하이브리드 광탄성법에 의한 혼합모드 광탄성 응력해석
Measurement of Mixed-mode Stress Intensity Factors by Using Hybrid Photoelasticity

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1. Introduction

Stress intensity factor is used in fracture mechanics to accurately predict the stress state near the tip of a crack caused by a remote load or residual stresses. Lots of theoretical and experimental studies show that the stress intensity factor is one of most important parameter for crack growth and propagation. In this paper, the hybrid method is employed[1][2]. At first, the isochromatic data of given points are calculated by finite element method and are used as input data of complex variable formulations. Then the numerical model of specimen is transformed from the physical plane to the complex plane by conformal mappings. The stress field is analyzed and mixed-mode stress intensity factors are calculated on this complex plane. The results are also calculated by finite element method and theoretical method and compared with each other.

2. Theory Formulation

2.1 Basic Equations

The present technique employs general expressions for the stress functions with traction-free conditions which are satisfied at the geometric discontinuity using conformal mapping and analytical continuation. In the absence of body forces and rigid body motion, the stresses under isotropy plane can be written as [3][4]

$$\sigma_x = 2\,\mathrm{Re}[\mu_1^2 \frac{\phi'(\xi_1)}{\omega_1'(\xi_1)} + \mu_2^2 \frac{\psi'(\xi_2)}{\omega_2'(\xi_2)}] \qquad (1a)$$

$$\sigma_y = 2\,\mathrm{Re}[\frac{\phi'(\xi_1)}{\omega_1'(\xi_1)} + \frac{\psi'(\xi_2)}{\omega_2'(\xi_2)}] \qquad (1b)$$

$$\tau_{xy} = -2\,\mathrm{Re}[\mu_1 \frac{\phi'(\xi_1)}{\omega_1'(\xi_1)} + \mu_2 \frac{\psi'(\xi_2)}{\omega_2'(\xi_2)}] \qquad (1c)$$

where the complex material parameters $\mu_j (j = 1,2)$ are the roots of the characteristic equation $S_{11}\mu^4 + (2S_{12} + S_{66})\mu^2 + S_{22} = 0$ for an isotropic material under plane stress and $S_{ij}(i, j = 1, 2, 6)$ are the elastic compliances. The two complex stress functions $\phi(\zeta_1)$ and $\psi(\zeta_2)$ are related to each other by the conformal mapping and analytic continuation. For a traction-free physical boundary, the two functions within sub-region Ω can be written as Laurent expansions, respectively. [3][4]

$$\phi(\zeta_1) = \sum_{k=-m}^{m} \beta_k \xi_1^k \qquad (k \neq 0) \qquad (2a)$$

$$\psi(\zeta_2) = \sum_{k=-m}^{m} (\overline{\beta_k} B \xi_2^k + \beta_k C \xi_2^k) \qquad (2b)$$

where complex quantities $B = (\overline{\mu_2} - \overline{\mu_1})/(\mu_2 - \overline{\mu_2})$, $C = (\overline{\mu_2} - \mu_1)/(\mu_2 - \overline{\mu_2})$, depending on material properties.

The coefficients of Eqs. (2) are $\beta_k = b_k + ic_k$, where b_k and c_k are real numbers. In addition to satisfying the traction-free conditions on the crack boundary Γ, the stresses of Eqs. (1) associated with these stress functions $\phi(\zeta_1)$ and $\psi(\zeta_2)$ satisfy equilibrium and compatibility.

Combining Eqs. (1), (2) gives the stress through regions Ω in matrix form $\{\sigma\} = [V]\{\beta\}$, where $\{\sigma\} = \{\sigma_x, \sigma_y, \tau_{xy}\}^T$, $\{\beta\}^T = \{b_{-m}, c_{-m}, \cdots, b_m, c_m\}$ and $[V]$ is a rectangular coefficient matrix whose size depends on material properties, positions and the number of terms m of the power series expansions of Eqs. (1).

$$V(i, j) = (-1)^{i-1}(2k)(\mathrm{Re}((\mu_1^{i-1})\frac{\xi_1^{k-1}}{\omega_1'(\xi_1)} + \mu_2^{i-1}\frac{(-B\xi_2^{-k-1} + C\xi_2^{k-1})}{\omega_2'(\xi_2)}) \qquad (3a)$$

$$V(i, j+1) = (-1)^{i-1}(2k)(\mathrm{Im}((-\mu_1^{i-1})\frac{\xi_1^{k-1}}{\omega_1'(\xi_1)} + \mu_2^{i-1}\frac{(-B\xi_2^{-k-1} - C\xi_2^{k-1})}{\omega_2'(\xi_2)}) \qquad (3b)$$

2.2 Stress Intensity Factor

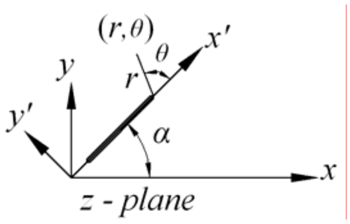


Fig.1 Coordinate system of the straight crack

As shown as Fig.1, the crack lies along the x'-axis in the physical z-plane and (r, Θ) are the local polar coordinates measured from the crack tip. When Θ=0 and $r<<a$, where a is the half of crack length, the stress intensity factor of mode I and Mode II is determined as follows:

$$K_I = \sigma_{y'}\sqrt{2\pi r} \qquad (4a)$$

$$K_{II} = \tau_{x'y'}\sqrt{2\pi r} \qquad (4b)$$

where $\sigma_{y'}$ and $\tau_{x'y'}$ are obtained from $\{\sigma\} = [V]\{\beta\}$ and coordinate transformation.

3. Experiment and Analysis

3.1 Photoelasticity Experiment

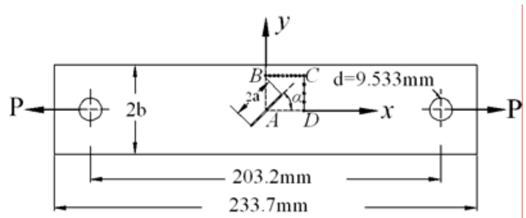
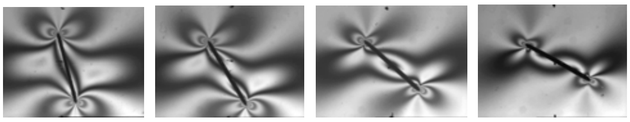
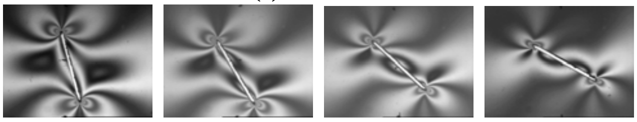


Fig.2 Finite-width uni-axially loaded tensile plate containing an inclined crack

In this experiment, a PSM-1 [5] plate shown as Fig.2 is subjected to the uni-axial tension P=3.05 MPa, the thickness of specimen is 3.175 mm, material fringe constant f_σ =7005N/m, Young's modulus E=2482 MPa, Poisson's ratio v= 0.38, the width of crack is 0.5 mm. Fig. 3 shows the original light-field and dark-`field fringe pattern of the loaded tensile plate containing an inclined crack.



(a) Dark field



(b) Light field

Fig.3 Original fringe patterns with different inclined angle of 15°, 30°, 45° and 60°

3.2 FEM Analysis

In this paper, a common FEM software ABAQUS[6] is used to discretize the specimen into two kinds of elements, CPS3 (3-node linear plane stress triangle element) and CPS4R (4-node bilinear plane stress quadrilateral element). The finite element model is shown as Fig. 4.

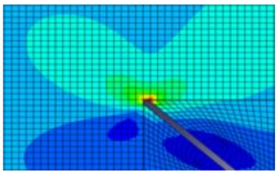
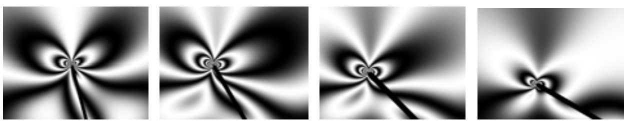


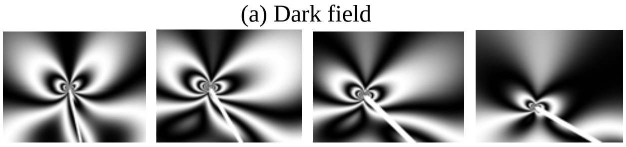
Fig. 4 ABAQUS discretization of the load tensile plate of Fig. 2

In order to obtain the input data of hybrid method, the isochromatic fringe order of given points on *B-C* line and *C-D* line are necessary. According to stress-optic law, the value of fringe order can be expressed by the stresses of those points. The stress intensity factor of Mode I and Mode II for the plate shown in Fig. 2 were calculated by ABAQUS.

3.3 Hybrid Method Analysis



(a) Dark field



(b) Light field

Fig.5 Regenerated fringe patterns with different inclined angle of 15˚ , 30˚ , 45˚ and 60˚

Using the stresses of given points calculated by FEM, the isochromatic fringe orders are obtained and the fringe patterns are regenerated. In order to accurately compare calculated fringes with experimental ones, both actual and regenerated photoelastic fringe patterns are two times multiplied and sharpened shown as Fig. 5 by digital image processing.

Table 1 Comparison of stress intensity factor

Stress Intensity factor	Angle	Hybrid	FEM	Equation[7]
$\frac{K_I}{\sigma_0\sqrt{\pi a}}$	15	0.982	0.986	0.994
	30	0.782	0.804	0.799
	45	0.492	0.526	0.532
	60	0.248	0.264	0.267
$\frac{K_{II}}{K_I}$	15	0.274	0.262	0.268
	30	0.558	0.574	0.577
	45	1.062	1.006	1.000
	60	1.648	1.746	1.732

From the figures, we can see that the regenerated fringes of *m*=1 by hybrid method are quite comparable to actual fringes. The mixed- mode stress intensity factor of crack with different inclined angle are obtained by hybrid method, FEM and theoretical formulation shown as table 1.

4. Discussions and Conclusions

In this study, we use only isochromatic data with their respective coordinates to easily obtain stress field distribution and stress intensity factor at the geometric discontinuity. From the tables and figures presented above, we can see that the results calculated by three kinds of different method are well agreed with each. Considering the experimental and calculated errors, the technique presented in this paper is effective and reliable.

Here we utilized FEM to obtain the isochromatic data and coordinate information of given points. The use of hybrid method has a potential future and the results attained in this study can be used for bench mark test in theoretical simulation and experiment.

Acknowledgement

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