A STUDY OF SUBDIVISION METHOD TO THREE AND FIVE SIDED FACES BASED ON REGULAR POLYGON

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ABSTRACT

3-D CAD (Computer Aided Design) system is an indispensable tool for manufacturing. A lot of engineers have studied for the methods to generate a curved surface on an N-sided shape, which is the basic technology of 3-D CAD systems. This surface generation, however, has three problems on the case of long and narrow shapes: the resultant surface is distorted, the surface is not continuous to adjacent surfaces, or additional user inputs are required to generate the surface. Conventional methods have not yet solved these problems at the same time.

In this paper, we propose the method to generate internal curves that divide a long and narrow shape into regular N-sided sections so as to divide the shape into an N-sided section and four-sided ones. Our method controls the shape of internal curves by dividing an N-sided long and narrow shape into an N-sided section and four-sided ones, and solves distortion of the generated curved surface. In addition, each of the generated sections is interpolated with G^1-continuous surfaces. This process does not require any user's further input. Therefore, the three problems mentioned above will be solved at the same time.

Keywords: Subdivision, Interpolation, G^1-continuous, N-Sided Face

1. INTRODUCTION

3-D CAD (Computer Aided Design) system is an indispensable tool for manufacturing. For modeling a 3-D shape using a CAD system, there are some methods to express the outlines of the shape with curves and a region enclosed with the curves with a free-form surface. The methods are classified roughly into the following two types:

(a) Method to generate a four-sided trimmed surface including a closed region with boundary curves.

(b) Method to generate a free-form surface whose boundary curves correspond to those of a closed region.

Method (a) is called " N-side filling "[1]. An N-sided filling expresses a closed region with a trimmed surface, although the continuity between the generated trimmed surface and its adjacent curved surfaces is not maintained. Moreover, if a trimmed surface is generated once, later modification operations may be restricted at high ratio.

Method (b) is called " surface interpolation "[2]. A surface interpolation maintains G^1-continuity[3] between the generated surface and its adjacent surfaces, although an expected surface shape may not be obtained depending on the shape of the boundary curves of the closed region. Considering the methods (a) and (b) from the viewpoint of the shape modification, it is preferable to use method (b) for N-sided regions in a lot of cases because it is easy to modify surface shapes. If, however, method (b) is applied to a closed region whose boundary curve lengths are extremely different, the generated surface will be greatly distorted.

Tokuyama solved the problem of method (b) by proposing the method to apply a B-Spline surface to a long and narrow three-sided area and an area without angles[1]. In Tokuyama's method, however, there is a problem of the continuity with adjacent surfaces and shape modification because trimmed surfaces are generated.

Muraki et al. enhanced surface interpolation, so that a tension parameter was added to the vector between the control points of a trimmed surface[4]. As the tension parameter controls the size of a tangent vector around the boundary, the surface shape and the continuity with adjacent surfaces are maintained. For this method, however, there is a problem that the user must set the tension parameter values to decide the location of the center point and internal curves and that it is difficult to decide the tension parameter values intuitively.

In this paper, we propose the method to control the shape of internal curves by dividing a long and narrow area into an N-sided section and four-sided ones. In our method, by dividing a long and narrow section with curves, curved surfaces can be generated without user's manual operation. Note that a long and narrow five-sided area mentioned in this paper is the shape generated by fillet operation as shown in Figure 1.

2. RELATED WORKS

2.1 Surface interpolation algorithm for N-sided area

Surface interpolation described in [2] is based on Catmull-Clark subdivision method[5], in which the center point and internal curves are generated in an N-sided area so that N pieces of four-sided faces are generated and interpolated. This surface interpolation may generate distorted surfaces

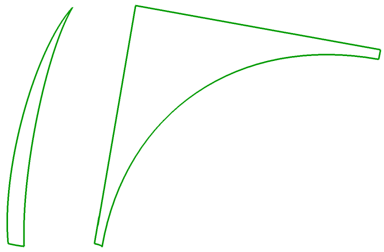

Fig. 1: Examples of long and narrow three-sided and five-sided areas

or the generated surfaces may interfere each other, since this method depends on the shape of the boundary curves of the closed regions.

For example, consider a long and narrow three-sided section and a five-sided one whose boundary curve lengths differ largely (Figure 2). Figure 2 shows the shapes of Figure 1 interpolated with the method of [2] and cross sections cut with planes. Seeing the cross sections, the surfaces are distorted. The cause of the distortion is the vector size between the control points. When the vector sizes between the control points of the boundary curves connected to the start and end points are extremely different, the generated surface will be distorted. The common center point, an end point of internal curves, lies outside the area because of a similar reason, which may result in generation of a torsional internal curve. Since surface interpolation depends on boundary curves of a source shape, the resultant surface shape will be distorted if the source shape is long and narrow.

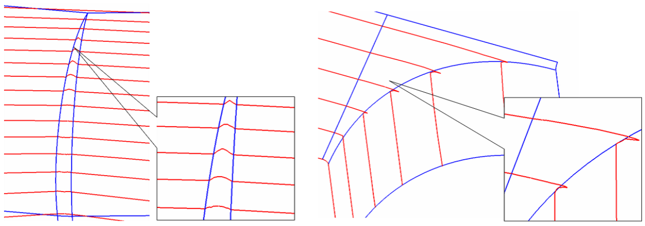

Fig. 2: Surface interpolation method

The N-side filling method[1] proposed by Tokuyama solves the problem of applying a curved surface to a long and narrow shape. This method applies a bicubic B-Spline surface to a convex N-sided area enclosed with free-form curves. When a surface is applied, the bounding box of a boundary curve is generated first, and a line segment is generated from a point and a crossing vector on the boundary curve, then a sequence of points are generated from the intersection between the line segment and the bounding box. Each of the generated sequence of points is interpolated with a B-Spline curve, and the internal control points of the B-Spline surface are obtained by using points with the surface fitting method[7]. As shown in Figure 3, this method can apply a non-distortional surface to a long and narrow shape, although the generated surface is discontinuous with the adjacent surfaces. In Figure 3, the cross sections are discontinuous on the boundary curve.

Muraki et al. enhanced the surface interpolation to solve the problems of shape distortion and discontinuity with adjacent surfaces, and introduced the tension parameter[4]. This new method controls the internal curve shape by adding the tension parameter to the vector between the control points connected to the boundary curve, and also controls distortion of curved surfaces. As Figure 4 shows, a smooth surface can be generated as the continuity with the adjacent surfaces are maintained. Despite this merits, there are problems that a user must set the tension parameter by manual operation with try and error and that it is difficult to decide the tension parameter value intuitively.

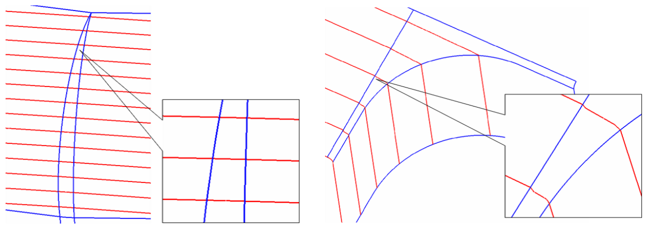

Fig. 3: N-side filling method

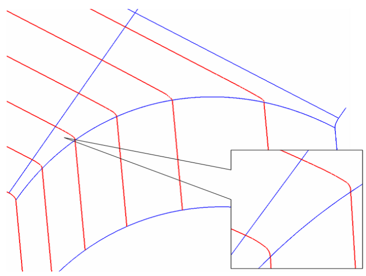

Fig. 4: Tension parameter method

2.2 Problems

Existing surface interpolation methods for N-sided shapes are understood that they have merits and demerits for a long and narrow shape for the reasons described in section 2.1. Table 1 shows the comparison of surface generation methods about the three issues: resultant surface shape, continuity with adjacent surfaces and automation of operation. When a long and narrow shape is interpolated, a curved surface is generated automatically to be G^1-continuous with adjacent surfaces, although the surface is distorted. The N-side filling method automatically generates a curved surface in a long and narrow area although the generated surface is discontinuous with the adjacent surfaces. The method with the tension parameter is necessary to set the parameter value by manual operation although it can generate a

G^1-continuous curved surface with adjacent surfaces. It is difficult for existing methods to solve the three issues at the same time.

Table 1: Comparison of surface generation method

	Shape	Continuity	Automation
Surface interpolation	N	Y	Y
N-side filling	Y	N	Y
Tension parameter	Y	Y	N

3. PROPOSED METHOD

In this section, we propose a new method to solve the problems of existing methods mentioned in section 2.

The new method generates internal curves in a long and narrow area to divide it into a regular N-sided polygon and four-sided areas, and interpolates each area with curved surfaces. In this paper, an internal curve that divides an area into an N-sided section and four-sided ones is called a “ division curve ”and both ends point of the division curve are called “ division points ”. Our method controls the center point location by narrowing a long and narrow N-sided area into a regular N-sided area. Since this method generates curved surfaces by surface interpolation with G^1-continuity, not by using a trimmed surface, the surface shape can be modified as G^1-continuity with adjacent surfaces. Moreover, this method does not task users for additional operations such as manual inputs to set tension parameters.

The new method can coexist with the original interpolation method. A curved surface generated with our method is more complex than the original method because two or more T-nodes[2] appear on a boundary curve. Because of this, it is complex to maintain continuity with adjacent surfaces. On this viewpoint, it is necessary to judge the case to apply our method. We describe first the conditions under which our method is applied, that is, we judge whether it is possible to express surfaces with the existing surface interpolation method. Next, when the new method is applied, the method to decide the suitable division curve and parameters is described. Then, the method to generate a division curve and a curved surface with T-node is described.

The outline of the area division method is as follows:

1. First, an original interpolation method[2] or the new one is decided. The new method is applied to a long and narrow shape.

2. The division edges are decided and the parameter to obtain a regular N-sided polygon is calculated.

3. The division curves are generated, and the long and narrow N-sided area is divided into four-sided areas and an N-sided one.

4. The curved surfaces are interpolated in each of the areas.

3.1 Applicable condition of proposed method

We describe first the condition under which proposed method is applied. The condition is decided according to the location of the center point, in either case of a long and narrow three-sided area or of a five-sided one.

When the original method is applied to a long and narrow three-side area, the center point $\mathbf{c}_1$ is generated on a rising spot as shown in Figure 5, so that the generated curved surface also rises. So, center point $\mathbf{a}(i)$ is generated with the original surface interpolation method as shown in Figure 6, and the center point is projected to edges A and B, which are adjacent to edge C whose length is the shortest, so as to obtain projection points $\mathbf{b}(i)$ and $\mathbf{c}(i)$. By connecting projection points $\mathbf{b}(i)$ and $\mathbf{c}(i)$, the division curve is generated. The method to generate the division curve is described in section 3.3. Next, center point $\mathbf{a}(i)$ is projected to the division curve, and projection point $\mathbf{a}\ (i)$ is generated. If the length of the straight line between $\mathbf{a}(i)$ and $\mathbf{a}\ (i)$ is longer than the threshold, in other words, if the center point is too distant from the division curve, the new method is applied. In our experiment, the threshold is set to $1/50$ of the length of the shortest edge. The smaller value the threshold is set, the narrower the resultant N-sided area will be.

If the existing method is applied to a long and narrow five-sided area, the center point $\mathbf{c}_2$ is generated outside the area as shown in Figure 5, so that a curved surface is also generated outside the area. So, the proposed method is applied to such a case, when the center point lies outside the area. Otherwise, the existing method[2] is applied.

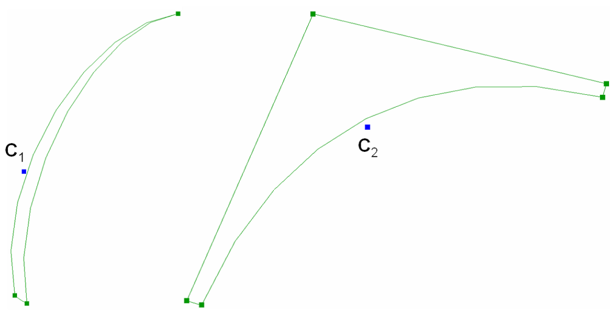

Fig. 5: Center point generated by existing method

3.2 Deciding divided edges and parameters

This section describes the method to decide the suitable edge and parameter for generating the division point. To control the center point location of a long and narrow shape, an edge is divided so as to shrink a long and narrow three-sided area or to generate a regular N-sided polygon from a long and narrow five-sided area. In the case of a long and narrow three-sided area, the edge and the parameter are obtained in the following procedure:

1. As Figure 6 shows, decide the shortest edge C and edges A and B adjacent to C as the edges on which the division point will be generated in the next step.

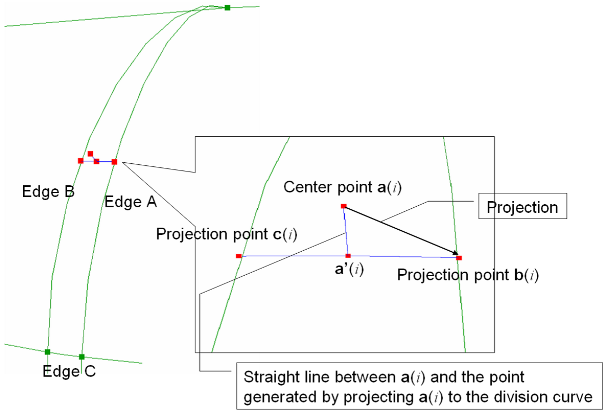

Fig. 6: Obtaining distance to tangent plane

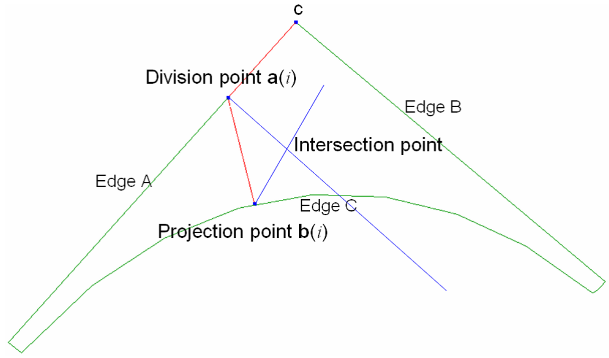

Fig. 7: Deciding parameter

2. Divide edge A equally into n sections by using the parameter i(i=0,1/n,2/n,...,(n-1)/n,1), and define the point corresponding to parameter i as $\mathbf{a}(i)$. Divide edge B in the same manner.

3. Generate a division curve as the division point is assumed to be $\mathbf{a}(i)$ and obtain a three-sided area.

4. To the obtained three-sided area, apply the judging method described in section 3.1. If the value is smaller than the threshold, decide it as division parameter i.

In the case of a long and narrow five-sided area, the edge and the parameter are obtained in the following procedure:

1. As Figure 7 shows, decide edges A and B of the second next to the longest edge C as the edges on which the division point will be generated.

2. Divide edges A and B equally into n sections and define the generated division point as $\mathbf{a}(i)$.

3. Project division point $\mathbf{a}(i)$ to the longest edge C to obtain projection point $\mathbf{b}(i)$.

4. Decide the parameter i so that the difference between the straight line from $\mathbf{a}(i)$ to C and the straight line from $\mathbf{a}(i)$ to $\mathbf{b}(i)$ will be smallest.

5. Decide parameter i for edge B in the same manner.

3.3 Generating division curves

As the division point is decided, this section describes the method to generate the division curve.

In the case of a long and narrow three-sided shape, the tangent vector around the boundary at the division points of edge A and B obtained in section 3.2 is extracted, and the straight lines are generated from the division points and the tangent vector around the boundary. The intersection point between the two straight lines is calculated, and the obtained intersection point is set to the control point of the division curve. Then, elevate the degree of the generated Bezier curve from quadratic to cubic and set the cubic Bezier curve to the division curve.

In the case of a long and narrow five-sided area shown in Figure 7, the tangent vector around the boundary at the division points $\mathbf{a}(i)$ and projection point $\mathbf{b}(i)$ are extracted, and the straight lines are generated from the division point of edge A or projection point of edge C and the tangent vector around boundary. As Figure 7 shows, the intersection point between the two straight lines is calculated, and the obtained intersection point is set to the control point of the division curve. In the same manner as the case of three-sided area, elevate the degree of the generated Bezier curve from quadratic to cubic and set the cubic Bezier curve to the division curve.

3.4 Connection method and curved surface generation

This section describes connection with adjacent surfaces and curved surface generation. Dividing an edge by using the new method generates a T-node, which is a node generated on T-shaped edges around the division point. In Figure 8, points encircled are T-nodes. As G^1-continuity with adjacent surfaces is considered, a base patch is divided. A base patch is a virtual patch generated on a boundary curve to connect two curved surfaces. Taking the long and narrow five-sided face of Figure 8 as an example, in order to interpolate the generated four-sided and five-sided areas, base patches $\mathbf{a_0}$ and $\mathbf{a_2}$ are divided according to the parameter value to obtain $\mathbf{a_1}$. For a four-sided area, divide $\mathbf{a_0}$ and $\mathbf{a_1}$ to generate base patch $\mathbf{a_3}$. For a five-sided area, divide $\mathbf{a_1}$ and $\mathbf{a_2}$ at T-nodes to obtain $\mathbf{a_4}$. Then, interpolate a surface in each four-sided area.

4. EXPERIMENT RESULT AND CONSIDERATION

The new method is applied to three-sided and five-sided faces A, B and C and the results are shown in Figures 9, 10, 11 and 12 where surfaces are shaded. In the case of a three-sided face shown in Figure 9, since the area is narrowed, the shape is not raised up. In the case of a five-sided face shown in Figures 10, 11, 12, since a regular pentagon is generated inside the face, the location of the center point is controlled and curved surfaces are generated inside the areas.

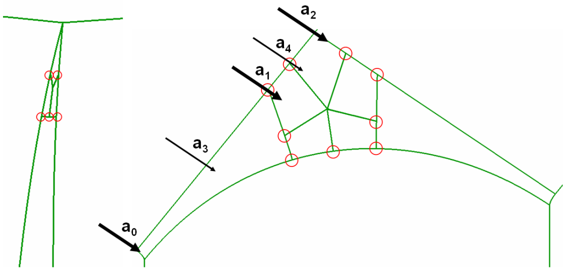

Fig. 8: Division curve

Figures 13, 14, 15, and 16 show the cross sections generated by cutting the surfaces with planes. As the figures show, smooth surfaces are generated. In addition, surfaces are connected smoothly without breaking the shape.

Figures 17, 18, 19, and 20 show the results of generating the normal vectors on the boundary curves. Since the normal vectors on each boundary curve sharing two surfaces coincide, the surfaces are G^1-continuous.

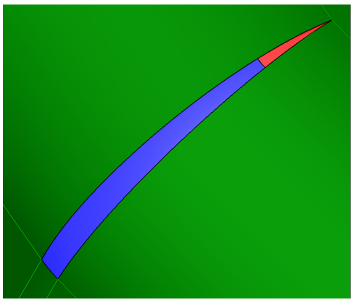

Fig. 9: Shaded image of three-sided face

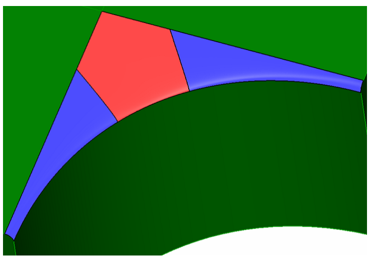

Fig. 10: Shaded image of five-sided face A

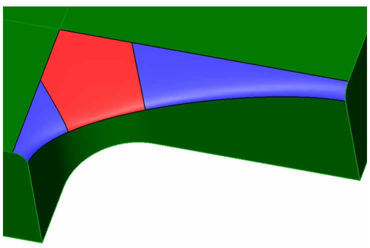

Fig. 11: Shaded image of five-sided face B

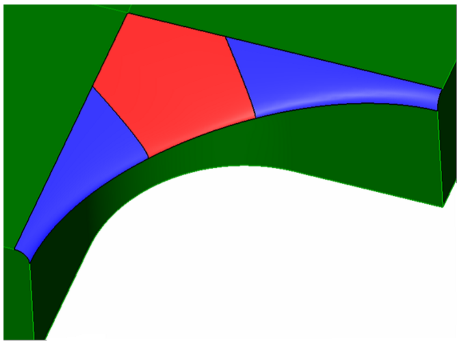

Fig. 12: Shaded image of five-sided face C

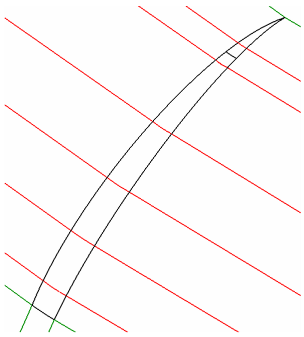

Fig. 13: Cross-section image of three-sided face

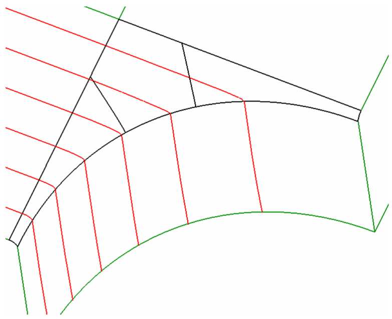

Fig. 14: Cross-section image of five-sided face A

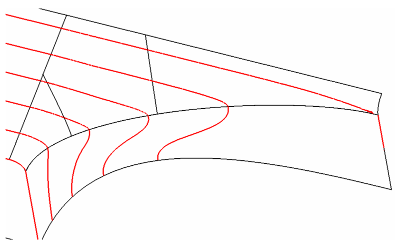

Fig. 15: Cross-section image of five-sided face B

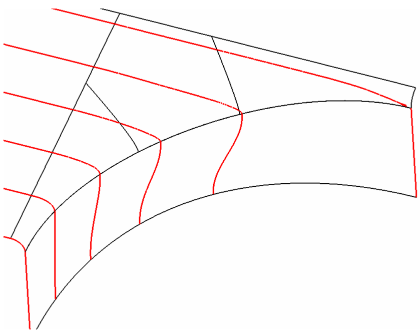

Fig. 16: Cross-section image of five-sided face C

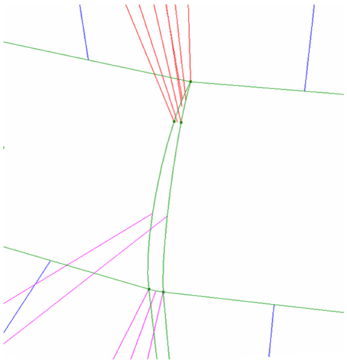

Fig. 17: Normal vectors of three-sided face

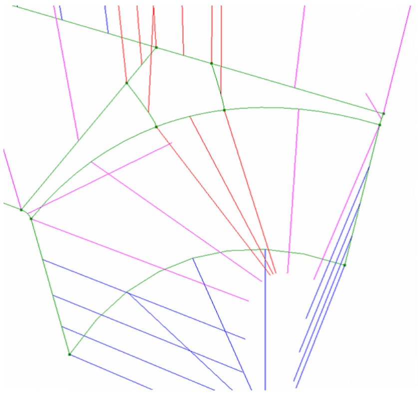

Fig. 18: Normal vectors of five-sided face A

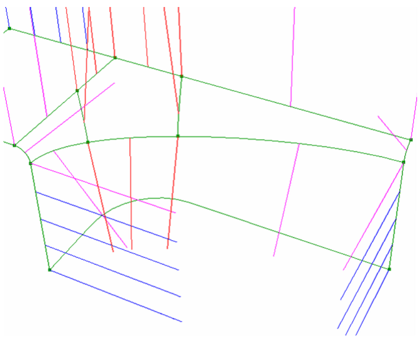

Fig. 19: Normal vectors of five-sided face B

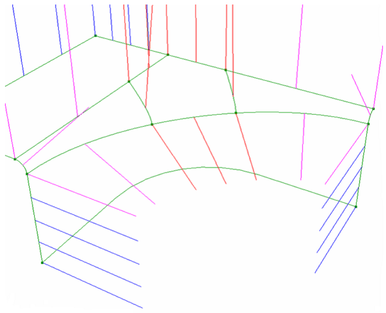

Fig. 20: Normal vectors of five-sided face C

5. CONCLUSION AND FUTURE ISSUES

In this paper, we examined the interpolation method according to area division based on a regular N-sided shape in a long and narrow area. As a long and narrow N-sided area is divided into an N-sided area and four-sided ones, the location of the center point is controlled and distortion of the generated curved surface is solved. As a base patch is divided according to T-nodes, G^1-continuous curved surfaces are generated. G^1-continuity of surfaces is proved as the normal vectors on the boundary curves sharing two surfaces are shown. Moreover, the new method does not require user's manual operation. With these reasons, the new method proposed in this paper solves three issues: "curved surface shape", "continuity" and "automation" at the same time.

Application to N-sided areas other than three- or five-sided ones is given as future works.

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