경사크랙 팁 주위의 광탄성 응력해석

Photoelastic Stress Analysis of an Inclined Crack Tip

첸레이 [1], *서 진 [1], 양민복 [1], 김명수 [2], #백태현 [3]

L. Chen[1], *C. Seo[1], M. B. Yang[1], M. S. Kim[2], #T. H. Baek (thbaek@kunsan.ac.kr)[3]

[1] 군산대학교 대학원 기계공학과, [2] 군산대학교 전자정보공학부, [3] 군산대학교 기계공학부

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1. Introduction

Many structural and mechanical parts with cracks fractured at the stress which is much lower than the ultimate strength because the stresses in the vicinity of the crack tip are always much higher than those far away from it. Due to the complexity of the engineering problems, it is difficult to obtain the stress field around the crack tip directly by theoretical derivation and photoelasticity is a conventional method. But it is a kind of experimental method and can not provide very high-precision results[1][2]. In this paper, the hybrid method is employed to calculate full-field stress around the crack tip in uni-axially loaded finite width tensile plate and to compare with experimental results and FEM results. In order to conveniently compare those values with each, both actual and regenerated photoelastic fringe patterns are two times multiplied and sharpened by digital image processing.

2. Theory Formulation

In the absence of body forces and rigid body motion, the stresses under isotropy plane can be written as [3][4]

$$\sigma_x = 2\,\mathrm{Re}[\mu_1^2 \frac{\phi'(\xi_1)}{\omega_1'(\xi_1)} + \mu_2^2 \frac{\psi'(\xi_2)}{\omega_2'(\xi_2)}] \tag{1a}$$

$$\sigma_y = 2\,\mathrm{Re}[\frac{\phi'(\xi_1)}{\omega_1'(\xi_1)} + \frac{\psi'(\xi_2)}{\omega_2'(\xi_2)}] \tag{1b}$$

$$\tau_{xy} = -2\,\mathrm{Re}[\mu_1 \frac{\phi'(\xi_1)}{\omega_1'(\xi_1)} + \mu_2 \frac{\psi'(\xi_2)}{\omega_2'(\xi_2)}] \tag{1c}$$

where the complex material parameters $\mu_j (j=1,2)$ are the roots of the characteristic equation $S_{11}\mu^4 + (2S_{12} + S_{66})\mu^2 + S_{22} = 0$ for an isotropic material under plane stress and $S_{ij}(i, j = 1, 2, 6)$ are the elastic compliances.

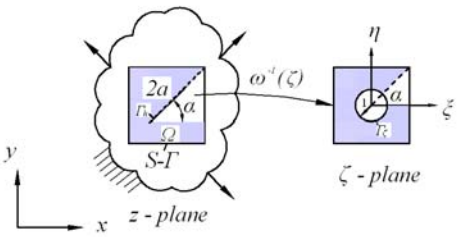


Fig.1 Conformal mapping of a crack in the physical z-plane into a unit circle in the ζ-plane.

The inverse of the mapping function ω namely ω^{-1}, maps the geometry of interest from the physical *z*-plane into the ζ-plane ($\zeta_l = \xi + \mu_l \eta$). For isotropic materials, the conformal transformations between unit circle in the ζ_l-plane and the crack in the *z*-plane of length $L = a/2$ are shown in Fig. 1 and are given by

$$\omega_j = \frac{a}{2}(\cos\alpha + \mu_j \sin\alpha)(e^{-i\alpha}\xi_j + e^{i\alpha}\xi_j^{-1}) \tag{2a}$$

$$\xi_j = \frac{e^{i\alpha}\left\{\omega_j \pm \sqrt{\omega_j^2 - a^2(\cos\alpha + \mu_j \sin\alpha)^2}\right\}}{a(\cos\alpha + \mu_j \sin\alpha)} \tag{2b}$$

3. Experiment and Analysis

3.1 Photoelasticity Experiment

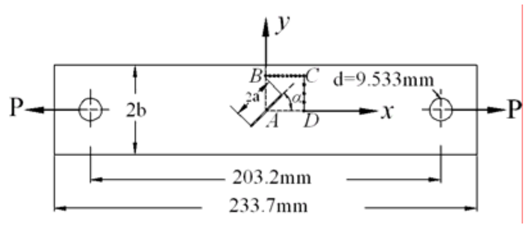
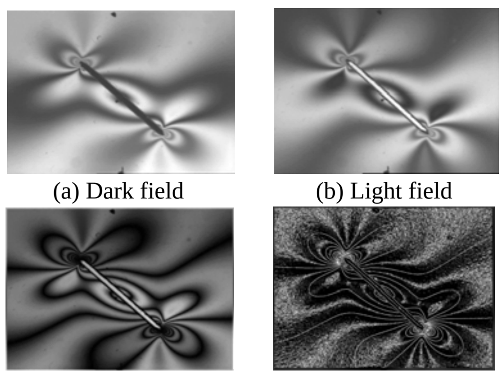


Fig.2 Finite-width uni-axially loaded tensile plate containing an inclined crack

In this experiment, a PSM-1 [5] plate shown as Fig.2 is subjected to the uni-axial tension *P=3.05 MPa*, the thickness of specimen is 3.175 *mm*, material fringe constant f_σ *=7005N/m*, Young's modulus *E=2482 MPa*, Poisson's ratio *v= 0.38*, the width of crack is 0.5 *mm*.



(a) Dark field (b) Light field

(c) Doubled pattern (d) Sharpened pattern

Fig.3 Experiment fringe patterns of 45° inclined crack

Fig. 3 shows the original light-field and dark-field fringe pattern of the loaded tensile plate containing an inclined crack. In order to accurately compare calculated fringes with experimental ones, those fringe patterns are two times multiplied and sharpened by digital image processing.

3.2 FEM Analysis

In this paper, a common FEM software ABAQUS[6] is used to discretize the specimen into two kinds of elements, CPS3 (3-node linear plane stress triangle element) and CPS4R (4-node bilinear plane stress quadrilateral element). The finite element model is shown as Fig. 4.

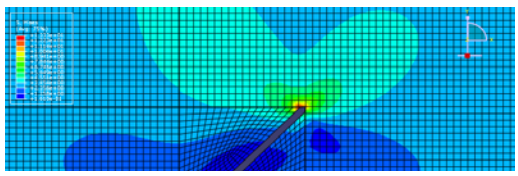
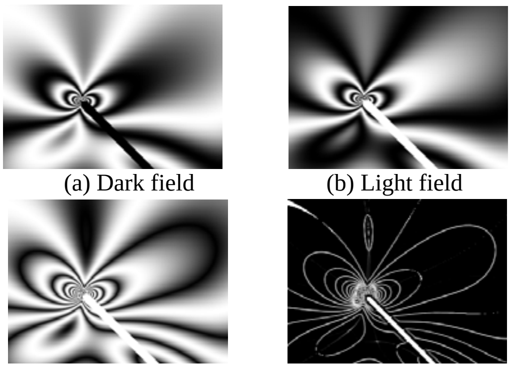


Fig. 4 ABAQUS discretization of the load tensile plate of Fig. 2

In order to obtain the input data of hybrid method, the stress of given points on *B-C* and *C-D* lines are necessary. By hybrid method, the unknown coefficients can be obtained and then we can know the stress field in this region. From ABAQUS output file, the value of the stress is conveniently obtained.

3.3 Hybrid Method Analysis



(a) Dark field (b) Light field

(c) Doubled pattern (d) Sharpened pattern

Fig.5 Regenerated Fringe patterns of 45° inclined crack

By the stress of given points calculated by FEM and equations presented above, the isochromatic fringe orders are obtained and reconstructed fringe pattern can be seen in the right haft of Fig. 5. From the figures, we can see that the regenerated fringes of *m*=1 by hybrid method are quite comparable to actual fringes [7][8].

Table 1 Comparison of input and calculated fringe orders

No	x (mm)	y (mm)	N-inp	N-cal	Error (100%)
1	-0.197	0.071	1.89	1.8969	0.1197
2	-0.197	0.106	1.97	2.1165	7.3507
3	-0.197	0.213	2.28	2.3616	3.6082
4	-0.197	0.248	2.38	2.3337	-1.96
5	-0.197	0.284	2.47	2.3252	-5.934
6	-0.197	0.319	2.55	2.4311	-4.71
7	-0.197	0.354	2.62	2.7754	6.041
8	-0.161	0.354	2.66	2.7542	3.4279
9	-0.126	0.354	2.72	2.7347	0.5557
10	-0.091	0.354	2.78	2.6803	-3.713
11	-0.055	0.354	2.86	2.689	-5.894
12	-0.02	0.354	2.95	2.8885	-2.038
13	0.016	0.354	3.05	3.1877	4.6401
14	0.051	0.354	3.15	3.2855	4.2231
15	0.087	0.354	3.27	3.2329	-1.084
16	0.122	0.354	3.39	3.362	-0.755
17	0.158	0.354	3.5	3.6966	5.5441
18	0.158	0.319	3.57	3.3971	-4.938

A quantitative check on the quality of fit between input and calculated displacements of given points shown in Table 1 is made by using a simple type of statistical parameter, such as the standard deviation (*SD*) of percentage error. For a predetermined point, the input fringe value (N_{inp}) is known. The calculated fringe value (N_{cal}) is also determined at the same point. Then, the percentage error (E) between the calculated and the input fringes at any point is $E = (D_{cal} - D_{inp}) / D_{inp} \times 100(\%)$.

For "*n*" data points, the standard deviations of the percentage error can be calculated from

$$SD = \sqrt{\frac{1}{n-1}\left[\sum_{i=1}^{n} E_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} E_i\right)^2\right]} \qquad (3)$$

4. Discussions and Conclusions

This paper presents a new attempt to use the hybrid method to evaluate stress concentration around crack tip in an isotropic tensile-loaded plate (PSM-1). The fringe order along A-B line is calculated through the stress of given points. This calculation has been made handy through complex power series representation (Laurent series) implemented on a computer program for high-speed processing.

In this study, we use only the stress of given points with their respective coordinates and easily obtain the stress field and isochromatic fringe order around crack tip. The results were obtained at m =1 with less than 4.3% difference from the results obtained from FEM. Comparing reconstructed fringe pattern with photoelasticity experiment images, we can see that they are nearly same with each other. Good agreement of the results indicates that the hybrid method utilized in this paper is properly reliable and the results can be used as bench mark in the theoretical studies and other experiments.

Acknowledgement

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