An Efficient Channel Estimation for Amplify and Forward Cooperative Diversity with Relay Selection

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Abstract **− In this paper, we propose a new channel estimation scheme for amplify and forward cooperative diversity with relay selection. In order to select best relay, it is necessary to know channel state information (CSI) at the destination. Most of the previous works, however, assume that perfect CSI is available at the destination. In addition, when the number of relay is increased it is difficult to estimate CSI through all relays within coherence time of a channel because of the large amount of frame overhead for channel estimation. In a proposed channel estimation scheme, each terminal has distinct pilot signal which is orthogonal each other. By using orthogonal property of pilot signals, CSI is estimated over two pilot signal transmission phases so that frame overhead is reduced significantly. Due to the orthogonal property among pilot signals, estimation error does not depend on the number of relays. Simulation result shows that the proposed channel estimation scheme provides accurate CSI at the destination.**

Index Terms **− Cooperative diversity, relay selection, channel estimation.**

I. INTRODUCTION

Multiple-input multiple-output (MIMO) system has been an emerging issue for a future wireless communication because it provides higher capacity and robustness on channel fading [1]. Although a MIMO system has a number of advantages, it is hard to implement a multiple antenna array at a mobile terminal due to the cost, power, and the size limitation. Recently, user cooperation or cooperative diversity has been proposed as a solution for those problems [2]-[4]. In cooperative diversity, terminals with a single antenna form a virtual antenna array so that the benefits of a MIMO system are obtained.

Two simple and efficient relaying protocols are widely accepted for cooperative diversity. One is decode and forward (DF) relaying. In DF relaying, each relay decodes source information and forwards re-encoded source information to the destination. Although it benefits from various coding schemes [5], [6], a complexity and power consumption for source information decoding at each relay are main drawbacks of DF relaying. In contrast to DF relaying, amplify and forward (AF) relaying does not need to decode the source information. AF relaying only forwards the amplified version of the source information so that it has distinct benefits in terms of a complexity and power consumption.

A number of cooperative diversity schemes such as distributed space-time codes, beam-forming, and radio resource allocation have been widely studied [7]-[12]. It was proved that cooperative diversity achieves a significant performance improvement as the number of potential relay is increased. However, the complexity for space-time code or beam design, and resource allocation is also increased. To reduce computational complexity and feedback signal, cooperative diversity with relay selection is proposed [13]. Among multiple relays, the best relay is selected and forwards the source information. It is shown that relay selection also achieves full diversity and the performance loss is negligible with low complexity [14]. However, most works for cooperative diversity with relay selection assume that perfect CSI is available at the destination. Additionally, when the number of relay is increased the more pilot signal transmission phases are required. It makes hard to obtain exact channel estimation within coherence time of a channel.

In this paper, we propose a new channel estimation scheme for cooperative diversity with relay selection. In a proposed scheme, only two pilot signal transmission phases are required to estimate CSI of all links. The estimation error is verified via numerical simulation.

This paper is organized as follows. Section II provides a system model and a brief overview of AF cooperative diversity with relay selection. Section III provides a new CSI estimation scheme for AF cooperative diversity with relay selection. Section IV provides simulation results and section V concludes this paper.

II. SYSTEM MODEL

Consider a cooperative diversity which consists of one source, one destination, and K multiple relays as shown in Fig. 1. Assume that each terminal is equipped with a single antenna and cannot transmit and receive simultaneously. Assume that a wireless channel is frequency-flat and quasi-static which the channel coefficient is constant over one data frame and vary independently from one frame to the other. Assume that all channel coefficients are modeled as a zero-mean circularly symmetric Gaussian random variable.

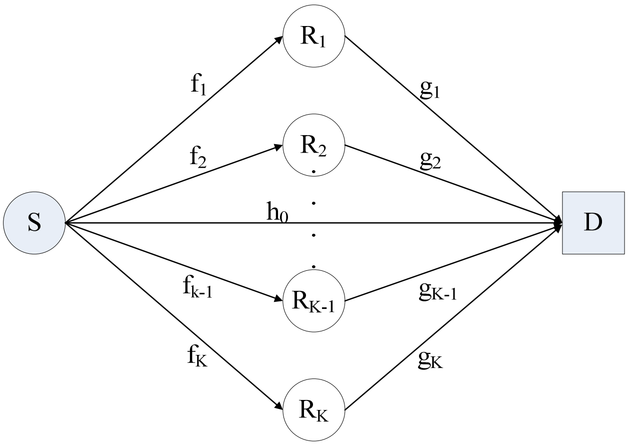

Fig. 1. System model for cooperative diversity with multiple relays.

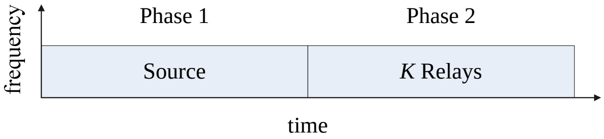

Fig. 2. Orthogonal subchannel allocation.

Assume that each terminal has half duplex constraint so that a source transmits an information signal over two orthogonal time slots. Orthogonal subchannel allocation is shown in Fig. 2.

In the first time slot, a source transmits an information signal to the destination and relays simultaneously. The information signal consists of N_D data symbols. The received information signal at the destination and relays are given by

$$\mathbf{y}_{D,1} = h_0\mathbf{x} + \mathbf{n}_{D,1} \tag{1}$$

$$\mathbf{y}_i = f_i\mathbf{x} + \mathbf{n}_i, \quad i = 1, 2, \ldots, K \tag{2}$$

where the information signal transmitted by a source is denoted by $\mathbf{x}$ which is a $1 \times N_D$ vector. The received source information signal at the destination and relay i is denoted by $\mathbf{y}_{D,1}$ and $\mathbf{y}_i$, respectively. Let h_0 and f_i denote the wireless channel between the source to the destination and the source to the relay i, respectively. Noise added at the destination and relay i in the first time slot are denoted $\mathbf{n}_{D,1}$ and $\mathbf{n}_i$, respectively. Assume that additive white Gaussian noise (AWGN) is modeled as an independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variable with the mean zero and the variance σ_i^2, $i = 1, 2, \ldots, K, D$. Assume that the noise variances for each terminal are the same.

In the second time slot, each relay forwards an amplified version of the received information signal to the destination after power normalization. The signal transmitted from the relay i is given by

$$\hat{\mathbf{x}}_i = \beta_i \cdot \mathbf{y}_i = \beta_i f_i \mathbf{x} + \mathbf{n}_i^{'} \tag{3}$$

where

$$\beta_i = \frac{\sqrt{E_R}}{\sqrt{E_S \left| f_i \right|^2 + \sigma_i^2}}, \quad \mathbf{n}_i^{'} = \beta_i \mathbf{n}_i .$$

Amplification factor for the relay i is denoted by β_i. The average transmit power of the source is denoted by E_S which satisfies $\mathbf{E}\left[\mathbf{x}^{\dagger}\mathbf{x}\right] = E_S \cdot \mathbf{I}_{N_D \times N_D}$. The average transmit power of the relay is denoted by E_R. Assume that the sum of the transmit power of all relays is normalized by 1 and total power for a relay is equally allocated to each relay so that $E_R = 1/K$.

The received information signal at the destination is given by

$$\begin{aligned} \mathbf{y}_{D,2} &= \sum_{i=1}^{K} g_i \hat{\mathbf{x}}_i + \mathbf{n}_{D,2} \\ &= \sum_{i=1}^{K} g_i f_i \beta_i \mathbf{x} + \sum_{i=1}^{K} g_i \beta_i \mathbf{n}_i + \mathbf{n}_{D,2} \\ &= \sum_{i=1}^{K} h_i \beta_i \mathbf{x} + n_e \end{aligned} \tag{4}$$

where $\mathbf{n}_{D,2}$ is additive noise at the destination in second time slot. We define the equivalent relay i channel denoted by h_i as the product of the channel coefficient between the source to the relay i and the channel coefficient between the relay i to the destination, *i.e.*, $h_i = g_i \cdot f_i$. In AF cooperative diversity, among K potential relays, the best relay is selected and forwards the source information signal.

By using relay selection, the received information signal at the destination is given by

$$\mathbf{y}_{D,2} = h_{i'} \beta_{i'} \mathbf{x} + n_e \tag{5}$$

where i' denotes the selected relay.

III. PROPOSED CSI ESTIMATION SCHEME

We assume a centralized relay selection, *i.e.*, the best relay is selected at the destination and informs which relay is selected by using feedback signaling. In order to select the best relay, it is needed to know equivalent relay CSI of all links at the destination.

In order to estimate equivalent relay i CSI, a source transmits a pilot signal and the relay i forwards the received pilot signal to the destination. By using the pilot signal from the relay i, the destination estimates equivalent relay i CSI. If K potential relays exist in a wireless network, K pilot signal transmission phases

Table 1

The percentage of frame overhead to estimate equivalent relay CSI. It is obtained by $100 \times K \cdot N_P / N_D$, where N_D is the number of total symbols in one data frame, and N_P is the number of pilot symbol. Assume that N_P is set to 8.

K N_D	2	3	4	5	6
128	12.8	18.75	25	31.25	37.5
256	6.25	9.38	12.5	15.63	18.75
512	3.13	4.69	6.25	7.81	9.38

are required to estimate all equivalent relay CSI. Note that pilot signal transmission phase is required as many as the number of relays, *i.e.*, the more channel estimation phases are required when the number of relay is increased. Table 1 shows the percentage of frame overhead to estimate equivalent relay CSI with the different number of relays.

Note that it is difficult to estimate exact channel state information within coherence time of a channel as the number of relay is increased. The study to reduce frame overhead had been carried out [9]. It reduces frame overhead in half. However, it also has a large amount of frame overhead in a wireless network having a large number of relays. Therefore, it is required to reduce frame overhead with reasonable size for exact channel estimation within coherence time of a channel. We propose a new channel estimation scheme using an orthogonal pilot signal.

In the first channel estimation phase, a source transmits the pilot signal $(\mathbf{p}_0)$ toward the destination and K multiple relays. Assume that each terminal has a distinct pilot signal which is orthogonal each other. Let $\mathbf{p}_i$ denotes $1 \times N_P$ pilot signal vector for terminal i and N_P denotes the length of a pilot signal. The received pilot signal at the destination and relay i are given by

$$\mathbf{y}_{D,1} = h_0 \mathbf{p}_0 + \mathbf{n}_{D,1} \qquad (6)$$

$$\mathbf{y}_i = f_i \mathbf{p}_0 + \mathbf{n}_i . \qquad (7)$$

Assume that the destination knows pilot signal of all terminals in the wireless network and all relays only know the pilot signal of the source. For the received pilot signal, the destination and each relay multiply a normalized conjugate transposition of the source pilot signal to estimate source-destination and source-relay CSI. Estimated CSI at the destination and the relay i are given by

$$\begin{aligned} \widehat{h}_0 &= \mathbf{y}_{D,1} \cdot \frac{\mathbf{p}_0^{\dagger}}{\|\mathbf{p}_0\|^2} = h_0 + \frac{\mathbf{n}_{D,1}\mathbf{p}_0^{\dagger}}{\|\mathbf{p}_0\|^2} \\ &= h_0 + e_{D,1} \end{aligned} \qquad (8)$$

$$\begin{aligned} \widehat{f}_i &= \mathbf{y}_i \cdot \frac{\mathbf{p}_0^{\dagger}}{\|\mathbf{p}_0\|^2} = f_i + \frac{\mathbf{n}_i\mathbf{p}_0^{\dagger}}{\|\mathbf{p}_0\|^2} \\ &= f_i + e_i . \end{aligned} \qquad (9)$$

where $e_{D,1}$ and e_i denote a channel estimation error occurred at the destination and each relay. Let $\mathbf{a}^{\dagger}$ denotes a conjugate transposition of vector $\mathbf{a}$ and $\|\cdot\|^2$ denotes a squared Euclidian distance.

After multiplying a normalized conjugate transposition of the source pilot vector, each relay obtains estimated source-relay CSI. For the estimated source-relay CSI, each relay multiplies its own pilot signal to obtain a new pilot signal which includes source-relay CSI. Then, each relay transmits a new pilot signal to the destination simultaneously[1]. We assume that the pilot signals assigned for each relay are orthogonal. The orthogonal property of each pilot signal is given by

$$E\left[\mathbf{p}_i\mathbf{p}_j^{\dagger}\right] = \begin{cases} 0, & i \neq j \\ 1, & i = j. \end{cases} \qquad (10)$$

A new pilot signal for the relay i is given by

$$\begin{aligned} \mathbf{p}_i^{'} &= \widehat{f}_i \cdot \mathbf{p}_i = \left(f_i + \frac{\mathbf{n}_i\mathbf{p}_0^{\dagger}}{\|\mathbf{p}_0\|^2} \right) \cdot \mathbf{p}_i \\ &= f_i \cdot \mathbf{p}_i + \frac{\mathbf{n}_i\mathbf{p}_0^{\dagger}\mathbf{p}_i}{\|\mathbf{p}_0\|^2} . \end{aligned} \qquad (11)$$

The received pilot signal at the destination is given by

$$\begin{aligned} \mathbf{y}_{D,2} &= \sum_{i=1}^{K} g_i \mathbf{p}_i^{'} + \mathbf{n}_{D,2} \\ &= \sum_{i=1}^{K} \left(g_i f_i \cdot \mathbf{p}_i + \frac{g_i \cdot \mathbf{n}_i\mathbf{p}_0^{\dagger}\mathbf{p}_i}{\|\mathbf{p}_0\|^2} \right) + \mathbf{n}_{D,2} . \end{aligned} \qquad (12)$$

Since pilot signals for each terminal are orthogonal, the equivalent relay i CSI is estimated by multiplying a normalized conjugate transposition of the pilot signal assigned to the relay i . Therefore, the proposed estimation scheme only requires two pilot signal transmission phases to estimate all equivalent relay CSI. In addition, channel estimation interference from multiple relays is efficiently canceled out so that estimation error does not depend on the number of relays. Estimated equivalent relay i CSI is given by

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[1] In this work, we assume a perfect synchronization. The synchronization is an important issue for a practical implementation of cooperative diversity. However, it is beyond the scope of this paper.

$$
\begin{aligned}
\widehat{h}_i &= \mathbf{y}_{D,2} \cdot \frac{\mathbf{p}_i^\dagger}{\|\mathbf{p}_i\|^2} \\
&= g_i \cdot f_i + \frac{f_i \cdot \mathbf{n}_i \mathbf{p}_0^\dagger}{\|\mathbf{p}_0\|^2} + \sum_{\substack{j=1 \\ i \neq j}}^{K} \left(g_j \cdot f_j + \frac{f_j \cdot \mathbf{n}_j \mathbf{p}_0^\dagger}{\|\mathbf{p}_0\|^2} \right) \mathbf{p}_j \cdot \frac{\mathbf{p}_i^\dagger}{\|\mathbf{p}_i\|^2} + \frac{\mathbf{n}_{D,2} \cdot \mathbf{p}_i^\dagger}{\|\mathbf{p}_i\|^2} \\
&= g_i \cdot f_i + \frac{g_i \cdot \mathbf{n}_i \mathbf{p}_0^\dagger}{\|\mathbf{p}_0\|^2} + \frac{\mathbf{n}_{D,2} \cdot \mathbf{p}_i^\dagger}{\|\mathbf{p}_i\|^2} = h_i + n_{D,2} \qquad (13)
\end{aligned}
$$

where $\widehat{h}_i$ denotes an equivalent relay i CSI and $n_{D,2}$ is channel estimation error due to the additive noise at the relay i and destination.

To verify approximate characteristic of the propose channel estimation scheme the variance of channel estimation error is given by

$$
\begin{aligned}
E\left[n_{D,2} \cdot n_{D,2}^\dagger\right] &= E\left[\left(\frac{g_i \cdot \mathbf{n}_i \mathbf{p}_0^\dagger}{\|\mathbf{p}_0\|^2} + \frac{\mathbf{n}_{D,2} \cdot \mathbf{p}_i^\dagger}{\|\mathbf{p}_i\|^2}\right) \cdot \left(\frac{g_i \cdot \mathbf{n}_i \mathbf{p}_0^\dagger}{\|\mathbf{p}_0\|^2} + \frac{\mathbf{n}_{D,2} \cdot \mathbf{p}_i^\dagger}{\|\mathbf{p}_i\|^2}\right)^\dagger\right] \\
&= \frac{|g_i|^2 \sigma_i^2}{\|\mathbf{p}_0\|^2} + \frac{\sigma_{D,2}^2}{\|\mathbf{p}_i\|^2}. \qquad (14)
\end{aligned}
$$

From (14), a channel estimation error is approximated as a circularly symmetric Gaussian random variable with a mean zero and a variance $|g_i|^2 \cdot \sigma_i^2 / \|\mathbf{p}_0\|^2 + \sigma_{D,2}^2 / \|\mathbf{p}_i\|^2$. Using Chebyshev inequality, it is verified that channel estimation error is negligible at high SNR. In addition, as the length of pilot signal is increased, estimation error is also reduced. Those two approximate characteristics are verified by numerical simulation in the following section.

IV. SIMULATION RESULTS

The performance of the proposed channel estimation scheme is verified via Monte Carlo simulation. Assume a symmetric network which channel fading variances between a source to each relay and each relay to the destination are the same, *i.e.*, $(\sigma_{Si} = \sigma_{iD}\ ,\ i = 1, 2, \ldots, K)$. Assume that, among K potential relays, the best relay is selected such that $\arg\max_i (h_i)$ for a relay selection with perfect CSI and $\arg\max_i (\widehat{h}_i)$ for that with a proposed channel estimation, respectively. We adopt a QPSK modulation. Suppose that one data frame is consists of 256 information symbols. Suppose that the noise variance of all the terminals are the same, *i.e.*, $\sigma_i^2 = \sigma_{D,1}^2 = \sigma_{D,2}^2\ ,\ \ i = 1, \ldots, K$.

Fig. 3 shows that the probability density function of phase estimation error for the equivalent relay channel. It is shown that accurate phase estimation for the equivalent relay channel is obtained using only two pilot signal transmission phases. It is also shown that, at SNR 15 dB, phase estimation error is with in [-0.5, 0.5] radian.

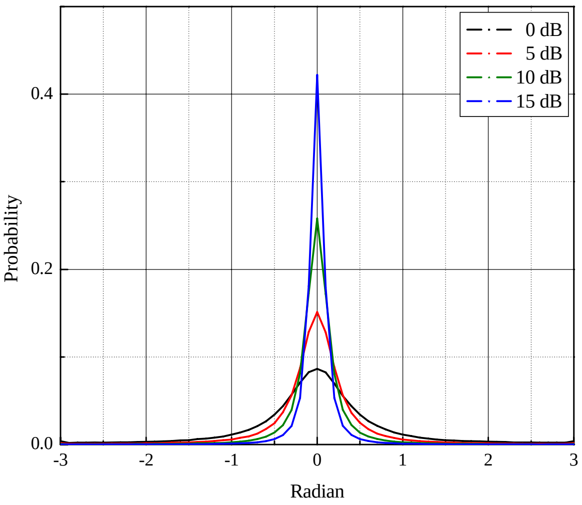

Fig. 3. Probability density function of phase estimation error for the proposed equivalent relay CSI estimation scheme, $K = 4, N_P = 8$.

Average bit error probability of AF cooperative diversity with relay selection is shown in Fig. 4 and Fig. 5, respectively. Compare with that having perfect CSI at the destination, it is shown that a proposed CSI estimation scheme has negligible performance degradation. It is shown that, at BER of 10^{-4}, 1 dB SNR loss is occurred when the length of pilot signal is 8. As the length of a pilot signal is increased, it is shown that the performance loss is reduced. It is also shown that channel estimation error does not depend on the number of relays in a wireless network. Additionally, all equivalent relay CSI is obtained only two pilot signal transmission phases.

Fig. 6 shows that normalized mean squared error (NMSE) of the proposed channel estimation scheme. It is shown that NMSE of estimation error is independent on the number of relays because of the orthogonal property of pilot signals.

V. CONCLUSIONS

In this paper, we propose a new channel estimation scheme for AF cooperative diversity with relay selection. It is required to know equivalent relay CSI at the destination to select the best relay among multiple relays. In order to obtain equivalent relay CSI at the destination, $K+1$ pilot signal transmission phases are required to estimate both a direct path and K equivalent relay CSI. However, as the number of relay is increased, the more pilot signal transmission phases are required. Increasing number of CSI estimation phases leads to an unavoidable increment of frame overhead. Consequently, exact estimation of a channel is difficult within coherence time of a channel.

We suggest a new channel estimation scheme for AF cooperative diversity with relay selection. By using orthogonal pilot signals, $K+1$ pilot signal transmit phases are reduced into two phases. Furthermore, estimation error does not depend on the number of relays. Numerical results show that the proposed channel estimation scheme provides accurate channel estimation for AF cooperative diversity with relay selection.

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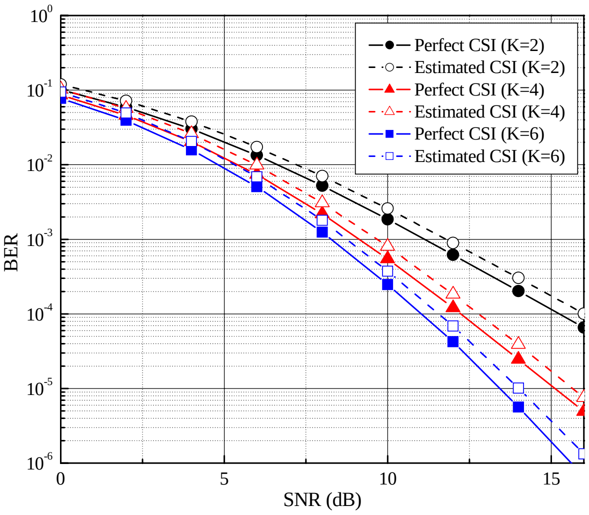

Fig. 4. Bit error rate of AF cooperative diversity with relay selection. $N_P = 8$, $N_D = 256$, and QPSK modulation.

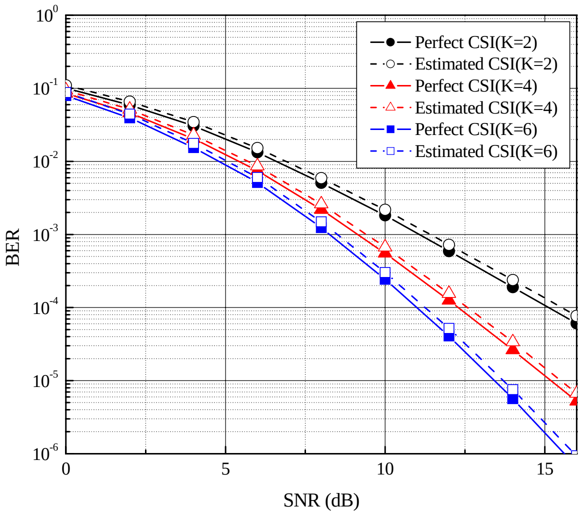

Fig. 5. Bit error rate of AF cooperative diversity with relay selection. $N_P = 16$, $N_D = 256$, and QPSK modulation.

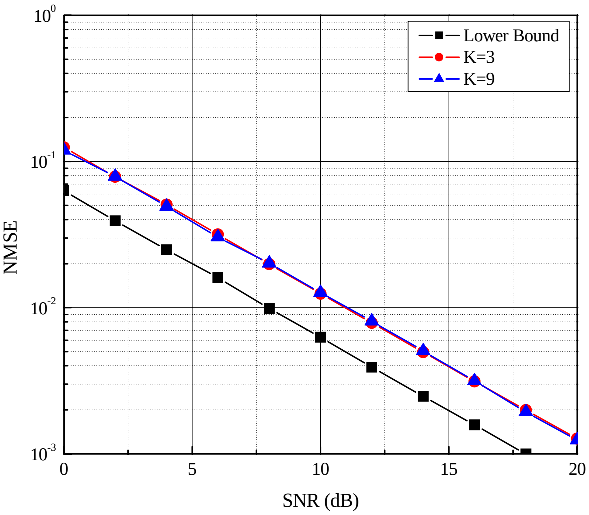

Fig. 6. Normalized mean squared error of the proposed channel estimation scheme. Lower bound denotes that the case of no additive noise at each relay.