ESPI 를 이용한 beam 의 포아송 비의 결정
Determination of Poisson's ratio of a beam by ESPI

*아흐터나심[1], 정현철[1], 장호섭[2], 김동수[3], #김경석[1]
*Akhter Naseem[1], H.C. Chung[2], H.S. Chang[3] , D.S. Kim[4] and #K.S.Kim(gsckim@chosun.ac.kr)[2]
[1]조선대학교 기계설계공학과, [2]조선대학교 레이저센터, [3]조선대학교 대학원 첨단부품소재공학과

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1. Introduction

It is important to determine the mechanical properties of thin materials in order to predict the performances of the electromechanical devices. The Poisson's ratio is used in almost all area of stress analysis and structural dynamics. Electronic speckle pattern interferometry is an appropriate method for the non-destructive inspection of technical and cultural objects [1, 2]. This method is based on the digital correlation of two speckled wave fronts representing two states of the object under test, loaded and unloaded condition [3].

Our study is centered over the determination of Poisson's ratio of a cantilever beam by electronic speckle pattern interferometry (ESPI). We described in this paper the theoretical background of the method of analysis and apply the theory to then out-of-plane schematic arrangements to obtain the resonance frequency for determine Poisson's ratio. We used the principle that one can determine the Poisson's of a material by means of its vibration characteristics which use the beam equation theory to link with the specimen's natural frequency of vibration. In principle, a harmonically vibrating object has the maximum surface displacement at a resonance frequency. The maximum vibrating amplitude at each vibration mode shape is a clue to find the resonance frequency.

The Poisson's ratio of the beam can be easily determined after the resonance frequencies of the bending and torsional vibration modes are measured by ESPI with the advantages of fast, high resolution, full-field non- destructive, non-contact and real-time measurement technique [4].

2. Review of ESPI

Speckle interferometry has been a powerful technique in the measurement of vibration mode shape and surface displacement. Their recording and reconstruction processes were fairly simple [5, 6]. In order to visualize a vibration mode shape, the interferometer is operated in the time-average mode that's why it is called time-averaged electronic speckle pattern interferometry. To understand the formation of fringe pattern, consider the intensity of before displacement I_{before} as given by

$$I_{before} = I_r + I_o + 2\sqrt{I_r I_o}\cos\phi \tag{1}$$

where, I_r, I_o and φ represent the reference beam, object beam, and the phase difference before the displacement respectively. The displacement $\{a(t) = a_o \sin(wt)\}$ is a periodic function of time where a_o is the amplitude of the vibrating object and at a given instant t, the irradiance in the image plane is given by

$$I(t) = I_r + I_o + 2\sqrt{I_r I_o}\cos[\phi + \frac{4\pi}{\lambda}a(t) \tag{2}$$

The intensity is averaged over time τ to obtain

$$I\tau = I_r + I_o + 2\sqrt{I_r I_o}\, J_o^2(\frac{4\pi}{\lambda}a_o)\cos\phi \tag{3}$$

where J_o is the zero-order Bessel function of the first kind. The value of I_τ averaged over many speckle patterns is constant over the whole image, but the constant of the speckle is seen to vary as the value of the J_o^2 function varies.

3. Determination of Poisson's ratio

The Young's modulus (E), Shear modulus (G) and Poisson's ratio (ν) are related by the following equation [7].

$$\upsilon = \frac{E}{2G} - 1 \tag{4}$$

According to the vibration analysis of Euler-Bernoulli beam model, the Young's modulus, and the nth natural frequency of the bending vibration of a cantilever beam, assuming that beam thickness and cross-sectional area is uniform throughout its length, has the following relationship [8]:

$$E = \frac{48\pi^2 \rho f_n^2}{(\beta_n)^4 h^2} \tag{5}$$

where n=1,2,3,4,…. and the number β_n depends upon boundary conditions of the clam-free beam equation, L is the length of the beam, E is the elastic modulus, ρ is the mass density and h represents the thickness of the beam. In addition, the shear modulus (G) and the natural frequency of the first torsional vibration mode (f_T) of the beam have the following relation [9]:

$$G = \frac{4\rho L^2 (b^2 + h^2) f_T^2}{3kh^2} \tag{6}$$

Where b is the width and 'k' is a coefficient that depends on the ration (b/h). Now the relation between Poisson's ratio and resonance frequencies of the beam can be expressed as using above equations

$$\upsilon = \frac{18\pi^2 k L^2 (f_n/f_T)^2}{(L\beta_n)^4 h^2 [1 + (b/h)^2]} - 1 \tag{7}$$

Therefore, one can determine the Poisson's ratio after the getting resonance frequencies of bending and torsional mode of the cantilever beam.

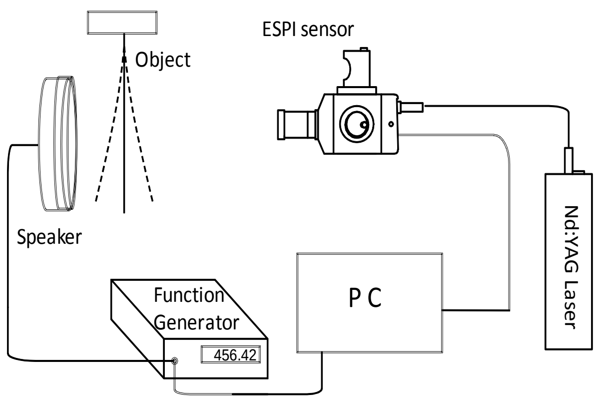


Fig.1 The schematic of experimental set-up

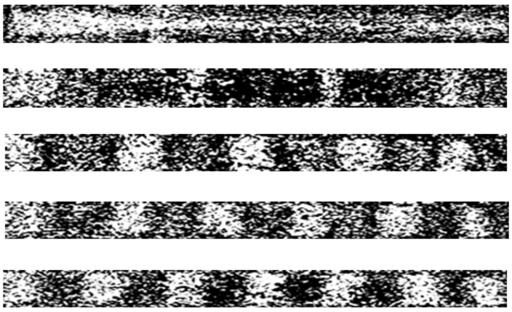
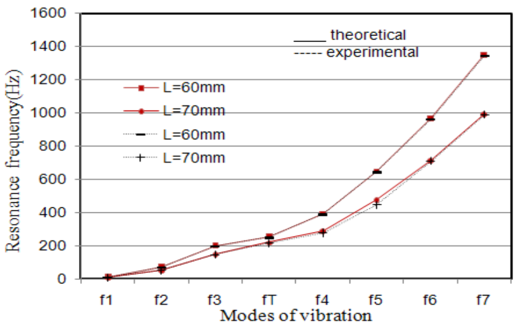
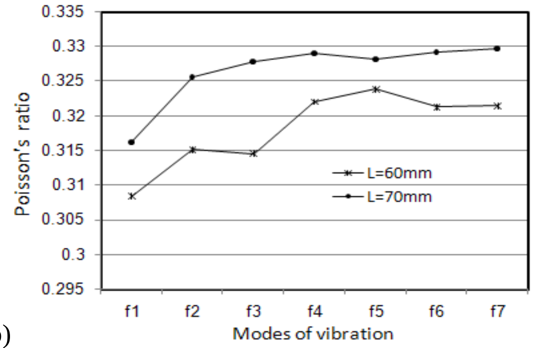


Fig.2 Vibration mode shapes by ESPI.



(a)



(b)

Fig.3. Comparison of resonance frequency, Poisson's ratio with vibration modes, respectively, (a) Comparison between theoretical and measured resonance frequencies (b) Variation of Poisson's ratio with vibration

4. Experiment and results

The working principle of the experimental set-up is shown in fig.1. It consists of Nd: YAG laser system of wavelength ($\lambda = 532\text{nm}$), ESPI sensor, Excitation device, Image processing program. The specimens were excited by a loud speaker from the back, and the exciting frequency was controlled by a function generator. The illuminating source was delivered by an optical fiber into a sensor, and the out of plane displacement sensitive interferometer was set up inside the sensor. The history of the vibration mode shape, according to the change of exciting frequency, was observed, and the resonance frequency at the maximum fringe order and the maximum displacement of the object were distinguished readily. In present research, the specimens used for experiment were pure Aluminium (99.99%) of lengths(L) 60mm and 70mm, respectively. The thickness (h) and width (b) of each specimen were 0.05mm and 5mm respectively. The Poisson's ratio, Elastic modulus, Shear modulus (G) and mass density of pure Al (99.99%) are taken to be 0.33, 70GPa, 26.316GPa and 2710 kg/m^3 respectively. Properties of materials generally vary depending upon manufacturing process, chemical composition, internal defects, temperature, and dimensions of test specimens, and other factors.

We used the principle that one can determined the elastic properties of a material by means of its vibration behavior which use the beam equation theory to link with the specimen natural frequency of vibration. The amplitude of the object is directly proportional to speckle pattern interferometry fringe order. The number of the speckle pattern interferometry fringe order is a clue to find the resonance frequency at each vibration mode shape. Fig.2 shows different modes of vibration shapes obtained by ESPI for the specimen of surface area (60mm x 5mm x 0.05mm). In this experiment, theoretically calculated resonance frequencies and measured by holography are determined and compared. The variation of resonance frequency with modes of vibration is shown in fig.3 (a). The Poisson's ratio has been investigated by using beam equation (7). The variation of Poisson's ratio with modes of vibration is shown in fig.3 (b). The average Poisson's ratio for (L=60mm, 70mm) were found to be 0.318 and 0.326, respectively. The average Poisson's ratio for specimen (60mm, 70mm) was found to be 0.321. There was some errors in the result and the variation of error was found to be 1~ 4%.

5. Conclusion

The mechanical properties, namely Poisson's ratio is a very important parameter to determine and analysis of stress and strain of a material. In this study, the method used laser ESPI to study and determine Poisson's ratio and its variation with vibration modes of a vibrating Aluminium beam. The Poisson's ratio can be easily determined by classical vibrating beam theory after getting resonance frequencies from laser holographic interferometry, although in many cases it is very difficult to determine precisely and conveniently. The laser holographic interferometry approach is very simple; it can be applied as a supplement to the measurement of materials properties.

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