/"GIRL " and "LENNA"(256 256pixels and 256 levels), and apply the following pulse search methods to them.

pulse amplitude optimization method[3]
pulse overlap search method[5]
correlation method[4]

The parameters of multipulse image coding scheme are shown in Table 1, and hardware conditions are shown in Table 2. We evaluate PSNR and computational complexity.

Table 1: Conditions for computer simulation.

Decimation	Lanczos3 [8]
Interpolation	Lanczos4 [8]
Reduction image size	32 32 pixels
The number of pulses	$m = 16$ 128[per line]
Weighting Coefficient	γ = 0.5 0.8
Synthesizer	LSP*synthesis filter[9]
Quantization	8bits

*LSP stands for Line Spectrum Pairs.

Table 2: Hardware conditions.

CPU	UltraSPARC III+ 900MHz
Memory	5120MB

5.2 Result and consideration

As the result of computer simulation, the PSNR versus the number of pulses is shown in Figs. 4 and 5. The reproduction image at 32 pulses of "GIRL" and "LENNA", and error image are shown in Fig. 6 as one example. The pulse location images for "LENNA" are shown in Fig. 7.

Moreover, Fig. 8 shows the relation between the number of pulses and the computational complexity.

In Fig. 4 (a) and Figs. 5 (a) and (b), the PSNR of pulse amplitude optimization method is about 1.0 dB higher than that of pulse overlap search method when $m = 100$ pulses. However, in Fig. 4 (b) for $\gamma = 0.8$, we can observe the saturation effect of PSNR in the pulse amplitude optimization method when $m \geq 80$ pulses, because the mutual interference among the adjoining pulses increases as the value of γ approaches 1.

Moreover, from Fig. 8, it is found that the calculation time of pulse amplitude optimization method is much more than that of other methods when the number of pulses increases, because the calculation order of other two methods is $O(N \quad m)$, while that of pulse amplitude optimization method is $O(N^2 \quad m)$ (where N is the length of processing frame and m is the number of pulses).

6. CONCLUSION

In this paper, we have examined the fundamental performance of image coding schemes based on multipulse model. From these results, it is found that the pulse amplitude optimization method takes considerable time to calculate, and that the method doesn't necessarily achieve the highest PSNR at the high pulse rate because of the saturation effect of PSNR, while there is a good balance between computational time and PSNR in the pulse overlap search method. Therefore, it is concluded that the pulse overlap search method is more suitable for images than the pulse amplitude optimization method.

Future works are to improve the performance of PSNR of pulse overlap search method and the quality of reproduction image by multipulse image coding.

7. REFERENCES

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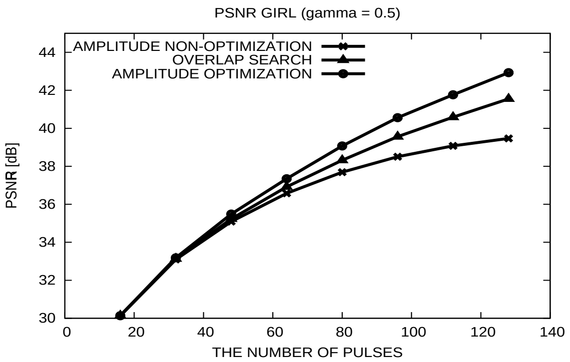
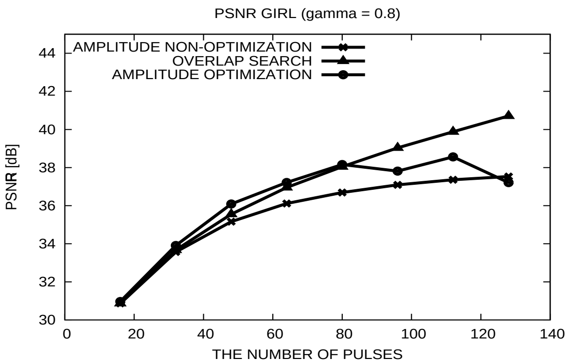
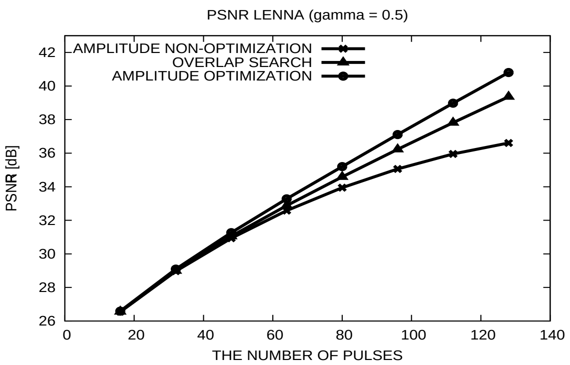


(a) $\gamma = 0.5$.

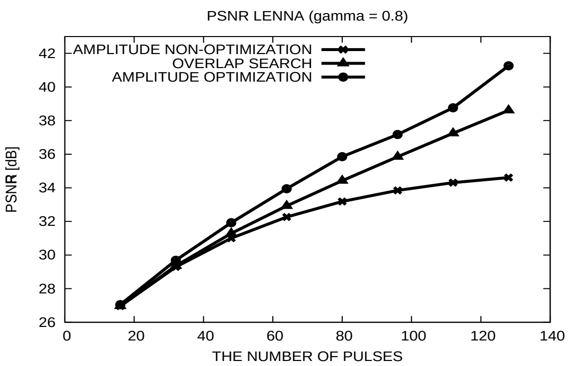


(b) $\gamma = 0.8$.

Fig. 4: PSNR versus the number of pulses (GIRL).



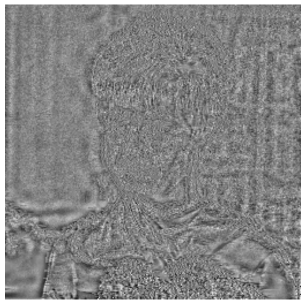
(a) $\gamma = 0.5$.



(b) $\gamma = 0.8$.

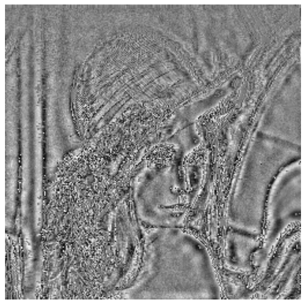
Fig. 5: PSNR versus the number of pulses (LENNA).

(a) The reproduced image (PSNR= 33.91[dB]).



(b) The error image (5).

(c) The reproduced image (PSNR= 29.60[dB]).



(d) The error image (5).

Fig. 6: The reproduced images and the error images for SIDBA/“GIRL”,“LENNA”($m = 32$).

(a) Plus pulses.

(b) Minus pulses.

Fig. 7: The pulse location images ($m = 32$).

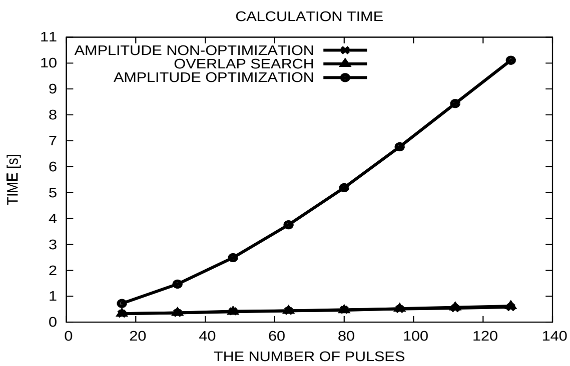


Fig. 8: Calculation time versus the number of pulses.

APPENDIX

The algorithm for pulse overlap search method follows.

$\langle S1 \rangle$ For 0 • $i < N$, compute

$$
\begin{aligned}
R(i) &:= \frac{1}{N} \sum_{n=i}^{N-1} y(n)h(n \quad i), \\
R_{hh}(i) &:= \frac{1}{N} \sum_{n=i}^{N-1} h(n)h(n \quad i).
\end{aligned}
$$

$\langle S2 \rangle$ Set

$$
\begin{aligned}
\texttt{PLflug}(i) &:= 0(1 \bullet \ i < N), \\
\texttt{Rmax} &:= 0, \\
n_1 &:= 0.
\end{aligned}
$$

$\langle S3 \rangle$ Choose j such that $\texttt{Rmax} := |R(j)|$ is maximum.

$\langle S4 \rangle$ Set

$$
\begin{aligned}
\beta_1 &:= \texttt{Rmax}/R_{hh}(0), \\
\texttt{PAtemp} &:= \texttt{Rmax}/R_{hh}(0), \\
n_1 &:= j, \\
\texttt{PLtemp} &:= j, \\
\texttt{PLflag}(\texttt{PLtemp}) &:= 1, \\
i &:= 2.
\end{aligned}
$$

$\langle S5 \rangle$ $\texttt{Rmax} := 0$, for 0 • $j < N$, compute

$$
R(j) := R(j) \quad \texttt{PAtemp} \cdot R_{hh}(|j \quad \texttt{PLtemp}|).
$$

$\langle S6 \rangle$ Choose j such that $\texttt{Rmax} := |R(j)|$ is maximum and $\texttt{PLflug(j)}$ • 10.

$\langle S7 \rangle$ Set

$$
\begin{aligned}
\texttt{PAtemp} &:= \texttt{Rmax}/R_{hh}(0), \\
\texttt{PLtemp} &:= j.
\end{aligned}
$$

$\langle S8 \rangle$ Determine pulse amplitude,
if $\texttt{PLflug}(\texttt{PLtemp}) = 0$ then

$$
\begin{aligned}
\beta_i &:= \texttt{Rmax}/R_{hh}(0), \\
\texttt{PLflag}(\texttt{PLtemp}) &+= 1.
\end{aligned}
$$

else choose t such that $\texttt{PLtemp} = n_t$,

$$
\begin{aligned}
\beta_t &:= \beta_t + \texttt{PAtemp}, \\
\texttt{PLflag}(\texttt{PLtemp}) &+= 1, \\
i &:= i \quad 1.
\end{aligned}
$$

$\langle S9 \rangle$ $i := i + 1$, repeat steps $\langle S5 \rangle$-$\langle S8 \rangle$ until $i = m$.