

## Coupling Particles Swarm Optimization for Multimodal Electromagnetic Problems

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## Coupling Particles Swarm Optimization for Multimodal Electromagnetic Problems

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**Abstract** - This paper proposes a novel multimodal optimization method, Coupling particles swarm optimization (PSO), to find all optima in design space. This method based on the conventional Particle Swarm Optimization with modifications. The Coupling method is applied to make a couple from main particle and then each couple of particles searches its own optimum by using non-stop-moving PSO. We tested out our method and other one, such as ClusteringParticle Swarm Optimization and Niche Particle Swarm Optimization, on three analytic functions. The Coupling Particle Swarm Optimization is also applied to solve a significant benchmark problem, the TEAM workshop benchmark problem 22

### 1. Introduction

In many electromagnetic designs, a stationary design which has small design sensitivities is preferred to an optimum design having big design sensitivities. It is because a design with big design sensitivities, when it is related with mass production process, may have a poor performance due to a manufacturing error. Finding all optima of an objective function, in this viewpoint, is necessary so that a designer may select a good robust design taking account of constraints. In recent years, therefore, there have been many attempts to improve particles swarm optimization (PSO) so that it may find not only a global optimum point but also all optima including local optimum point [1]-[2]. The Clustering PSO and Niche PSO, among them, are popular.

The Clustering PSO has its base on clustering algorithm which classifies particles into different groups and allows each group to move toward not a global best point but a group best point [1]. On the other hand, the Niche PSO basically follows the conventional PSO at the beginning of iterations, and divides the whole particles into several groups and, then, follows the idea of Clustering PSO[1]-[2]. This is known more effective than the Clustering PSO, however, it failed to find all optima, or it requires a huge number of objective function calculations when it is applied to engineering optimization problems [3].

In this paper, a robust and efficient PSO algorithm, Coupling PSO, is developed to locate all optima of a multimodal function with less number of objective function calculations. The proposed algorithm is applied to some analytic functions to test its effectiveness and applied to TEAM problem 22 [4]

### 2. Coupling Particles Swarm Optimization Algorithm

In the conventional PSO, the velocity of  $i$ -th particle is updated as follows [2]:

$$\mathbf{v}_i(t) = \omega \mathbf{v}_i(t-1) + \alpha_1 r_1 (\mathbf{p}_i(t-1) - \mathbf{x}_i(t-1)) + \alpha_2 r_2 (\mathbf{g}(t-1) - \mathbf{x}_i(t-1)) \quad (1)$$

$$\mathbf{x}_i(t) = \mathbf{x}_i(t-1) + \mathbf{v}_i(t), \quad i = 1, 2, \dots, N \quad (2)$$

where  $\mathbf{x}_i$ ,  $\mathbf{v}_i$ ,  $\mathbf{p}_i$  are the position, velocity and personal best position of the  $i$ -th particle, respectively, and  $\mathbf{g}$  is the global best position,  $\omega$  is the inertia,  $\alpha_1$ ,  $\alpha_2$  are cognitive and social coefficients, and  $r_1, r_2$  are uniform random number within [0,1], and  $N$  is the total number

of the particles. When the coefficient 2 is set to zero, equation (1) is called as *cognition-only* model.

The overall flow of the proposed Coupling PSO is summarized as follows:

#### Step 1. Initial particles

Initially  $N$  main particles are randomly generated, and allowed to move according to the cognition only model. Each main particle is expected to move toward not a global optimum point but its nearest local optimum point.

#### Step 2. Coupling

When a main particle updates its personal best position, i.e., when  $F(\mathbf{x}_i(t)) < F(\mathbf{p}_i(t-1))$  is satisfied, it forms a  $i$ -th couple by generating a new particle near by itself as shown in Fig. 1. The position of the new particle is given randomly as follows:

$$\mathbf{c}_i(t) = \mathbf{x}_i(t) + r_i (\mathbf{x}_i(t) - \mathbf{p}_i(t-1)) \quad (2)$$

where  $r$  is a uniform random number within [0,1].

#### Step 3. Movement of a couple

The movement of a couple is just like that of a group in the Clustering PSO. The two particles in this  $i$ -th couple will move according to the following rule:

$$\mathbf{v}_{k,i}(t+1) = \omega \mathbf{v}_{k,i}(t) + c_1 r_1 (\mathbf{p}_{k,i}(t) - \mathbf{x}_{k,i}(t)) + c_2 r_2 (\mathbf{c}_k(t) - \mathbf{x}_{k,i}(t)) + \delta r_3 \epsilon \quad i = 1, 2. \quad (3)$$

where  $c_k$  is the couple best position of  $k$ -th couple and  $r_3$  is a uniform random number within [0,1], and  $\epsilon$  is a very small number (for example, 10-4% of the design space). The last term is for non-stop-moving and  $\delta$  becomes 1 only when the condition,  $\mathbf{c}_k(t) = \mathbf{p}_{k,i}(t) = \mathbf{x}_{k,i}(t)$  is satisfied for 3 consecutive iterations.

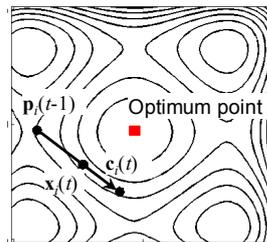


Fig. 1. Generation of a new particle.

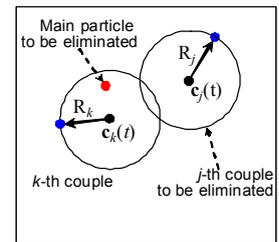


Fig. 2. Elimination.

#### Step 4. Elimination of a couple and main particle

When a couple or a main particle is very near to another couple, the couple or the main particle is eliminated as shown in Fig. 2. This process will increase the numerical efficiency of the proposed Coupling PSO.

#### Step 5. Stopping criterion

The  $k$ -th couple will stop its moving when its movement is very small (for example, less than 10<sup>-4</sup>% of the design space) for 10 consecutive iterations and its couple best position will be an optimum. The algorithm will be terminated when all couples stop without regard to main particles.

### 3. Numerical Results and Conclusion

#### 3.1 Analytic Problems

NPSO was order to check the validity and numerical efficiency of the proposed Coupling PSO, the following three analytic functions with two design variables are considered:

$$F_1(\mathbf{x}) = (x_1^2 + x_2 - 11) + (x_1 + x_2^2 - 7) - 200, \mathbf{x} \in [-5, 5]^2 \quad (4)$$

$$F_2(\mathbf{x}) = \sum_{i=1}^2 \{x_i^2 - 10 \cos(2\pi x_i) + 10\}, \mathbf{x} \in [-1.3, 1.3]^2 \quad (5)$$

$$F_3(\mathbf{x}) = \frac{1}{4000} \sum_{i=1}^2 x_i^2 - \prod_{i=1}^2 \cos\left(\frac{x_i}{\sqrt{i}}\right) + 1, \mathbf{x} \in [-9.5, 9.5]^2 \quad (6)$$

The performances of three methods are compared in Table I by their percentages of successful trials of finding all optima. It is clearly shown that, the Coupling PSO and Niche PSO successfully locate all optima while the Clustering PSO can not, and the Coupling PSO requires less number of function calls than the Niche PSO. It is thought because the Coupling PSO eliminates the couples and main particles which would locate the same local optimum. It is also shown that the number of function calls in the proposed Coupling PSO is proportional to the number of optima found. In each test the number of initial particles is set to 50, and the maximal iteration is limited to 800.

TABLE I

		Experiments Results				Average function calls	
Function	Optima	% of all optima found					
		Clustering	Niche	Proposed	Niche	Proposed	
F1(x)	4	100	100	100	40,000	20,546	
F2(x)	9	83	100	100	40,000	26,144	
F3(x)	17	14	100	100	40,000	38,352	

#### 3.2 Optimization in Electromagnetic Device

TEAM problem 22 was chosen to show the application of Coupling PSO in electromagnetic optimization. This problem consists on the minimization of the magnetic flux density at a certain distance from a superconducting magnetic energy storage device. The design parameters are three variables which define the size and position of the outer coil of the device. The range of search space and other parameters are shown in Table II. The problem is defined as the minimization of the value of  $B_{stray}$ :

$$B_{stray}^2 = \frac{1}{22} \sum_{i=1}^{22} B_{stray,i}^2 \quad (7)$$

subject to:

$$B_{max} \leq 4.92 T \quad (8)$$

$$\frac{Energy - E_{ref}}{E_{ref}} \leq 0.05 \quad (9)$$

where the stray field  $B_{stray}$  is evaluated at 22 equidistant points along line a and, as shown in Fig.3,  $B_{max}$  is the maximum magnetic flux density at the outer coil,  $B_{max}$  is defined as the following critical curve of NbTi superconductor as shown in Fig.4, and Energy is the stored magnetic energy. In this paper, the tolerance for the energy constraint has been set at 5% of the reference value  $E_{ref} = 180MJ$ .

The Coupling PSO has been applied to solve this problem with number of particles is 50, maximum iteration is 1,000. There are 12 optima which are located after 42,000 function calls. There are 6 optima which are satisfied both constraints (8) and (9) among 12 optima found. Three of the best points found are shown in Table III.

From the Table III, the candidates obtained by Coupling PSO are compared to the result from TEAM 22. The first optimum (Opt1)

presents a very low value for  $B_{stray}$ , the second shows a slightly larger value for  $B_{stray}$  and the third is the biggest  $B_{stray}$  value. However, the deviation from the reference value for the Energy of the Opt1 is the biggest one. The deviation of Opt3 is the second and the Opt2 has the smallest deviation. So, by using Coupling PSO, there are multiple optima which could be located. The additional optima provide a range of design options for the designer, who may decide, for example, for a solution with lower sensitivity to variations in the design parameters, even if it presents a slightly higher value for the objective. The Opt2 is selected as the solution.

In this paper, Coupling PSO is proven that it can successfully locate all optima, including global optimum. Moreover, Coupling PSO required less number of objective function evaluations than that of Clustering PSO and Niche PSO. These advantages make the algorithm suitable for electromagnetic problems.

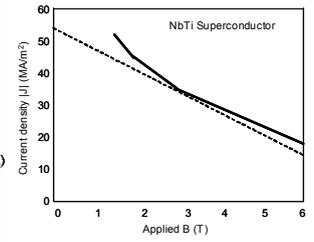
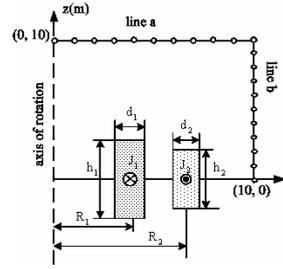


Fig.3. Design variables

Fig.4. Critical curve of the material

TABLE II

Variable Ranges and Values Used

Variable	$R_1$	$h_1/2$	$d_1$	$R_2$	$h_2/2$	$d_2$	$J_1$	$J_2$
[Unit]	[m]	[m]	[m]	[m]	[m]	[m]	[MA/m <sup>2</sup> ]	[MA/m <sup>2</sup> ]
Min	-	-	-	2.6	0.204	0.1	-	-
Max	-	-	-	4.5	1.1	0.6	-	-
Value	2.0	0.8	0.27	-	-	-	22.5	-22.5

TABLE III

Optimization Results

Optima	$R_2$	$h_2/2$	$d_2$	$B_{stray}$	Energy
	[m]	[m]	[m]	[mT]	[MJ]
Opt1	2.994	0.355	0.280	0.7818	172.01
Opt2	3.268	0.376	0.222	1.0891	183.99
Opt3	3.415	0.458	0.169	1.2234	186.30
TEAM[4]	3.080	0.239	0.394	0.8896	180.03

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