# 전기 구동 이동 로봇을 위한 적응 관측기 설계

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## Adaptive observer design for electrically driven mobile robots

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**Abstract** - In this paper, we propose an adaptive observer for electrically driven mobile robots in the presence of parametric uncertainties. To estimate the unmeasured velocities of the mobile robot, we develop an adaptive observer using the transformation matrices which can solve the difficulty caused by the quadratic velocity terms depending on the uncertain parameters. Using the Lyapunov stability theory, we prove that all signals in a closed-loop system converge to zero. Finally, the simulation results demonstrate the effectiveness of the proposed observer system.

#### 1. Introduction

The tracking control of nonholonomic mobile robots is a challenging problem due to the nature of nonholonomic constraints. Thus, many studies have been reported on the tracking control of nonholonomic mobile robots. The kinematic model was firstly used to design the controller whose input is the velocity [1]-[3]. One method using the dynamic model has been proposed to obtain the torque as an input [4]-[6] because the controller based on the kinematic model requires the realization of the perfect velocity tracking. However, most of these works have ignored the dynamics coming from electric motors which should be required to implement the mobile robots in the real environment. That is, since the wheels of the mobile robot are driven by actuators, it is more realistic that the control input is the actuator input voltages. Hence some results using the actuator dynamics were reported [7],[8], but all these schemes, which are state-feedback control, require the velocity measurements which increase the cost and impose constraints on the achievable bandwidth.

In order to overcome the limitations of the state-feedback control, the output-feedback controllers for the robot manipulators have been proposed [9],[10]. However, many solutions proposed for the robot manipulators cannot directly be applied to the mobile robots due to the quadratic cross terms of unmeasured velocities and the nonholonomic constraints of the mobile robots. To solve these problems, Besancon [11] proposed an output-feedback controller relied on the solution of the differential equation, which may not exist, and Do et. al [12] proposed a coordinate transformation to cancel the velocity cross terms in the mobile robot dynamics. However, all these schemes require the precise knowledge of the dynamic parameters which may not be available. Thus the adaptive output-feedback controller, which is robust to the uncertain system parameters, is required, but the proposed methods in [11], [12] cannot be applied to the adaptive version because the quadratic velocity terms depend on the parameters.

In this paper, we propose an adaptive observer for electrically driven mobile robots with uncertainties, which is necessary for implementing the adaptive output-feedback controller. To estimate the unmeasured velocities, we develop an adaptive observer which is robust to the parametric uncertainties. In the design of the adaptive observer, the transformation matrices are introduced to solve the difficulty caused by the quadratic velocity terms depending on the uncertain parameters.

This paper is organized as follows: Section II introduces the model of nonholonomic mobile robots incorporating actuator dynamics. In Section III, we propose an adaptive observer for electrically driven mobile robots with uncertainties. In Section IV, the stability of the proposed observer system is analyzed based on the Lyapunov stability theory. Section V presents some simulation results and Section VI gives some conclusions.

#### 2. Problem Statement

The kinematics and dynamics of nonholonomic mobile robots are described by the following differential equations:

$$\dot{q} = J(q)z = 0.5r \begin{bmatrix} \cos \phi \cos \phi \\ \sin \phi \sin \phi \\ R^{-1} R^{-1} \end{bmatrix} \begin{bmatrix} z_r \\ z_l \end{bmatrix}$$
(1)

$$\dot{Mz} + C(\dot{q})z + Dz = \tau \tag{2}$$

where  $q = [x, y, \phi]^T$ ; x, y, and  $\phi$  are the position and orientation of the mobile robot,  $z = [z_r, z_l]^T$ ;  $z_r$  and  $z_l$  represent the angular velocities of right and left wheels, respectively. R is the half of the width of the mobile robot and r is the radius of the wheel,

$$\begin{split} &M\!=\!\begin{bmatrix}m_{11}\ m_{12}\\m_{12}\ m_{11}\end{bmatrix},\ C\!\left(\!\dot{q}\!\right)\!=\!0.5R^{-1}r^2m_cd\!\begin{bmatrix}0&\dot{\phi}\\-\dot{\phi}&0\end{bmatrix},\\ &D\!=\!\begin{bmatrix}d_{11}\ 0\\0\ d_{22}\end{bmatrix},\\ &m_{11}\!=\!0.25R^{-2}r^2\left(mR^2\!+\!I\right)\!+\!I_w,\\ &m_{12}\!=\!0.25R^{-2}r^2\left(mR^2\!-\!I\right),\\ &m\!=\!m_c\!+\!2m_w,I\!=\!m_c\!d^2\!+\!2m_wR^2\!+\!I_c\!+\!2I_m,\\ &\tau\!=\!\left[\tau_r,\ \tau_l\right]^T. \end{split}$$

In these expressions, d is the distance from the center of mass  $P_c$  of the mobile robot to the middle point  $P_0$  between the right and left driving wheels. $m_c$  and  $m_w$  are the mass of the body and the wheel with a motor, respectively.  $I_c$ ,  $I_w$ , and  $I_m$  are the moment of inertia of the body about the vertical axis through  $P_c$ , the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively. The positive terms  $d_{ii}$ , i=1,2, are the damping coefficients.  $\tau$  is the control torque applied to the wheels of the mobile robot. In addition, the actuator dynamics, which is assumed that both motors are dc motors, can be represented as follows (suffix r stands for right wheel actuator):

$$\begin{cases} \tau_{mr} = K_{Tr} i_r \\ u_r = R_{ar} i_r + L_{ar} \dot{i}_r + K_{Er} \dot{\theta}_r \end{cases}$$
(3)

where  $\tau_{mr}$  is the torque generated by the right motor,  $K_{Tr}$  is the motor torque constant,  $i_r$  is the current,  $u_r$  is the input voltage,  $R_{ur}$  is the resistance,  $L_{ar}$  is the inductance,  $K_{Er}$  is the back electromotive force coefficient, and  $\dot{\theta}_r$  is the angular velocity of the right motor. The relationship between the dc motor and the mobile robot wheel can be written as

$$N = \frac{\theta_r}{z_r} = \frac{\tau_r}{\tau_{mr}} \tag{4}$$

where N is the gear ratio. Using (4), the dynamic model of dc motors (3) can be rewritten as

$$\begin{cases} \tau = NK_T i \\ u = R_a i + L_a i + NK_E z. \end{cases}$$
(5)

In (5), it is assumed that  $K_{Tr} = K_{Tl} = K_T$ ,  $R_{ar} = R_{al} = R_a$ ,

$$\begin{split} L_{ar} = L_{al} = L_a, ~~ K_{Er} = K_{El} = K_E ~~,~~ i = \begin{bmatrix} i_r, ~~ i_l \end{bmatrix}^T ~~ \text{is the current vector} \\ \text{and} ~~ u = \begin{bmatrix} u_r, ~~ u_l \end{bmatrix}^T ~~ \text{is the actuator input voltage vector.} \end{split}$$

### 3. Adaptive observer design

In order to design an adaptive observer, we transform (1), (2), and (5) into more appropriate representations as follows:

$$\dot{q} = \overline{J}(q)\eta \dot{Q}\eta = -C_q(\eta)\eta - D_q\eta + K_q i L_p i = -R_p i - H_1\eta + Pu$$
(6)

Let us define the state variables as  $x_1 = q$ ,  $x_2 = \eta$ , and  $x_3 = i$ . Then, (6) can be expressed in the following state-space form:

$$\begin{split} \dot{x}_1 &= \overline{J}(x_1)x_2 \\ Q\dot{x}_2 &= -C_q(x_2)x_2 - D_q x_2 + K_q x_3 \\ L_p x_3 &= -R_p x_3 - H_1 x_2 + P u \end{split}$$
 (7)

where  $x_1 = \begin{bmatrix} x_{11}, x_{12}, x_{13} \end{bmatrix}^T$ ,  $x_2 = \begin{bmatrix} x_{21}, x_{22} \end{bmatrix}^T$ , and  $x_3 = \begin{bmatrix} x_{31}, x_{32} \end{bmatrix}^T$ . We now design an adaptive observer for electrically driven mobile robots. Under the new z-coordinates, we propose the adaptive observer for the system (7) as follows:

$$\begin{split} &z_1 = J(x_1)(z_2 + l_1H_2(x_1 - z_1)) + L_1(x_1 - z_1) \\ &\dot{z}_2 = \hat{Q}^{-1}[-\hat{C}_q(z_2 + l_1H_2(x_1 - z_1))(z_2 + l_1H_2(x_1 - z_1)) \\ &-\hat{D}_q(z_2 + l_1H_2(x_1 - z_1)) + \hat{K}_qx_3 + L_2(x_1 - z_1) \\ &+ L_3(x_3 - z_3)] + l_1H_2L_1(x_1 - z_1) \\ &\dot{z}_3 = \hat{L}_p^{-1}[-\hat{R}_px_3 - H_1(z_2 + l_1H_2(x_1 - z_1)) + \hat{P}u + L_4(x_3 - z_3)] \end{aligned}$$

where  $L_i (i = 1, \dots, 4)$  are the observer gain matrices.

# 4. Simulations

In the simulation, we assume the parameters as follows: R=0.75 m, d=0.3m,  $m_c=30$ kg,  $m_w=1$ kg,  $I_c=15.625$ kg,  $I_w=0.005$ kg·m<sup>2</sup>,  $I_m=0.0025$ kg·m<sup>2</sup>,  $d_{11}=d_{22}=5$ m,  $R_a=1.6\Omega$ ,  $L_a=0.48$ H,  $K_E=0.019$  Vs/rad,  $K_T=0.2639$ N·m/A, and N=62.55. The simulation results are shown in Figs. 1 and 2. Fig. 1 shows the linear and angular velocities of the mobile robot. The observer errors are plotted in Fig. 2. From the simulation results, we can demonstrate the effectiveness of the proposed adaptive observer in the presence of uncertainties.



<Fig. 1> Linear and angular velocities of the mobile robot.

### 5. Conclusion

In this paper, an adaptive observer has been presented to observe the unmeasured velocities of electrically driven mobile robots with uncertainties. To solve the main difficulty caused by the quadratic velocity terms depend on the system parameters, we have transformed the mobile robot dynamics including actuator dynamics. Then, we have designed an adaptive observer to estimate the velocities of the mobile robots with uncertainties. From the Lyapunov stability theory, we have proved that all signals in the closed-loop system are converge to zero. Simulation results have verified the effectiveness of the proposed observer system in the presence of uncertainties.



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