PROPOSAL OF AMPLITUDE ONLY LOGARITHMIC RADON DESCRIPTER — A PERFORMANCE COMPARISON OF MATCHING SCORE —

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ABSTRACT

Amplitude-only logarithmic Radon transform (ALR transform) for pattern matching is proposed. This method provides robustness for object translation, scaling, and rotation. An ALR image is invariant even if objects are translated in a picture. For the object scaling and rotation, the ALR image is merely translated. The objects are identified using a phase-only matched filter to the ALR image. The ratio of size, the difference of rotation angle, and the position between the two objects are detected. Our pattern matching procedure is described, herein, and its simulation is executed. We compare matching scores with the Fourier-Mellin transform, and the general phase-only matched filter.

Keywords: Pattern recognition, Radon transform, log-polar mapping, phase-only correlation, Fourier-Mellin transform

1. INTRODUCTION

Pattern matching is an important technology for object recognition. Recognition performance that does not depend on the object attitude and external environment is required. Various methods are proposed and applied to many systems. Herein, we discuss a method of pattern matching that does not depend on object scaling, translation, or rotation.

When object movement is restricted solely to translation, a method using a phase-only matched filter proposed by C. Kuglin and D. Hines is effective for pattern matching [1]. Their method calculates a correlation between two pictures using phase spectra. Because this method is robust for object translation and external environment, it is applied to various systems for computer vision. However, when the objects are scaled and rotated, the matching precision decreases remarkably.

A rotation and scaling invariant method called log-polar mapping has been studied. Log-polar mapping is based on a polar coordinate system and logarithmic scale expression. A picture is described in a polar coordinate system; its radius is expressed on a logarithmic scale. Here, in the polar coordinate system, the vertical axis shows the radius; the horizontal axis shows the argument. When an object is rotated, the log-polar image is translated horizontally. Object scaling is substituted for the vertical translation in the log-polar image. This method achieves robust pattern matching which does not depend on object scaling and rotation using a phase-only correlation on log-polar images. However, object translation produces complicated movement on the log-polar image. This method does not have robustness for object translation.

Y. Sheng and J. Duvernoy proposed a method called Fourier-Mellin transform [2]. Their method is based on a polar coordinate system. The radius is converted using a Mellin transform; the argument is converted using a Fourier transform. Chen and co-workers proposed a method applying a phase-only matched filter to Fourier-Mellin image [3]. Their method extracts an amplitude spectrum using a Fourier transform to extract amplitude spectrum. The amplitude image is robust for object translation. Subsequently, they execute a log-polar mapping and a phase-only matched filter. Thereby, they can achieve robustness for object translation, rotation, and scaling. In this paper, we use Chen's method as a typical one for the Fourier-Mellin transform.

T. Tsuboi and S. Hirai proposed a method using a Radon transform and a one-dimensional phase-only matched filter [4]. The Radon transform is based on a polar coordinate system. This method converts a picture using the Radon transform, and applies a one-dimensional phase-only matched filter along the radius in the Radon image. For object rotation, the Radon image is translated horizontally. When object is translated, one-dimensional phase-only matched filter on each column provides the object position. Consequently, in the case of objects translation and rotation, this method can achieve robust pattern matching. However, object scaling decreases the matching precision remarkably.

We discuss a method using the Radon transform, logarithm mapping, and a phase-only matched filter [5, 6, 7]. Our method converts a picture using the Radon transform first. We extract the amplitude spectrum and execute logarithmic mapping along the radius of the Radon image. We obtain a processed image, which is call the amplitude-only logarithmic Radon image (ALR image). The ALR image is invariant for object translation. For object scaling and rotation, the ALR image is translated vertically and horizontally. We identify objects and detect the ratio of size, in addition to the difference of rotation angle between two objects by a phase-only matched filter to the ALR image.

(a) (b)

Fig. 1: Original images: (a) "AVION07" and (b) "AVION14."

To adjust the size and rotation angle, we correct the Radon image using vertical scaling and horizontal translation. The difference of position between the two objects is detected using one-dimensional phase-only matched filter to each column in the corrected Radon transform image.

In the following section, we mention about the Radon transform. We propose the amplitude-only logarithmic Radon transform in Section 3, and describe our pattern matching procedure in Section 4. In Section 5, we execute experiments and discuss matching scores through comparison with the Fourier-Mellin transform and the general phase-only correlation.

2. RADON TRANSFORM

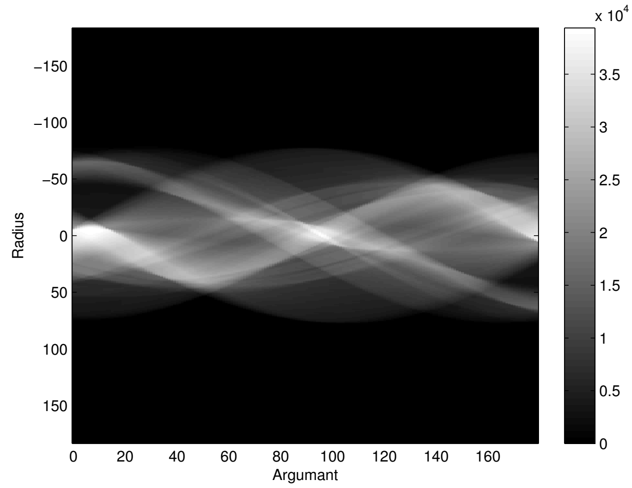
We explain the Radon transform; A coordinate (x, y) in the two-dimensional xy plane is represented as **x**; A picture such as that of Fig. 1 is represented as $f(\mathbf{x})$. Consequently, the Radon transform of $f(\mathbf{x})$ is defined as

$$r(\ ,\rho) = \int f(\mathbf{x})\delta(\mathbf{x}\cdot \quad \rho)d\mathbf{x}, \tag{1}$$

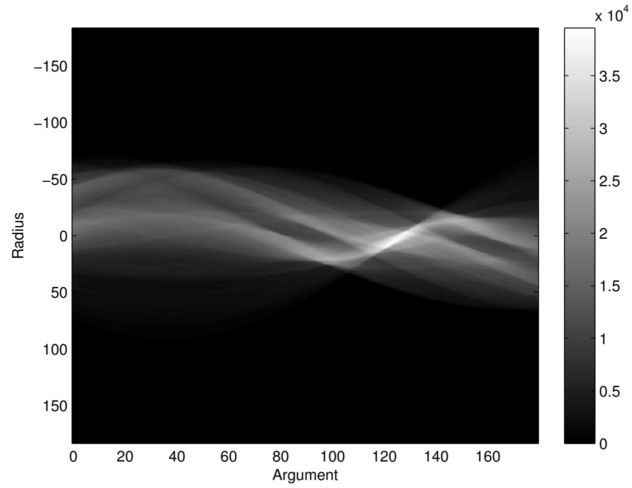
where $\ = (\cos\ ,\sin\ )$, and $\delta(\ )$ is a delta function. We obtain a Radon image $r(\ ,\rho)$. We assume a straight line l_0 which passes through a point located on a Cartesian coordinate (x_0, y_0). A line segment from (x_0, y_0) to the origin and line l_0 are mutually perpendicular. Then, the distance between line l_0 and the origin is ρ_0. The argument that is created by the line segment and the x axis is $_0$. The coefficient on the radon transform $r(\ _0, \rho_0)$ means the result of line integral along line l_0. The Radon image is shown as Fig.2. The horizontal axis is argument and the vertical axis is radius.

When an object is rotated in the picture, the Radon image is translated horizontally. If the object is scaled, the Radon image is scaled vertically. When we translate the object toward vector $\mathbf{x}_0$, the original picture is represented as $f(\mathbf{x} \quad \mathbf{x}_0)$. The Radon image is

$$\begin{aligned} r(\ ,\rho) &= \int f(\mathbf{x} \quad \mathbf{x}_0)\delta(\mathbf{x}\cdot \quad \rho)d\mathbf{x} \\ &= \int f(\mathbf{y})\delta((\mathbf{y}+\mathbf{x}_0)\cdot \quad \rho)d\mathbf{x} \end{aligned}$$



(a)



(b)

Fig. 2: Radon images of the original images: (a) "AVION07" and (b) "AVION14."

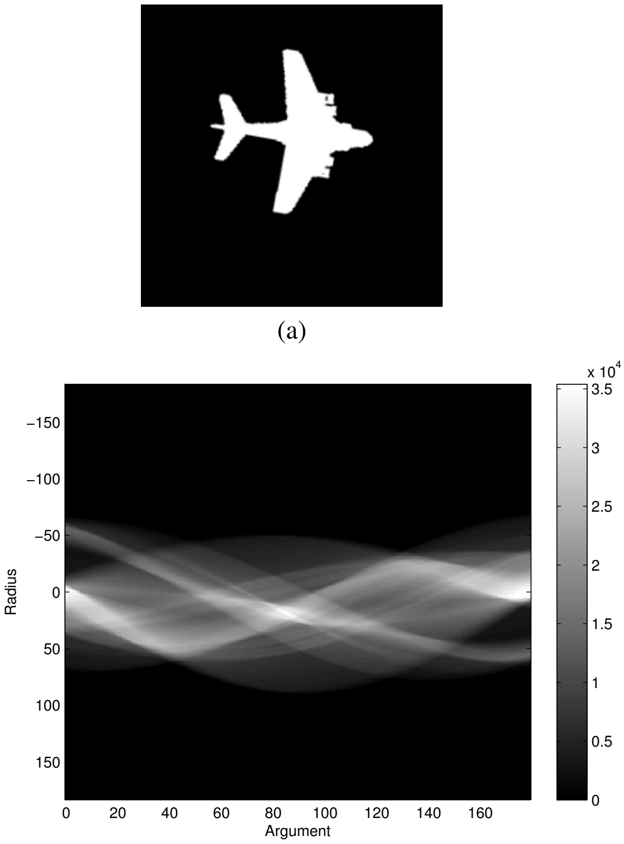
$$= \quad r(\ ,\rho \quad \mathbf{x}_0\cdot\ ), \tag{2}$$

where $\mathbf{y} = \mathbf{x} \quad \mathbf{x}_0$. Then, each column in the Radon image is translated vertically. However, the respective movement quantities of columns differ. In the case of object translation, rotation, and scaling, as in Fig. 3(a), we obtain a Radon image such as that depicted in Fig. 3(b). Each column of the Radon image is translated vertically. In addition, the Radon image is translated left and reduced vertically.

3. AMPLITUDE-ONLY LOGARITHMIC RADON TRANSFORM

The amplitude spectrum in each column in the Radon image is extracted using a Fourier transform as

$$R(\ ,\omega) \quad = \quad \int r(\ ,\rho)e^{-i\omega\ } d\rho$$



(a)

(b)

Fig. 3: (a) Translated, rotated, and scaled "AVION07." (b) Its Radon image.

$$\hat{r}(\ ,\rho) \;=\; \frac{1}{2\pi}\int\left[R(\ ,\omega)\overline{R(\ ,\omega)}\right]^{\frac{1}{2}} e^{i\omega}\ d\omega, \quad (3)$$

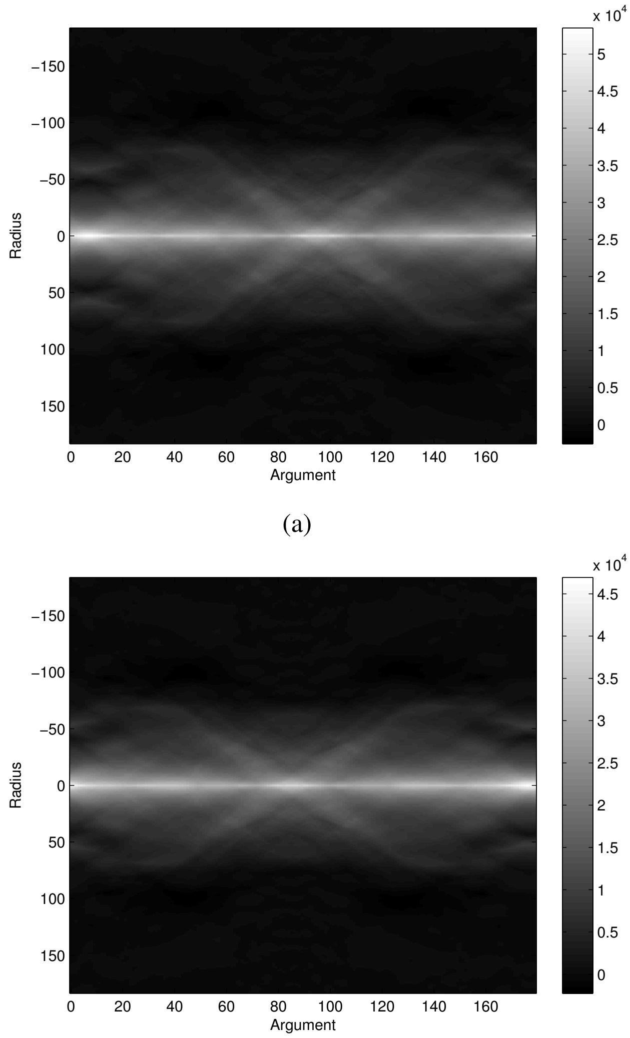
where $\overline{R(\ ,\omega)}$ is a complex conjugate of $R(\ ,\omega)$. We obtain an amplitude-only radon image $\hat{r}(\ ,\rho)$, which is shown as Fig. 4. The phase spectrum is removed, so that the amplitude-only Radon image $\hat{r}(\ ,\rho)$ is invariant for object translation. Object rotation is substituted for horizontal translation of the processed image. Object scaling is substituted for vertical scaling. The amplitude-only radon image is symmetric vertically; then we handle only the downside.

Next, we execute logarithmic mapping along the radius of $\hat{r}(\ ,\rho)$ as

$$\begin{aligned} \ln\rho &= \\ \tilde{r}(\ ,\ ) &= \hat{r}(\ ,e^{\lambda}). \end{aligned} \quad (4)$$

For object scaling, the logarithmically mapped image $\tilde{r}(\ ,\ )$ is translated vertically.

We must devote attention to the upper side of the processed image. The upper side does not close values; The upper side are not zero. Consequently, the translation for object scaling of $\tilde{r}(\ ,\ )$ is not simple. Non-zero values well out into the upper-side. We must therefore close the upper



(a)

(b)

Fig. 4: Amplitude extracted radon image: (a) "AVION07" and (b) the relocated "AVION07."

side using partial differentiation as

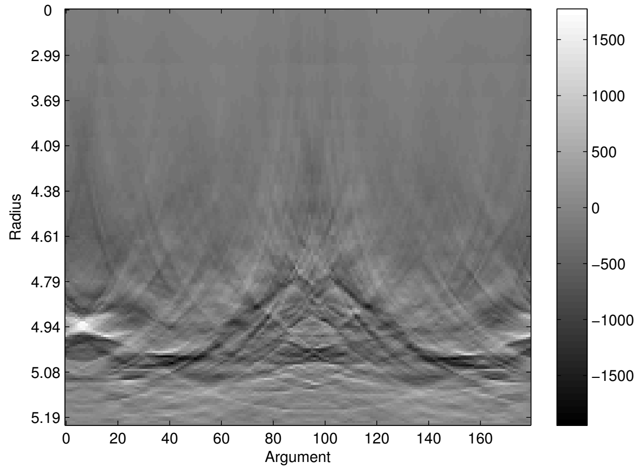
$$h(\ ,\ ) \;=\; \frac{\partial\tilde{r}(\ ,\ )}{\partial}. \quad (5)$$

We obtain amplitude-only logarithmic Radon image $h(\ ,\ )$, as is shown in Fig. 5. Object rotation and scaling are substituted respectively for horizontal and vertical translation.

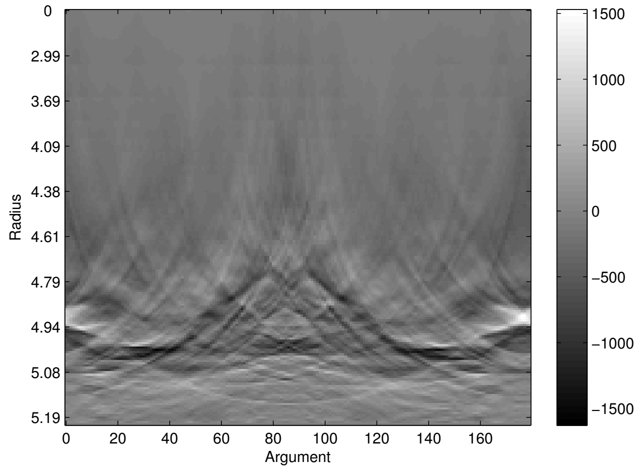
4. PATTERN MATCHING PROCEDURE

We retrieve template pictures showing an object with equal to the object in the input picture. We summarize our procedure as the following.

Step 1 Convert the input picture $f_0(\mathbf{x})$ and the template pictures $f_n(\mathbf{x})$ using the Radon transform as Function (1), and extract their amplitude spectra using Function (3). We obtain amplitude only Radon images $\hat{r}_0(\ ,\rho)$ and $\hat{r}_n(\ ,\rho)$, respectively.



(a)



(b)

Fig. 5: Amplitude-only logarithmic Radon image: (a) “AVION07” and (b) the relocated “AVION07.”

Step 2 Execute logarithmic mapping using Function (4), and differentiate using Function (5). We respectively obtain amplitude-only logarithmic Radon images $h_0(\ , \ )$ and $h_n(\ , \ )$.

Step 3 Execute the phase-only matched filtering between $h_0(\ , \ )$ and $h_n(\ , \ )$ as

$$
\begin{aligned}
H(u,v) &= \iint h(\ , \ )e^{-i(u \ \ +v\lambda)}d \ d \\
poc_h(\ , \ ) &= \\
&\frac{1}{(2\pi)^2}\iint \frac{H_0(u,v)\overline{H_n(u,v)}}{|H_0(u,v)\overline{H_n(u,v)}|}e^{i(u \ \ +v\lambda)}dudv.
\end{aligned} \tag{6}
$$

If the input object is equal to a template object, we obtain a sharp peak on $poc_h(\ , \ )$. However, when objects are different, we cannot obtain a sharp peak. The peak position provides the ratio of size and the difference of rotation angle between the two objects.

Step 4 Adjust the size and the rotation angle in the Radon image $r_0(\ , \rho)$. We expand $r_0(\ , \rho)$ vertically, and translate it horizontally for adjustment.

Step 5 Detect the difference of object position between the two objects using the phase-only matched filtering on each column in the Radon image as

$$
\begin{aligned}
R(\ , u) &= \int r(\ , \rho)e^{-iu} \ d\rho \\
poc_r(\ , \rho) &= \frac{1}{2\pi}\int \frac{R_0(\ , u)\overline{R_n(\ , u)}}{|R_0(\ , u)\overline{R_n(\ , u)}|}e^{iu} \ du.
\end{aligned} \tag{7}
$$

We obtain a peak on each column, then there is a sinusoidal curve in the processed image. We convert the processed image using inverse Radon transform. Thereby, we obtain a general phase-only correlation image with a peak. The peak position provides the difference of the object position between the two objects.

We describe our pattern matching procedure in Fig. 6.

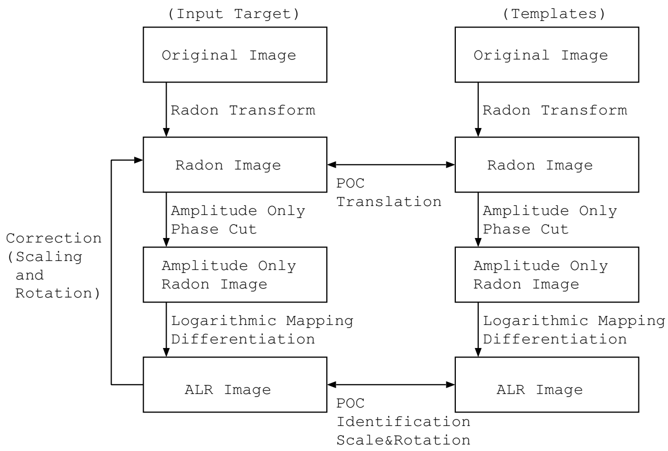


Fig. 6: Pattern matching procedure.

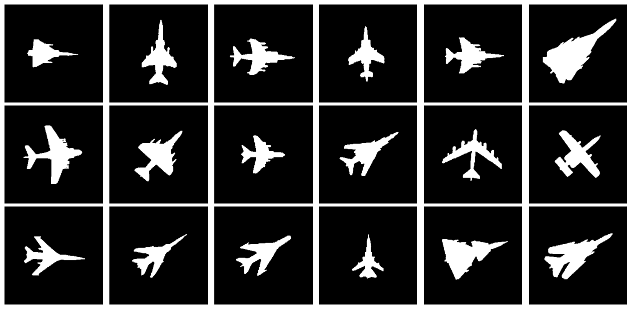


Fig. 7: Original test images “AVION01” - “AVION18”.

Table 1: Matching scores.

	(A) ALR (Proposed)	(B) FMT	(C) POC
AVION07	1.0000	1.0000	1.0000
AVION07 (Translated)	0.9614	1.0000	1.0000
AVION07 (Rotated)	0.9302	0.4851	0.0372
AVION07 (Scaled)	0.7070	0.3635	0.0678
AVION01	0.0410	0.0532	0.0377
AVION02	0.0400	0.0449	0.0288
AVION03	0.0393	0.0604	0.0399
AVION04	0.0450	0.0517	0.0229
AVION05	0.0363	0.0470	0.0303
AVION06	0.0409	0.0408	0.0263
AVION08	0.0369	0.0437	0.0224
AVION09	0.0376	0.0487	0.0400
AVION10	0.0404	0.0509	0.0286
AVION11	0.0359	0.0464	0.0301
AVION12	0.0452	0.0490	0.0279
AVION13	0.0420	0.0387	0.0329
AVION14	0.0374	0.0566	0.0407
AVION15	0.0408	0.0609	0.0309
AVION16	0.0389	0.0473	0.0293
AVION17	0.0379	0.0721	0.0356
AVION18	0.0357	0.0568	0.0332

5. EXPERIMENTAL RESULTS

Fig. 7 shows 18 pictures provided from Granada University [9] . Each picture is a binary image which has 256 256 pixels. We create various relocated pictures by object translation, rotation, and scaling.

Three different pattern matching methods are compared and estimated; (ALR) A phase-only matched filter to the amplitude only logarithmic Radon image; it is our proposed method; (FMT) A phase-only matched filter to the Fourier-Mellin image; (POC) a general phase-only matched filter.

Here, we call the matching using same objects is "genuine", and the matching using different objects is "impostor."

5.1 Matching Scores

The value of a correlation peak on the processed image gives a good similarity measure for our pattern matching. Then, we estimate these peak values as a matching score. However, in the case of impostor, we cannot get a clear peak. Then, the maximum value in the processed image is assumed to be a matching score.

Examples of matching score is shown in Table 1. We execute object matching using "AVION07" and other various test pictures. We create three relocated picture; We translate "AVION07" 10 pixel right, rotated it 10 degree clockwise, and expand it 1.1 times.

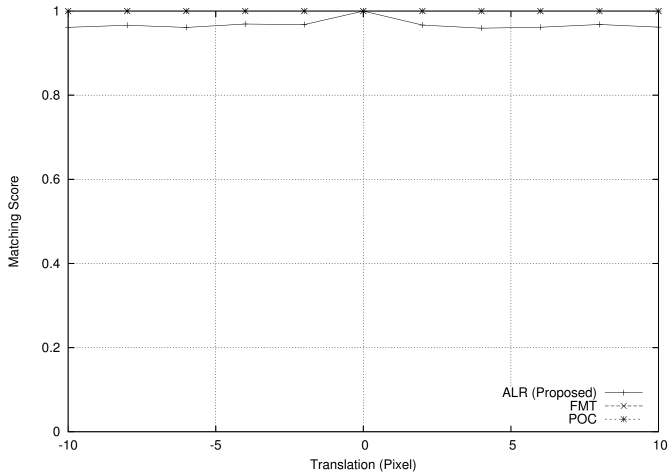


Fig. 8: Influence of object translation.

In the case of our proposed method and the Fourier-Mellin transform method, the genuine matching scores are higher than the impostor matching scores. Then, we can recognize objects without depending on translation, rotation, and scaling. The difference between the genuine matching scores and the impostor matching scores of our proposed method is relatively larger than the case of the Fourier-Mellin transform. Therefore, our proposed method score is better than the Fourier-Mellin transform for the pattern matching. In the case of the general phase-only filter using rotated and scaled object, the matching score is relatively low. Then, the general phase-only filter cannot apply under object rotation and scaling.

5.2 Influence of Object Dislocation

We create relocated 10 pictures of "AVION07"; We translate the object by a pixel gradually right or left; We rotate it by 2 degrees gradually clockwise or anticlockwise; We reduce it by 0.90, 0.92, 0.94, 0.96 0.98 times, and expanded 1.02, 1.04, 1.06, 1.08, 1.10 times. We execute object matching between "AVION07" and relocated picture, and estimate the influence of object dislocation. The experimental results are shown in Fig. 8, 9, and 10.

There is no degradation by translation in both the Fourier-Mellin transform and the general phase-only matched filter. However, our propose method has a little constant degradation under the object translation. It is necessary to use the Fourier-Mellin transform and the general phase-only correlation when object movement is restricted solely to translation.

In the case of object rotation and scaling, our degradation is the smallest of the three methods. Therefore, our proposed method provides the best recognition performance under the object rotation and scaling. In the case of our proposed method and the Fourier-Mellin transform, the intensely up and down in the figure is caused by the sampling in the logarithmic mapping.

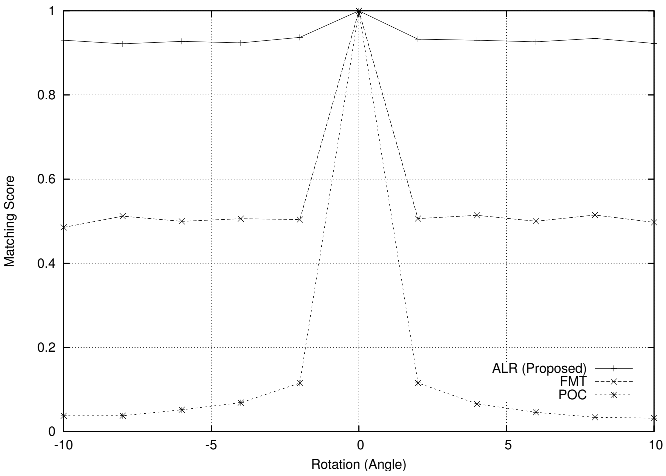


Fig. 9: Influence of object rotation.

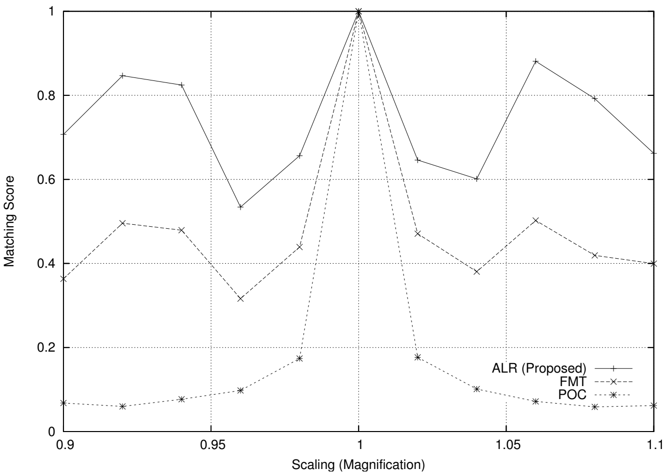


Fig. 10: Influence of object scaling.

6. CONCLUSION

Amplitude-only logarithmic Radon transform is proposed. The ALR image is invariant even if objects are translated. For object scaling and rotation, the ALR image is translated. Objects are identified using a phase-only matched filter to the ALR image, and the ratio of size, the difference of rotation angle, and the position between two objects is detected. We describe our pattern matching procedure. We execute an pattern matching simulation, and compare matching scores. Our degradation of matching score under the object rotation and scaling is the smallest of the three methods. We confirm our robustness for object rotation and scaling.

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