STREAM PATTERN GENERATION USING PDE BY CONSIDERING VISCOSITY

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ABSTRACT

This paper reports a non-photorealistic rendering method for creating stream pattern from an input image. Our method extracts potential stream pattern in the given image. The proposed approach uses a shock filter based on a partial difference equation(PDE) which is implemented by applying a selective dilation and erosion processes. However, unlike the traditional first order solution to the PDE, we employ a second order scheme and compensate for the undesired diffusive effects caused by a viscosity form. The selection of dilation or erosion for a pixel is based on an edge detector computed from a structure tensor. By adding noises on to the input image, our method also can generate stream pattern even if there is less texture in some area. The experimental results show that the stream pattern is extracted very well.

1. INTRODUCTION

In 1990, Haeberli reported a famous, pioneer work about non-photorealistic rendering(NPR)[1]. Unlike photorealistic rendering in traditional computer graphics, NPR emphasizes artistic effects to express subjective mood. There exists many NPR techniques to produce these kinds of artistic effects such as watercolor[2], oil-painting[3], colored pencil drawing[4] etc. There are so many papers related to NPR techniques to be listed here. Here we recommend two books[5, 6] for interesting reader about the introduction to non-photorealistic rendering. A survey paper[7] can be found as a good tutorial material.

Stream pattern has many applications[8, 9]. Shock filter is a useful tools to extract stream pattern included in an image. Originally, shock filter is often used for image edge enhancement[10, 11, 12]. The filter uses non linear time dependent partial differential equation and its discretization to compute dilation and erosion of an input image. The dilation or erosion process at a pixel is selected according to its sign of the Laplacian at this pixel. The shock filter deblurs image and create shocks at inflection points. However, a main problem of Osher-Rudin scheme[10] is the sensitivity to noise. Although an improved approach[12] is proposed, the algorithm uses complex diffusion process and is a little slowly.

In this paper we propose a method of generating stream pattern using shock filter. Dilation and erosion processes are adapted, which are less blurring at discontinuity of shock. The dilation and erosion contains a viscosity term which is responsible for the undesired diffusive effects which are compensated by a subsequent stabilized inverse diffusion process[13]. Since we need to add noises to the image and also avoid these noises to be dilated, the input image is smoothed by using Gaussian filter to compute the Laplacian. Moreover, to acquire reliable orientation of stream, a local structure tensor is computed. The eigenvectors describes the local, perhaps, potential orientation[14, 15] which is useful for anisotropic filtering. In region with less texture on an image, since there does not exist edges it is difficult to extract stream pattern. Even if there has, the edges may be very weak. To solve this problem, we add Gaussian noises to the input image that help us generating stream pattern.

2. SHOCK FILTER

Shock filter has been introduced by Osher and Rudin in 1990 for image enhancement[10]. The filter is based on a partial differential equation. Solving the PDE equals to use a dilation process near a maximum and an erosion process around a minimum.

Given an image $f(x, y)$, a series of filtered images $f(x, y, t)$ may be created by evolutions under the following process

$$f_t = -sign(\Delta f)\|\nabla f\|_2, \tag{1}$$

where the initial condition when $t = 0$, $f(x, y, 0) = f(x, y)$ begins the evolution process. $f_t = \partial f(x, y, t)/\partial t$ is the first derivative to the time of a pixel at (x, y) in image. The Laplacian is denoted as $\Delta f = f_{xx} + f_{yy}$, while $sign(\Delta f)$ denotes the sign of Δf. And $\nabla f = (f_x, f_y)^\top$ means the spatial gradient of $f(x, y, t)$ at pixel (x, y) at the time t, while $\|\cdot\|$ denotes the norm of a vector. It is well known that the Laplacian is negative near a local maximum. In this case the evolution is a dilation process. While the Laplacian is positive near a local minimum, the evolution becomes a erosion process. Hence, the sign of Laplacian determines whether dilation or erosion process is proceeded at a pixel.

For an image $f(x, y)$, each pixel belongs to an influence zone of a minimum or a minimum. The iterations of dilation and erosion will make maximum zone and minimum zone meet at the inflection curve and form a shock. The image will become piece constant on both sides of the shock.

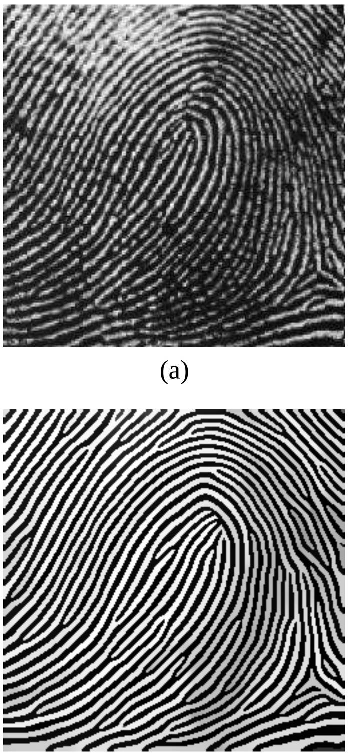

Fig. 1: Generation of stream pattern: (a) Finger print image; (b) Generated finger stream pattern by applying dilation and erosion evolutions.

3. DILATION AND EROSION SCHEME

The evolution of an image is in fact to solve the partial differential equation of shock filter which is solved mainly by using prominent numerical method with upwind scheme proposed by Osher[10] and Rouy-Tourin[16]. However these are mainly first order numerical schemes, both suffer from undesirable blurring effect, also called numerical viscosity. This problem is overcome by the flux corrected transport(FCT) technique by Breub-Weikert[13].

We use the FCT scheme to implement the above mentioned shock filter, because of its excellent features. For simplicity, here we briefly introduce the 1-D FCT scheme. Only 1-D dilation is shown here, the corresponding scheme for erosion can be established analogously.

We denote a 1-D signal as $f(x)$, which will be evoluted as $f(x,t)$ at the time t. For the 1-D dilation, the equation 1 can be written as $\partial_t f = \partial_x f$. To solve this partial equation, there have two discrete evolution principle of the dilation process on the discrete level. These says that (1) in regions of strictly monotone data, the flow is directed from lower to high value; (2) local minima are increased, while local maxima are maintained.

Rouy-Tourin upwind scheme[16] uses the first-order difference to approximate the spatial difference, while the difference about time is a forward difference. The basic idea behind the FCT scheme is to rewrite the spatial first-order difference as the sum of a second-order approximation and a so called viscosity term. Under the evolution principle, the rewritten upwind scheme is formulated as follow:

$$f_j^{n+1/2} = \begin{cases} f_j^n, & \text{for } \Delta f_{j-1/2}^n \geq 0 \text{ and } \Delta f_{j+1/2}^n \leq 0, \\ f_j^n + \frac{\lambda}{2}\left|\Delta f_j^n\right| + \underbrace{\frac{\lambda}{2}\Delta f_{j+1/2}^n - \frac{\lambda}{2}\Delta f_{j-1/2}^n}_{\text{viscosity terms}}, & \text{else} \end{cases} \tag{2}$$

where $\lambda = \Delta t/\Delta x$. Here we let $\Delta x = 1$. The numerical viscosity term is responsible for the undesirable blurring effects in the results. This viscosity term should be subtracted to acquire a shock in the signal.

Simply subtracting the viscosity term , for example, deleting the this term from the equation 2 would result in unstable evolutions. Here the correction formula is used

$$f_j^{n+1} = f_j^{n+1/2} - g_{j+1/2} + g_{j-1/2} \tag{3}$$

where the correction terms are

$$g_{j+1/2} = minmod(\Delta f_{j-1/2}^{n+1/2}, \tfrac{\lambda}{2}\Delta f_{j+1/2}^{n+1/2}, \Delta f_{j+3/2}^{n+1/2}) \tag{4}$$

$$g_{j-1/2} = minmod(\Delta f_{j-3/2}^{n+1/2}, \tfrac{\lambda}{2}\Delta f_{j-1/2}^{n+1/2}, \Delta f_{j+1/2}^{n+1/2}) \tag{5}$$

The middle arguments of the minmod-function in Eq.4 and 5 correspond to the numerical viscosity terms, while the left and right arguments are responsible for numerical stabilization.

An evolution process is consisted by two steps: (i) a predictor step performed by using equation 2, then (ii) a correction step performed by using equation 3. Similarly, the erosion scheme can be established analogously. The detailed description can be found in the reference[13].

We combine the above dilation and erosion procedure to filter an image to generate stream pattern potentially existed in the image. The dilation and erosion is applied to a pixel selectively, that is to say, we first compute the Laplacian at that pixel, if the Laplacian is less than zero, dilation procedure is used; if the Laplacian is greater than zero, the erosion procedure is applied. After all pixels are processed, one step evolution is completed. When several evolutions are finished, the image will become a piecewise constant image. Figure 1 shows (a) an original finger print image; (b) stream pattern generated from (a). From this figure we can see that the finger print is well detected.

4. LOCAL STREAM ORIENTATION

It is well known that Laplacian is very sensitive to noises in the image. This will have a heavy influence on the selection of dilation and erosion process. To solve this problem and to improve the performance of stream pattern extraction, one solution is to smooth the image before computing the Laplacian, that is

$$v = K_\sigma * f(x,y,t), \tag{6}$$

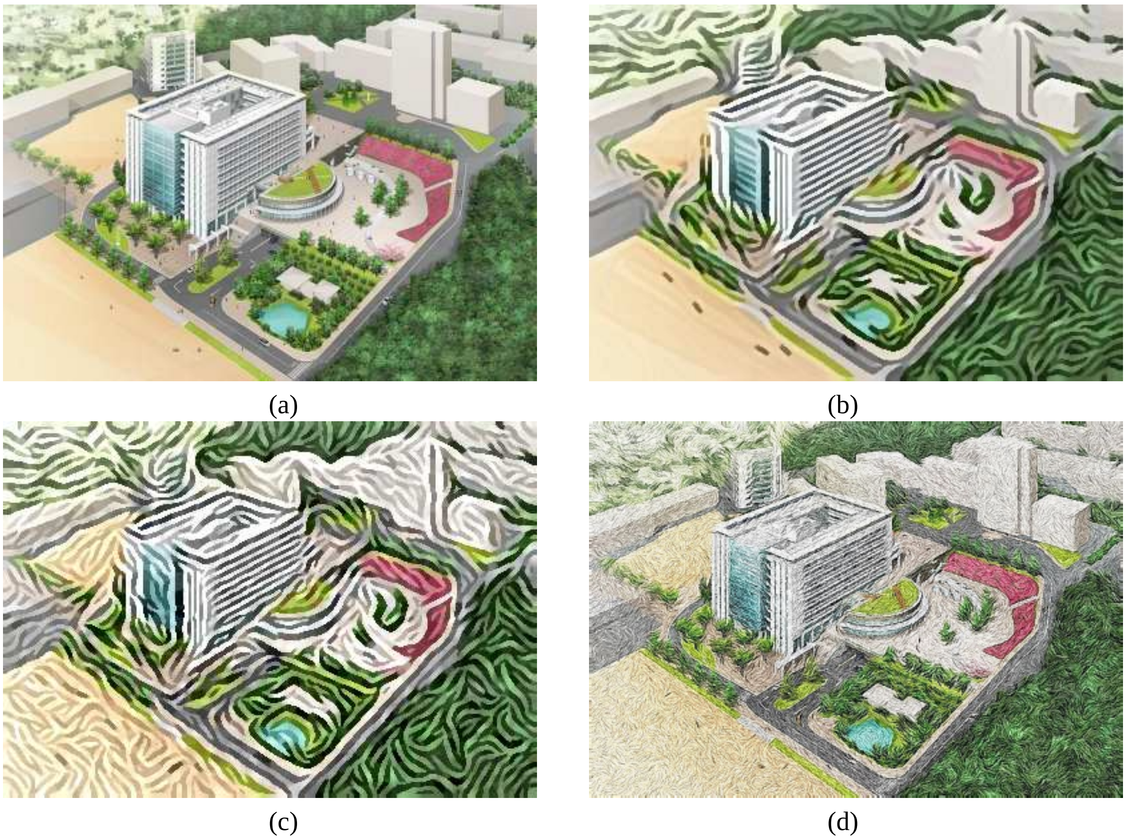

Fig. 2: Generating stream patterns from a vision of a building: (a) An original vision image of a new building; (b) Generated stream pattern, note, less pattern is generated in some region with less texture such as the ground at the left-bottom corner; (c) By adding noise to the region with less texture, this time the stream patterns are well generated in all regions; (d) stream patterns with thin lines.

where K_σ is a Gaussian kernel with a standard deviation σ. Moreover, a second derivative $v_{\eta\eta}$ of image in the direction of gradient ∇f is a better version of Laplacian recommended by Osher[10], where η is parallel to ∇f, the gradient at a pixel in the image.

However, since parallel lines may have opposite gradient direction, the Gaussian smoothed image does not provide reliable information to the orientation. To solve this problem we use a structure tensor[14] to compute local stream pattern orientation. The tensor product for each pixel is

$$J(\nabla f) = \nabla f \nabla f^T = \begin{pmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{pmatrix}. \quad (7)$$

After all tensor products at each pixel are computed, they are Gaussian smoothed with a standard deviation ρ:

$$J_\rho(\nabla f) = K_\rho * \begin{pmatrix} f_x^2 & f_x f_y \\ f_x f_y & f_y^2 \end{pmatrix}. \quad (8)$$

The Gaussian smoothed tensor product is a positive semidefinite matrix. Its two orthogonal eigenvectors describes the maximum and minimum directions of intensity variations. The normalized dominant vector $w = (s, t)$ of $J_\rho \nabla f$ corresponding to the largest eigenvalue can be computed easily in an analytical way. Then the second-order derivative along the dominant eigenvector is calculated as $v_{ww} = s^2 v_{xx} + 2st v_{xy} + t^2 v_{yy}$. This second-order derivative substitutes the Laplacian in the filtering process. For a color image, the evolution is processed for each channel independently. However whether dilation and erosion are applied is based on the same v_{ww}, which is computed from a joint structure tensor $J_p(\nabla f) = K_\rho * \Sigma_{i=1}^3 \nabla f_i \nabla f_i^\top$. In this case the second-order derivatives v_{xx}, v_{xy}, v_{yy} are computed from the gray scale of the color image.

For an image with abundant texture, the stream pattern is generated very well as shown in figure 1. In this figure, although the finger print is faintly with disconnection, the finger stream pattern is extracted clearly. However, if there is less texture in the regions with uniform intensity or color, it is difficult to extract stream pattern in that region.

To generate stream pattern in a uniform region, we added gaussian noises[17] to the region with less textures in the image. This is implemented by adding noises with deviation proportional to the norm of gradient at that pixel. Then using the mentioned dilation and erosion process to shock-filter the noisy image. Figure 2(a) is an original image which is a vision of a new building in our university. Figure 2(b) is a resultant image with extracted stream patterns by applying the algorithm described in section 3 and 4. However almost no stream pattern is generated on the ground at the left-bottom. Figure 2(c) and (d) are other result images in

(a)

(b)

(c)

(d)

Fig. 3: Generating stream patterns from a natural image: (a) An original image with a large sky region with less texture; (b) While the stream patterns are generated in the region containing trees and flowers, almost no stream patterns are produced in the sky region; (c) Adding noise to the original image; (d) Stream patterns are well generated even in the sky region.

which the stream pattern is well generated because of added noises. Figure 3 is an other example. (a) is an original image with a large sky area containing less texture. When applying the algorithm of dilation and erosion process to this image, only in the region containing trees and flowers the stream patterns are well produced, while in the sky region with less texture almost no stream patterns are generated as shown in figure 3(b). After added gaussian noise to the original image as shown in figure 3(c), and the same algorithm is applied, this time the stream patterns are well created shown in figure 3(d).

5. EXPERIMENTAL RESULTS

To verify our analysis and the algorithm, we carried out experiments on several images. Fig.1(a) is a fingerprint image and (a) is a filtered result. By observing the two images, one can know that the finger pattern are well recovered. Even the weak patterns are also reconstructed and the discontinuity is disappeared and become continuous due to the tensor space analysis of local stream orientation. Figure 4 shows the comparison of results with or without considering viscosity during shock-filtering. The red curve shows a part of data taken out from the finger image. The blue curve is

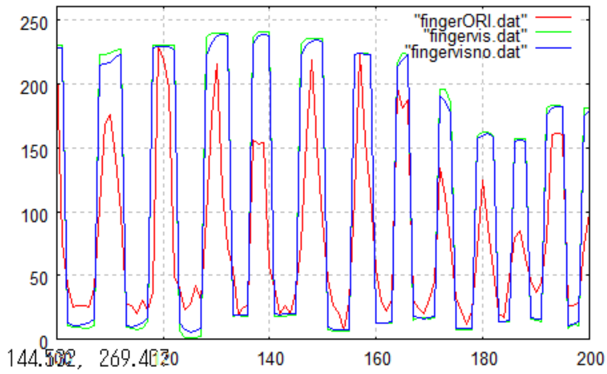

Fig. 4: Comparison of the results with or without considering viscosity

the shock-filtered result without considering viscosity term, while the green one is the shock-filtered result by considering viscosity. It is known from the comparison the red curve is sharper the blue curve.

Figure 2(a) is an original image of new building in our campus. By applying the above-mentioned algorithm to (a), we obtained wide stream patterns shown in (b) with a large

σ in Equation 6. However, the ground and up-right building are less textured, almost no stream patterns are generated. To create stream pattern even no texture existed, we add Gaussian noised to the image, and then applied the algorithm the noise added image. The result is shown in Fig.2(c). (d) is a result by adding Gaussian noise to the image and applying the algorithm with small σ in Equation 6 therefore thin patterns are generated. Figure 3(a) is an image of Sakura with blue sky. (b) is a resulted image with generated patterns. Again since almost no textures existed in the blue sky, no pattern is created. We added Gaussian noise to the image shown in (c), this time the patterns are well generated (d). We also show another interesting result in figure 5. (a) is an original image of an oil-paint *the walk lady with a parasol* by Claude Monet in 1875. We filtered this image by using anisotropic bilateral filter [19], the result is shown in 5(b). (c) is shock filtered result by using the method described in section (3) and (4). (d) is a result by applying anisotropic bilateral filter to the shock filtered result (c).

6. CONCLUSION AND FUTURE WORK

We have presented a technique to create stream patterns from an image. When applying to an image, the algorithm is stable and converges to a stable state. Even without texture in some region, the stream patterns are also be generated by adding Gaussian noise to the image. We also experimented on combining the shock filter and other filters such as anisotropic bilateral filter. We are now considering the combination of shock filter with blur filter and hope to obtained interesting results.

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Fig. 5: Generating stream patterns: (a) An image of *the walk lady with a parasol* (1875) by Claude Monet; (b) A filtered image by using anisotropic bilateral filter; (c) Shock filtered image; (d) A resulted by applying anisotropic bilateral filter to the shock-filtered result (c).