# 두 평판 전극간에 놓인 하전된 마이크로 입자에 작용하는 힘에 대한 해석

**김승택**<sup>\*,\*\*</sup>, 이상호<sup>\*\*</sup>, 김용권<sup>\*</sup> 서울대학교<sup>\*</sup>, 한국생산기술연구원<sup>\*\*</sup>

# Analysis of forces on a charged micron-sized particle between two parallel-plate electrodes

Seungtaek Kim\*\*\*, Sangho Lee\*\*, Yong-Kweon Kim\* Seoul National University\*, Korea Institute of Industrial Technology\*\*

**Abstract** - This paper investigates forces on a charged particle used in the e-paper application. The particles were inserted into the pixels and according to the applied voltage, particles moves up or down to the electrodes. So, to design the e-paper, the force analysis is very important for the stable operation of e-paper. For these purposes, we divided forces into 4 different forces and numerically evaluated each force. From the simulation results, we confirmed that the minimum voltage to detach the particle from the bottom electrode can be obtained for the given condition.

#### 1. Introduction

The force analysis of a charged particle resting on a plane conductor is important in a wide variety of applications including the electrostatic transfer of toner in electro-photography[1] or fine powder removal [2], gas-insulated system (GIS)[3] in transmission system, and recently the electrostatic transition of pixel particles between top and bottom electrodes in e-paper like a quick-response liquid powder display (QR-LPD) [4]. Many studies [5,6] have been reported about the force analysis on the toner adhesion or detachment containing the electrostatic attractive force, van de Waals force, gravitational force, and electrostatic image force for electrophotography.

A QR-LPD is the attractive e-paper proposed by Bridgestone Ltd, like Fig.2 because of fast response time and existence of threshold voltage. In this e-paper, the black positively-charged particles and white negatively-charged particles were inserted into each pixel, together. The each particle tend to move in opposite direction according to the built-up electric field, resulting in the change of surface color at each pixel. Therefore, the force analysis on a charged particle under uniform electric field is necessary to estimate the threshold voltage, to select the charge or the size of the particles and to predict the consumption power for the portable applications of e-paper.

In this study, the various forces on a charged particle are investigated including Coulomb force, van der Waals force, electrostatic image force and gravitational force, when the particle is in the uniform electric field built up by the voltage applied to between the top plane electrode and the bottom plane. The adhesion force at each electrode and the minimum voltage supported by the driving circuit, will be considered for the e-paper operation.

#### 2. Force Analysis and Simulation Results

# 2.1 Theoretical Model

Fig.1 is theoretical model depicting the forces on a charged particle at (a) top and (b) bottom electrode. The first is the Coulomb force. Because of the external electric field, the Coulomb force have to be considered. The direction of the force is determined by the applied voltage. The second is the van der Waals force. Hamaker calculated total interaction between two particles [7], and various configuration was also reported in Tadmor's report [8], such as sphere-sphere, sphere-plate, plate-plate, spherical shell-spherical shell. sphere-spherical shell, and so on. From the paper [8], the equation for the van der Waals force between a sphere and a plate have been driven. The third is an electrostatic image force. When the charge is on the conducting electrode, the image charge method is normally used to obtain the electric field. Then, the electrostatic image force from the image charge -Q, can be also calculated. The last is the gravitational force. From the gravity, the direction of the gravitational force is always same being independent on the particle position. Therefore, two different equations were obtained for the detachment force of the charged particle at top and bottom electrode. In the following paragraph, each force will be discussed in detail.



## 2.2 Coulomb Force (F<sub>q</sub>)

When a charged particle is present in an electric field like Fig.3, it is acted on by the Coulomb force. If the length of two electrodes were finite and the specific voltage, V, was applied to between two electrodes, the electric field generated in the medium were can be expressed as

$$E = \frac{V}{d} \tag{1}$$

where d is the distance between two electrodes. The electric coulomb force,  $F_{\rm Q}$  exerted on the particle is

$$F_Q = QE \tag{2}$$

where Q is a charge and E is the built-up electric field from equation (1).



# 2.3 Van der Waals Forces (F<sub>vdw</sub>)

Attractive forces between dry and uncharged particles are mainly attributed to van der Waals forces produced by the interaction of fluctuating molecular dipole fields. The van der Waals force for a sphere near a surface as shown in Fig. 1 is given by

$$F_{vdw} = \frac{A_{132}R}{6z_o} \tag{3}$$

where  $A_{132}$  is the Hamaker constant for substances "1" and "2" in presence of medium "3" and  $z_{\rm o}$  is separation distance [7]. For a particles attached to a substrate,  $z_{\rm o}$  is about 4 to 10 Å. Typically  $z_{\rm o}$  =4Åis used [9].



<Fig. 4> Van der Waals force

#### 2.4 Electrostatic Image Forces

For simplifying the problem, we assumed that a point charge Q was resting in contact with a grounded plate electrode. When a point charge Q is located at a distance R from the plate electrode like Fig.5, the force was usually analyzed by image charge method. An general expression for the electrostatic image force is

$$F_{image} = -\frac{Q^2}{4\pi\epsilon_o\epsilon_r(2R)} \tag{4}$$

where  $\epsilon_o$  is the permittivity of free space and  $\epsilon_r$  is the relative permittivity of medium.



<Fig. 5> Electrostatic Image Force

#### 2.5 Gravitational Force (F<sub>G</sub>)

The gravitational force always acts on the particles in the z-direction shown in Fig.1. The gravitational force is

$$F_G = ma = \frac{4}{3}\pi R^3 \left(\rho_{particle} - \rho_{air}\right)g\tag{5}$$

where R denotes the particle radius, g the gravitational acceleration,  $\rho_{particle}$  and  $\rho_{air}$  the particle and the medium density, respectively.

#### 2.6 Total Force at each electrode

When the charged particle is attached to the top electrode shown in Fig.1 (a), the total adhesion force on the charged particle can be expressed as

$$F_{adh @top} = F_{IMAG} + F_{vdw} - F_G. \tag{6}$$

Thus, when  $F_Q$  is greater than  $F_{adh \oplus top}$ , equation (6), the particle start to jump down to the bottom electrode. On contrary, at the bottom electrode shown in Fig.1 (b), it can be written as equation (7).

$$F_{adh @bottom} = F_{IMAG} + F_{vdw} + F_G.$$
<sup>(7)</sup>

Although the amount of the gravitational force on the particle is not dominant among these forces, the total adhesion force of the particle at the bottom electrode is always greater than it at the top electrode. So, the required voltage was decided to detach the particle from the bottom electrode resulting in voltage limit of the operation condition for e-paper. Furthermore, although there is no external electric field, the charged particle have to be sticked to the top electrode for maintaining the surface color. Thus, at the top electrode, the electrostatic image force plus the van der Waals force on the particle have to be greater than the gravitational force. From the above discussion, the operation voltage was limited by the detachment force at the bottom electrode and the size or charge of particle was determined by the adhesion force at the top electrode.

# 2.7 Numerical Simulation Result

Fig.6 are the simulation results of these forces for 4 different radius of particle. The charge of the particle is 10 fC. The gap between two electrode is 100µm. The applied voltage range is from 0 to 200 V DC. The threshold voltage at each graph of Fig.6 was about 145, 85, 65 and 45 V, respectively. If the power supply used in

e-paper can not support to generate above 50V, only for (d) case (radius is  $7\mu$ m), e-paper can be operated appropriately and for the other cases (radius is 4, 5 and,  $6\mu$ m), it won't work due to the insufficient electric field. As the radius of particle decreased, the electrostatic image force increased dominantly more than the other forces, resulting in the enhanced adhesion force.



## 3. Conclusion

E-paper is one of the most attractive displays for the low power consumption application where the charged particle is inserted into the pixels surrounded by walls and electrodes. So, the force analysis on the particle is important to design the size or the charge of particle or to select the power supply voltage. We divided the forces on the particles used in e-paper into 4 types of force such as coulomb force, electrostatic image force, van der Waals force and gravitational force. Using equations for each force, the numerical values was calculated for the given conditions at 4 different radius of particle and the threshold voltages for each case were compared. From the simulation results, we confirmed that the lowest voltage to transit the particle from bottom to top electrode can be determined through the force analysis at the bottom electrode for the stable operation of e-paper.

#### [References]

[1] L.B. Schein et al, "Proximity theory of toner adhesion," J. of Imaging science and technology, Vo.48, no.5, pp.412-416, 2004.

[2] Satoru Watano et al, "Removal of fine powders from film surface. I. Effect of electrostatic force on the removal efficiency," Chem. Pharm. Bull. Vol.50, no.9 pp.1258-1261, 2002.

[3] M. Hara et al, "A method for prediction of gaseous discharge threshold voltage in the presence of a conducting particle", Journal of Electrostatics, Vol. 2, pp.223-239, 1976.

[4] Ryo Sakurai et al, "Color and flexible electronic paper dipslay using QR-LPD technology," SID 06 Digest pp.1922-1925, 2006.

[5] Dan A. Hays, "Adhesion of charged particles," J. of Adhesion Science Technology, vol.9 no.8 pp.1063-1073, 1995.

[6] James Q. Feng, "Electric field detachment of a nonuniformly charged dielectric sphere on a dielectric coated electrode," J. of Electrostatics, 40&41, pp.289–294, 1997.

[7] H.C. Hamaker, "The london-van der Waals attraction between spherical particles," Physica IV, no.10, IC58, 1937.

[8] Rafael Tadmor,"London van der Waals interaction energy between objects of various geometries," J. of phy.; Condes Matter 13 No.9 pp.195-202, 2000.

[9] M.A.S. Quintanila, et al, "Adhesion force between fine particles with controlled surface properties," Vol.52, No.5, pp.1715–1782, 2006.