

## 강인 적응성 슬라이딩을 이용한 PMSM 서보드라이브 시스템 제어기

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### Robust Adaptive Sliding Mode Controller for PMSM Servo Drives System

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**Abstract** - Dynamic friction and force ripple are the most predominant factors that affect the positioning accuracy of permanent magnet synchronous motor(PMSM) servo drives system, and it is desirable to compensate them in finite time with a continuous control law. In this paper, based on LuGre dynamic friction model, a robust adaptive skidding mode controller is proposed to compensate the nonlinear effect of friction and force ripple. The controller scheme consists of a PD component and a robust adaptive sliding mode controller for estimating the unknown system parameter. Using Lyapunov stability theorem, asymptotic stability analysis and position tracking performance are guaranteed. Simulation results well verify the feasibility and the effectiveness of the proposed scheme for high0precision motion trajectory tracking.

#### 1. Introduction

Nowadays, permanent magnet synchronous motors are receiving increasing attention for high performance applications because of their high torque to inertia ratio, superior power density, and high efficiency. In a PMSM servo drives system, the most predominant nonlinear effects are friction and force ripple. Recently, a large number of adaptive friction compensation techniques have been proposed to compensate nonlinear friction in servo drives systems. An adaptive compensation approach was developed for stick-slip friction. A nonlinear compensation technique that has a nonlinear proportional feedback control force was proposed for the regulation of one-degree freedom mechanical system. A ripple compensation scheme was proposed for the position control loop based on an identified first-order approximation of the ripple forces. In this paper, an adaptive sliding mode dynamic friction compensation method is proposed to compensate the friction and force ripple for tracking control with guaranteed high precision and smoothness of motion. To verify the performance of proposed compensation approach, tracking experiments are conducted for a ship-borne servo drives system. Experimental results show that our proposed controller can meet the requirements of both lower tracking error and higher positioning accuracy. This paper is organized as follows. Section II introduces our studied PMSM motor servo drives system used in ship-borne gun. Section III is dedicated to design an adaptive sliding mode compensation controller based on LuGre dynamic friction model. Section IV gives the simulation results. And section V concludes the paper.

#### 2. Dynamic model formation

##### 2.1 Dynamic System Model

The gun servo drives system we studied, which is used to track the trajectory of the target accurately, is an important segment of the ship-borne integrative fire command and control system. The performance of the servo drives system has direct effect on the performance, efficiency, and reliability of the whole fire command and control system. Therefore, the servo drives system with high precision positioning, high frequency response, and low velocity tracking is a key factor of improving ship-borne gun warfare capability. Taking various factors into account, we choose a PMSM system as our servomechanism, Generally, there are many nonlinear factors and uncertainties affecting the performance of the servo drive system. First major factor is force ripple, which comes from cogging force and magnet reluctance force, second major factor is friction, and third major factor is the disturbance of load force. In our servo drives system, compared with these three effects, other effects are

negligible and can be ignored. The simplified second-order differential equation for the servo drives system under the study is considered as a simple mass system with the dynamic friction and force ripple.

$$\ddot{x} = -\frac{K_m C_e}{JR} \dot{x} + \frac{K_m K_{PWM}}{JR} u - \frac{F_{friction}}{J} - \frac{F_{ripple}}{J} - \frac{F_{load}}{J} \quad (1)$$

where J is the moment of inertia, x the angular displacement and assumed to be second-order differentiable, u the control force, friction F the friction force, ripple F force ripple, load F applied load force, e C back EMF voltage, m K amount of force produced by the motor, R total resistance between any two phases, PWM K amplifier coefficient of the pulse-width modulation. Let us first define

$$b = \frac{JR}{K_m K_{PWM}}, \quad a = -\frac{C_e}{K_{PWM}}, \quad F_f = \frac{R}{K_m K_{PWM}} F_{friction}$$

$$F_f = \frac{R}{K_m K_{PWM}} F_{friction}, \quad F_{load} = \frac{R}{K_m K_{PWM}} F_{load} \quad (2)$$

Then Eq. (1) can be rewritten as

$$b\ddot{x} = a\dot{x} + u - F_f - F_r - F_l \quad (3)$$

The load force is assumed to be bounded within the unknown upper bound as follows

$$|F_{load}| < F_{IM}, \quad \forall t > 0 \quad (4)$$

Usually, the force ripple can be described by a sinusoidal function of the load position with a period of  $\omega$  and amplitude of r A

$$F_r = A_r \sin(\omega x + \phi) = A_{r1} \cos(\omega x) + A_{r2} \sin(\omega x) \quad (5)$$

##### 2.2 LuGre Friction Model

The LuGre model has been established to capture the various physical phenomena such as the downward bend in the low-velocity region, the pre-sliding displacement, and the frictional lag[2]. The dynamic model is described as follows

$$F_f = \sigma_0 z + \sigma_1 \frac{dz}{dt} + \sigma_2 \dot{x} \quad (6)$$

$$\frac{dz}{dt} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x})} z \quad (7)$$

where  $0 < \sigma_1$  is the bristle stiffness,  $1 > \sigma_2$  the bristle damping coefficient,  $2 > \sigma_3$  the viscous damping coefficient,  $x \rightarrow$  relative velocity between the matting surfaces, z average deflection of the bristles.  $g(x \rightarrow)$  models the Stribeck effect and the most common form of  $g(x \rightarrow)$  is as follows

$$\sigma_0 g(\dot{x}) = F_c + (F_s - F_c) e^{-\left(\frac{\dot{x}}{\dot{x}_s}\right)^2} \quad (8)$$

where c F is the Coulomb friction level, s F the level of static friction, and  $s \dot{x} \rightarrow$  characteristic Stribeck velocity. Furthermore, Canudas et al. suppose that  $g(x \rightarrow)$  is always strictly positive real and bounded. Set  $dz/dt=0$ . Inserting Eqs. (7),(8) into Eq. (6) yields the relation between friction force and angular velocity for steady-state motion

$$F_{ss}(\dot{x}) = \left[ F_c + (F_s - F_c) e^{-\left(\frac{\dot{x}}{\dot{x}_s}\right)^2} \right] \text{sgn}(\dot{x}) + \sigma_2 \dot{x} \quad (9)$$

On the other hand, when the velocity is constant, we obtain

$$F_{ss}(\dot{x}) = u \quad (10)$$

Note that if the control force u holds constant, the velocity reaches the steady state. When this happens, Eq. (10) can be satisfied. Thus, we can obtain a Stribeck curve between friction force and velocity by repeating such experiments for various control force u. Then, four static parameters in LuGre model can be identified with various identification algorithms.

### 3. Robust adaptive sliding mode controller

#### 3.1 Definition of Related terms

Before present the proposed adaptive sliding mode controller, we define some related terms that will be used in the following subsections. We first define tracking errors as follows

$$e = x_d - x \quad (11)$$

$$\dot{e} = \dot{x}_d - \dot{x} \quad (12)$$

where the items with subscript "d" represent their corresponding desired values.

To achieve tracking control, we define a sliding surface

$$s = ce + \dot{e} \quad (13)$$

where c is a positive constant. Another error metric,  $s(t)\Delta$ , is defined as

$$s_\Delta(t) = s(t) - \delta \text{sat}(s(t)/\delta) \quad (14)$$

where  $\text{sat}(i)$  is a saturation function defined as

$$\text{sat}(t) = \begin{cases} x, & \text{if } |x| < 1 \\ \text{sgn}(x), & \text{otherwise} \end{cases} \quad (15)$$

The function  $s\Delta$  has the following useful properties

The problem to be addressed is thus to design an adaptive control law  $u(t)$  which can ensure that tracking error metric  $s$  lies in the predetermined boundary  $\delta$  for all time  $t > 0$ .

#### 3.2 Controller Design

Substituting Eqs. (5)(8) into Eq. (3), we have

$$\begin{aligned} \ddot{x} = & \dot{a}\dot{x} - \sigma\dot{x} + u - \sigma_0 z + \sigma_1 \Phi(\dot{x})|x|z \\ & - A_{r1} \cos(\omega x) - A_{r2} \sin(\omega x) - F_l \end{aligned} \quad (17)$$

where  $1 \ 2 \ \sigma = \sigma + \sigma > 0$ ,  $\varphi(x) = 1/g(x)$ . Since  $g(x)$  is strictly positive real and bounded,  $\varphi(x)$  is also strictly positive real and bounded, so we assume  $\varphi(x) - \rho$  without loss of generality, where  $\rho$  is a positive constant. State variable  $z$  in LuGre friction model is unknown and immeasurable, which makes the realization much difficult. Here, we use a dual-observer to estimate friction state  $z$ , the observers are written as follows

$$\frac{d\tilde{z}_1}{dt} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x})} \tilde{z}_1 + \tau_1 \quad (18)$$

where  $0^z, 1^z$  are the estimates for the friction state  $z$ , and  $0 \ \tau, 1 \ \tau$  are observer dynamic terms that are yet to be designed. The corresponding observation errors can be computed as

$$\frac{d\tilde{z}_0}{dt} = \dot{x} - \frac{|\dot{x}|}{g(\dot{x})} \tilde{z}_0 - \tau_1 \quad (19)$$

$$\frac{d\tilde{z}_1}{dt} = -\frac{|\dot{x}|}{g(\dot{x})} \tilde{z}_1 - \tau_1 \quad (20)$$

where  $z^\Delta = z - z^\wedge$ ,  $z^\Delta = z - z^\wedge$ . Since the parameters are unknown in Eq. (16), The following controller is constructed based on Eq. (3) for the nonlinear servo drives system as

$$\begin{aligned} u = & -\hat{a}\dot{x} - \hat{b}u_c + \hat{\sigma}\dot{x} + \hat{\sigma}_0 \tilde{z}_0 - \hat{\sigma}_1 \phi(\dot{x})|\dot{x}|\tilde{z}_1 \\ & + \hat{A}_{r1} \cos(\omega x) + \hat{A}_{r2} \sin(\omega x) + \hat{F}_l \text{sgn}(s_\Delta) \end{aligned} \quad (21)$$

where  $c \ u$  is an additional control given by

$$u_c = -c\dot{e} - \dot{x}_d - k_s s_\Delta \quad (22)$$

The update laws for the parameter estimates are determined as follows

$$\dot{\hat{a}} = -k_a \dot{x} s_\Delta \quad (23)$$

$$\dot{\hat{b}} = -k_b u_c s_\Delta \quad (24)$$

$$\dot{\hat{\sigma}} = -k_\sigma \dot{x} s_\Delta \quad (25)$$

$$\dot{\hat{\sigma}}_0 = -k_{\sigma_0} \tilde{z}_0 s_\Delta \quad (26)$$

$$\dot{\hat{\sigma}}_1 = -k_{\sigma_1} \phi(\dot{x})|\dot{x}|\tilde{z}_1 s_\Delta \quad (27)$$

$$\dot{\hat{A}}_{r1} = k_{r1} s_\Delta \cos(\omega x) \quad (28)$$

$$\dot{\hat{A}}_{r2} = k_{r2} s_\Delta \sin(\omega x) \quad (29)$$

$$\dot{\hat{F}}_l = k_f |s_\Delta| \quad (30)$$

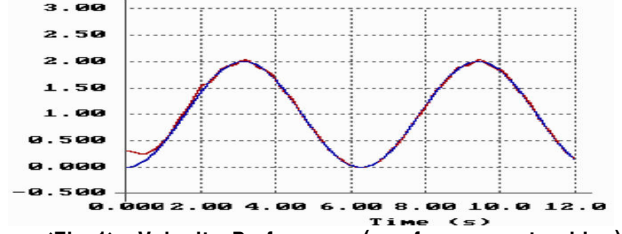
$$\tau_0 = s_\Delta \quad (31)$$

$$\tau_1 = -\phi(\dot{x})|\dot{x}|s_\Delta \quad (32)$$

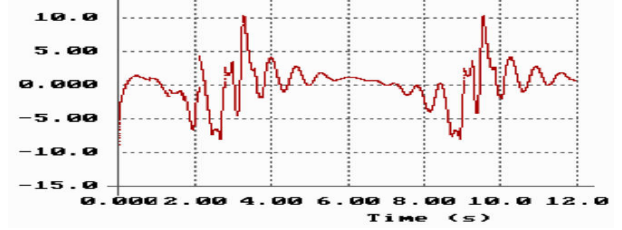
#### 4. Simulation and discussion

In this section, we will illustrate the effectiveness of the proposed compensation approach in a ship-borne gun servo drives system

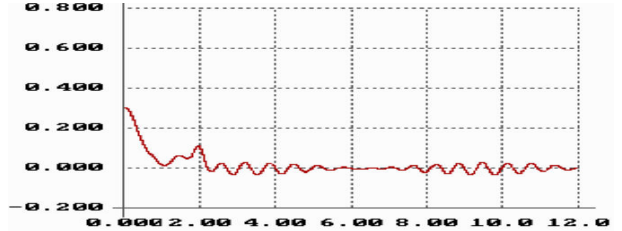
using experiments. It can be concluded that, in sinusoidal case, adaptive sliding mode compensation controller based on LuGre friction model and force ripple can achieve much less tracking error than PID controller, and can improve performance of gun servo drives system greatly, especially smoothness. However, when velocity and acceleration get higher, the dynamic performance of the servo drives system becomes worse. The reasons is that in high velocity and acceleration case, besides friction and force ripple, there are many other factors affecting the system's performance.



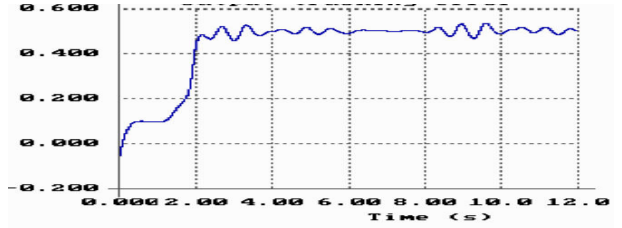
<fig 1> Velocity Performance(- reference, - tracking)



<fig 2> Control Effort



<fig 3> Control Error



<fig 4> Approximation Parameter

#### 3. Conclusions

In this paper, by designing state observers and parameter adaptive laws, a nonlinear adaptive approach based on sliding mode is proposed for compensation of friction and force ripple arising in permanent magnet synchronous motor servo drives system. The simulation results have demonstrated the effectiveness of the proposed compensation approach. We find that proposed controller can achieve better performance than PID controller with much smaller tracking errors and steady-state errors, improve tracking performance of the system in both low velocity and high velocity with high acceleration. Performance of the servo drives system can meet the requirements of ship-borne gun.

#### [References]

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