# REGISTRATION OF MICROSCOPIC SECTION IMAGES BASED ON A RADIAL DISTORTION MODEL 

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#### Abstract

Registration of microscopic section images from an organism is of importance in analyzing and understanding the function of an organism. Microscopes usually suffer from the radial distortion due to the spherical aberration. In this paper, a correction scheme for the intra-section registration is proposed. The correction scheme uses two corresponding feature points under the radial distortion model. Proposing several variations of the proposed scheme, we extensively conducted experiments for real microscopic images. Iterative versions of the correction from multiple feature points provide good performance for the registration of the optical and scanning electron microscopic images.


## 1. INTRODUCTION

In the area of biology, observing the distribution of some features in the three dimensions, e.g., the cell nuclei in the rat cerebellum, is of importance for analyzing the mutation or the transition of an organism due to stimuli, such as disease. In order to construct a three dimensional structure, a conventional approach is using a series of microscopic section images of the organism. Here, the section images are obtained by sectioning the object using the microtome. We then carefully stack the section images by matching the adjacent section images. While matching the images, we may need some operations of rotation, translation, scaling, and even nonlinear transformations, due to the deformation of the sectioned samples occurred during the sample preparation and the image acquisition. Such operations for matching is called the registration of the images [3]. The registration of a series of section images is referred to the intersection registration.

In order to obtain detail microscopic images of the sectioned organism, we usually increase the magnification factor of the microscope. However, because of the magnification, we may lost the position of details with respect to the hole body of the organism. Hence, we need to mosaic the magnified images to investigate the global position of the interesting features, which is called the intra-section reg-
istration of the images. However, the microscopic images usually suffer from an image deformation problem caused by the spherical aberration [2]. The deformation eventually yields the registration error. There are a few works on the lens distortion calibration [2],[5]. However, since these works mainly focus on three dimensional reconstructions, a different deformation model is used. Therefore, research on deriving a simple algorithm, which is specially for the microscopic images, is required.

In this paper, we study the intra-section registration of the microscopic section images. In Section 2, we first consider two compensation schemes based on a radial distortion model in order to alleviate the image deformation problem caused by the spherical aberration. We then apply the compensation techniques to a grid image to verify the performance of the suggested schemes. In Section 3, we conduct the simulation for real optical microscopic section images obtained from trichoptera and the rat cerebellum. Further, we apply the schemes to the scanning electron microscope (SEM) images of the drosophila eyes.

## 2. COMPENSATION BASED ON A RADIAL DISTORTION MODEL

In this section, we first formulate a radial distortion model, which is simple and deterministic, but can faithfully describe the image deformation caused by the spherical aberration. We then introduce two simple schemes for compensating the image deformation based on a reference grid and a set of feature points, respectively.

Let $(x, y) \in \mathbb{R}^{2}$ denote a point in the corrected image, which has no image deformation due to the spherical aberration, and $(X, Y) \in \mathbb{R}^{2}$ denote the corresponding point in the deformed image, which is the observed image. The radial distortion model [4] is then given by

$$
\begin{align*}
x & =X\left(1+k_{1} r^{2}+k_{2} r^{4}\right) \\
y & =Y\left(1+k_{1} r^{2}+k_{2} r^{4}\right), \tag{1}
\end{align*}
$$

where $r^{2}:=X^{2}+Y^{2}$, and $k_{1}$ and $k_{2}$ are the radial coefficients. Note that the radial coefficients are dependent on the


Fig. 1. Rectangular grid as a reference pattern.
microscope and its magnification. Here, we suppose that the microscope does not have the axial astigmatism and the optical axis of the microscope is on the center $((x, y)=(0,0))$ of the image coordinates. Note that, in the intra-section registration, we may only consider the rotation and translation of the images [4].

Using a reference pattern, a calibration algorithm can calculate the radial distortion coefficients, and correct the images from the relationship (1). Such a calibration algorithm is introduced in the following subsection using a rectangular grid as a reference pattern.

### 2.1. Compensation Based on Reference Pattern

We consider a grid, where each side has the same length of $e$ as shown in Fig. 1. In order to estimate the two radial coefficients, we need four independent equations from a property of the grid in Fig. 1 as

$$
\begin{equation*}
x^{\prime}=x+e \text { and } y^{\prime}=y \tag{2}
\end{equation*}
$$

Suppose that, in the deformed image, the corresponding points of $(x, y)$ and $\left(x^{\prime}, y^{\prime}\right)$ are $(X, Y)$ and $\left(X^{\prime}, Y^{\prime}\right)$, respectively. We then have $x=X\left(1+k_{1} r^{2}+k_{2} r^{4}\right), y=$ $Y\left(1+k_{1} r^{2}+k_{2} r^{4}\right), x^{\prime}=X^{\prime}\left(1+k_{1} r^{2}+k_{2} r^{\prime 4}\right)$, and $y^{\prime}=Y^{\prime}\left(1+k_{1} r^{\prime 2}+k_{2} r^{\prime 4}\right)$, where $r^{2}=X^{2}+Y^{2}$ and $r^{\prime 2}=X^{\prime 2}+Y^{\prime 2}$. Hence, from (2), we can obtain the radial coefficients as follows:
$k_{1}=\frac{\left(Y X^{\prime}-X Y^{\prime}-Y e\right) r^{4}+\left(X Y^{\prime}-X^{\prime} Y+Y^{\prime} e\right) r^{\prime 4}}{\left(Y X^{\prime}-X Y^{\prime}\right) r^{4}+\left(X Y^{\prime}-Y X^{\prime}\right) r^{4} r^{\prime 2}}$
and
$k_{2}=\frac{\left(X Y^{\prime}-Y X^{\prime}+Y e\right) r^{2}+\left(Y X^{\prime}-X Y^{\prime}-Y^{\prime} e\right) r^{\prime 2}}{\left(Y X^{\prime}-X Y^{\prime}\right) r^{4}+\left(X Y^{\prime}-Y X^{\prime}\right) r^{4} r^{\prime 2}}$.
A simulation on the compensation of the grid image is shown in Fig. 2. We may notice that we can successfully correct the deformed image based on the simple model in (1) especially for low-magnification microscopic images.

### 2.2. Compensation Based on Corresponding of Feature Points

In practical applications, taking an image of a reference pattern from a high-magnification microscope is very difficult.


Fig. 2. Compensation of the grid image . (a) Deformed image (sigmoid distortion obtained by an optic-microscope with $\times 100$ ). (b) Corrected image ( $k_{1} \approx-2.1 \times 10^{-6}$ and $k_{2} \approx-9.9 \times 10^{-9}$ ).

Hence, the scheme in the previous subsection, which uses a grid as shown in Fig. 1, is not practical, especially for highmagnification microscopy. Therefore, we suggest a scheme, which exploits several feature points in the section images. Note that the feature points are obtained by a supervised learning method and each point in a section image has its corresponding point in the other section image to be registered. Here, an expert chooses the corresponding feature points. However, for an automatic feature point selection, adopting a corner point detection and a robust point matching algorithms is in progress.

We now introduce the correction scheme by introducing an example of registration of two deformed images. We first consider a deformed image to be registered. Let points $\left(X_{1}, Y_{1}\right)$ and $\left(X_{1}^{\prime}, Y_{1}^{\prime}\right), \in \mathbb{R}^{2}$, denote feature points in the image. Let points $\left(x_{1}, y_{1}\right)$ and $\left(x_{1}^{\prime}, y_{1}^{\prime}\right), \in \mathbb{R}^{2}$, denote the corresponding points, which are corrected based on the model (1) from the radial coefficients $k_{1}$ and $k_{2}$. Note that these coefficients should be calculated from the feature points. In a manner similar to the first image case, we now consider the other images and denote the points as $\left(X_{2}, Y_{2}\right)$ and $\left(X_{2}^{\prime}, Y_{2}^{\prime}\right)$ for the feature points in the second image, and $\left(x_{2}, y_{2}\right)$ and $\left(x_{2}^{\prime}, y_{2}^{\prime}\right)$ for the corrected image. We then obtain a relationship as

$$
\begin{equation*}
\left(x_{1}, y_{1}\right)-\left(x_{1}^{\prime}, y_{1}^{\prime}\right)=\left(x_{2}, y_{2}\right)-\left(x_{2}^{\prime}, y_{2}^{\prime}\right) \tag{3}
\end{equation*}
$$

since the feature points $\left(X_{1}, Y_{1}\right)$ corresponds to $\left(X_{2}, Y_{2}\right)$, and so does the points $\left(X_{1}^{\prime}, Y_{1}^{\prime}\right)$ to $\left(X_{2}^{\prime}, Y_{2}^{\prime}\right)$. From (1), we obtain the following equations:

$$
\begin{aligned}
& \left(X_{1} r_{1}^{2}-X_{1}^{\prime}{r_{1}^{\prime}}^{2}-X_{2} r_{2}^{2}+X_{2}^{\prime}{r_{2}^{\prime}}^{2}\right) k_{1} \\
& +\left(X_{1} r_{1}^{4}-X_{1}^{\prime} r_{1}^{\prime 4}-X_{2} r_{2}^{4}+X_{2}^{\prime} r_{2}^{\prime 4}\right) k_{2} \\
& \quad=-X_{1}+X_{1}^{\prime}+X_{2}-X_{2}^{\prime}
\end{aligned}
$$

and

$$
\begin{aligned}
& \left(Y_{1} r_{1}^{2}-Y_{1}^{\prime}{r_{1}^{\prime}}^{2}-Y_{2} r_{2}^{2}+Y_{2}^{\prime}{r_{2}^{\prime}}^{2}\right) k_{1} \\
& +\left(Y_{1} r_{1}^{4}-Y_{1}^{\prime} r_{1}^{\prime 4}-Y_{2} r_{2}^{4}+Y_{2}^{\prime} r_{2}^{4}\right) k_{2} \\
& \quad=-Y_{1}+Y_{1}^{\prime}+Y_{2}-Y_{2}^{\prime},
\end{aligned}
$$



Fig. 3. Section images of trichoptera obtained from an optic-microscope (Laboratory of Cell Engineering and 3D Structure, Korea University). (a) Low-magnification image ( $\times 100$ ). (b)-(d) High-magnification images to be mosaicked $(\times 200)$.
where $r_{1}^{2}=X_{1}^{2}+Y_{1}^{2},{r_{1}^{\prime}}^{2}={X_{1}^{\prime}}^{2}+Y_{1}^{\prime 2}, r_{2}^{2}=X_{2}^{2}+Y_{2}^{2}$, and ${r_{2}^{\prime}}^{2}={X_{2}^{\prime}}^{2}+Y_{2}^{\prime 2}$. Hence, we can calculate the radial coefficients $k_{1}$ and $k_{2}$ from the two pairs of feature points. In the following section, we will see the performance of this scheme through extensive experiments for various images. Further, some modified application of the scheme will be introduced.

## 3. EXPERIMENTS

In order to check the performance of the proposed scheme, which is using the feature points, we first use section images of trichoptera obtained from an optical microscope at magnifications of $\times 100$ and $\times 200$ as shown in Fig. 3. The three high-magnification images of Figs. 3(b)-(d) are to be performed the registration between the low-magnification image of Fig. 3(a), which is used as a reference frame. Note that the sampling space in the magnification of $\times 200$ is approximately $0.33 \mu \mathrm{~m}$. Before performing the registration, the reference image is corrected based on the radial coefficients obtain in Fig. 2, which is shown in Fig. 4(a). Fig. 4(b) shows a registration result without any correction of the high-magnification images. Here, we may notice a serious registration error. However, after a correction using the coefficients obtained from the feature points in Fig. 4(c), we can faithfully perform the registration as shown in Fig. 4(d). In the simulations, the registration error is represented by the mean square error (MSE) in decibel between the reference frame as shown in Fig. 4(a) and the registration result. Note that the performance of the registration is quite dependent on choosing the corresponding feature points between


Fig. 4. Correction and registration of the trichoptera image . (a) Corrected low-magnification image ( $\times 100$ ). (b) Registration of high-magnification images ((b)-(d) of Fig. 3) without correction (MSE $=26.6 \mathrm{~dB})$. (c) Example of feature points a and b ((c) and (d) of Fig. 3). (d) Registration of corrected images (coefficients from the feature points in (c) $(\mathrm{MSE}=24.6 \mathrm{~dB})$.
two images. Hence, we need a modification to the correction scheme by exploiting the information obtained from the feature points in an iterative approach.

We first correct the two images using the two pares of the feature points in a manner similar to Fig. 4(c) and (d). We then calculate the radial coefficients from the corrected two images in the same way, and correct again the corrected images. In this manner, we may iteratively correct the two images until the radial coefficients go to zero. An experimental result is shown in Fig. 6 using the two images of Fig. 5. Compared to the registration using uncorrected images as shown in Fig. 6(a), using corrected images based on the feature points as in Fig. 4 shows lower registration errors. Iteratively correcting the images using the same feature points can further reduce the registration error as shown in Fig. 6(b). Here, by iteratively applying the correction, we can reduce the registration error. However, the increasing performance gain can be obtained only when we choose the corresponding feature points accurately. For inappropriate feature points, further iteration usually cannot produce evident improvement. As shown in (3), two feature point on an image is enough to calculate the radial coefficients $k_{1}$ and $k_{2}$. If we choose three feature points on an image, then we can calculate three sets of the radial coefficients, since the maximum number of combination of choosing two points are $\binom{3}{2}=3$. Using more than two feature points, we can alleviate the problem caused by inappropriately choosing the feature points. The modified correction procedure is as follows. First, we choose a combination of the feature points
and then perform the correction based on (3). We perform the correction again using another combination of the feature points. Hence, we perform the corrections $\binom{n}{2}$ times, where $n$ is the number of the feature points on an image. Fig. 4(c) shows an example of the $n=4$ case.


Fig. 5. Registration of the trichoptera images. (a)-(b) Highmagnification images ( $\times 200$ ).

In practical cases, we should perform the registration using more than two images. Hence, we can calculate the radial coefficients for different pairs of the images. Exploiting the all possibility of obtaining the radial coefficients, we can correct the corrected images for several times. Though experimental results, we may notice a decreasing MSE by applying further corrections using many feature points.

In Fig. 7, we also performed the registration based on the correction using the drosophila eye section images obtained from SEM. We may notice that the correction exploiting the feature points can enable successful registration.

## 4. CONCLUSION

In order to perform the registration of microscopic section images, we conducted a correction by calculating the radial coefficients based on the radial distortion model. Iterative version of the proposed scheme could provide a good registration performance. Further research, which can more increase the registration accuracy, should be conducted especially by compared to other conventional methods.

## 5. REFERENCES

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Fig. 6. Registration of the trichoptera images of Fig. 5. (a) Registration without correction. (b) Registration with the iterative correction using the same feature points.


Fig. 7. Registration of the drosophila eye section image (scanning electron microscope with $\times 1650$ from the Laboratory of Cell Engineering and 3D Structure, Korea University).

