Basic study on correction method of measured earthing resistance by 4-potential method

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Abstract

The fall of potential method is generally used for measuring the earthing-resistance. This method needs two auxiliary electrodes and it is required that two auxiliary electrodes are placed at enough distances from the earthing electrode, e.g. building foundation. However, in urban area there is not enough space for taking measurement. So, it is very difficult to measure the earth resistance precisely. However, there is the 4- potential method when it stand on the theory of the fall of potential method. It is the purpose of this paper to basic study on correction method of measured earthing resistance by 4-potential method.

1. INTRODUCTION

In the measurement of the earthing resistance of building foundation, the precise measurement is difficult when there is not enough distance of earth electrode for auxiliary current electrode. However, there is 4-potential method when it is stand on the theory of the fall of potential method.

The paper describes verification of the correction by 4-potential method using the hemispherical electrode.

2. 4-POTENTIAL METHOD

(1) Expression of 4-potential method

4-potential method is correction method of measured earthing resistance. It is stand on the theory of the fall of potential method. In this method choose the position relations of the potential electrode (P) 4 points voluntarily for a current electrode (C) and measure earthing resistance.

It is the way of getting true resistance based on an earthing resistance value.

The resistance is given by the fall of potential method eqn.(1).

$$R=\frac{\rho}{2\pi}\left(\frac{1}{r}-\frac{1}{P}-\frac{1}{C}+\frac{1}{C-P}\right) \quad \ldots(1)$$

As already mentioned, one difficulty is to find the exact point from which P and C should be measured. Suppose that there is an error X in the measurement of P, and Y in the measurement of C. Then eqn.(1) becomes

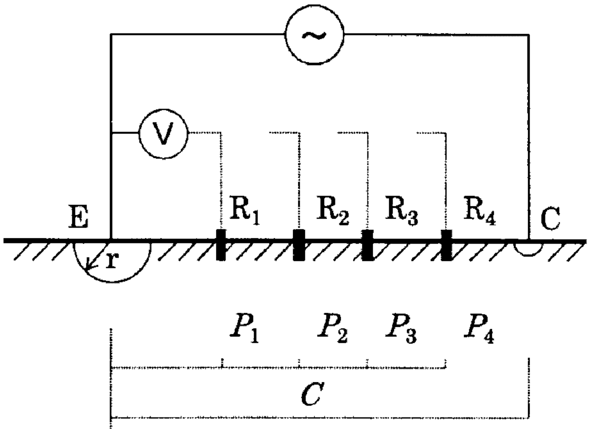

Fig.1 Model of 4-potential method

$$R=\frac{\rho}{2\pi}\left(\frac{1}{r}-\frac{1}{P+X}-\frac{1}{C+Y}+\frac{1}{C+Y-P-X}\right) \quad \ldots(2)$$

$R_\infty=\dfrac{\rho}{2\pi r}$ and so eqn.(2) can be rewritten.

$$R=R_\infty+\frac{\rho}{2\pi C}\left(-\frac{1}{\frac{P}{C}+\frac{X}{C}}-\frac{1}{1+\frac{Y}{C}}+\frac{1}{1+\frac{Y}{C}-\frac{P}{C}-\frac{X}{C}}\right) \quad \ldots(3)$$

Now, put

$\dfrac{\rho}{2\pi C}=K$, $\dfrac{P}{C}=\lambda$, $\dfrac{X}{C}=\beta$, $\dfrac{Y}{C}=\gamma$ then

$$R=R_\infty+K\left(-\frac{1}{\lambda+\beta}-\frac{1}{1+\gamma}+\frac{1}{1+\gamma-\lambda-\beta}\right) \quad \ldots(4)$$

$$R = R_\infty + K\left(-\frac{1}{\lambda\left(1+\frac{\beta}{\lambda}\right)} - \frac{1}{1+\gamma} + \frac{1}{(1-\lambda)\left(1+\frac{\gamma-\beta}{1-\lambda}\right)} \right) \quad \ldots(5)$$

If $\frac{\beta}{\lambda}$, γ and $\frac{(\gamma-\beta)}{(1-\lambda)}$ are sufficiently small for the fractions to be expanded by the binominal theorem,

$$R = R_\infty + K\left(\frac{-\left(1-\frac{\beta}{\lambda}\right)}{\lambda} - (1-\gamma) + \frac{\left(1-\frac{\gamma-\beta}{1-\lambda}\right)}{(1-\lambda)} \right) \quad \ldots(6)$$

Which simplifies to

$$R = R_\infty + K(f\beta + g\gamma + h) \quad \ldots(7)$$

Where

$$f = \frac{1-2\lambda+2\lambda^2}{\lambda^2(1-\lambda)^2} \quad \ldots(8)$$

$$g = \frac{\lambda^2-2\lambda}{(1-\lambda)^2} \quad \ldots(9)$$

$$h = \frac{\lambda^2+\lambda-1}{\lambda(1-\lambda)} \quad \ldots(10)$$

Eqn.(7) can be written

$$R = R_\infty + fK\beta + gK\gamma + hK \quad \ldots(11)$$

There are four unknowns in this equation: R_∞, $K\beta$, $K\gamma$, K Suppose four values of λ are chosen, resulting in corresponding values of f,g,h and R. The result is the four equations:

$$R_1 = R_\infty + f_1K\beta + g_1K\gamma + h_1K \quad \ldots(12)$$

$$R_2 = R_\infty + f_2K\beta + g_2K\gamma + h_2K \quad \ldots(13)$$

$$R_3 = R_\infty + f_3K\beta + g_3K\gamma + h_3K \quad \ldots(14)$$

$$R_4 = R_\infty + f_4K\beta + g_4K\gamma + h_4K \quad \ldots(15)$$

Solving these equations for R_∞ leads to the result

$$R_\infty = \frac{\phi_1(\lambda_1)R_1 + \phi_2(\lambda_2)R_2 + \phi_3(\lambda_3)R_3 + \phi_4(\lambda_4)R_4}{\phi_1(\lambda_1) + \phi_2(\lambda_2) + \phi_3(\lambda_3) + \phi_4(\lambda_4)} \quad \ldots(16)$$

Where

$$\phi_1(\lambda_1) = -[f_2(g_3h_4 - g_4h_3) - f_3(g_2h_4 - g_4h_2) + f_4(g_2h_3 - g_3h_2)] \quad \ldots(17)$$

$$\phi_2(\lambda_2) = [f_1(g_3h_4 - g_4h_3) - f_3(g_1h_4 - g_4h_1) + f_4(g_1h_3 - g_3h_1)] \quad \ldots(18)$$

$$\phi_3(\lambda_3) = -[f_1(g_2h_4 - g_4h_2) - f_2(g_1h_4 - g_4h_1) + f_4(g_1h_2 - g_2h_1)] \quad \ldots(19)$$

$$\phi_4(\lambda_4) = [f_1(g_2h_3 - g_3h_2) - f_2(g_1h_3 - g_3h_1) + f_3(g_1h_2 - g_2h_1)] \quad \ldots(20)$$

(2) Correction technique by 4-potential method

1) Measurement of earthing resistance

In Fig.1, P is input to the arbitrary part between EC. The earthing resistance in the various place point is measured.

2) Calculation of position λ

The position λ is shown from the driving part in P and the isolation distance in C in the ratio of the distance in P and C.

Hereafter, details of the distance are shown.

P=distance of auxiliary potential electrode (P) from centre of earth electrode system

C=distance of auxiliary current electrode (C) from centre of earth electrode system

$\lambda = P/C$

λ is possible becomes $r/C<\lambda<1.0$.

3) Correction calculation by 4-potential method

$\lambda=\lambda_1$, λ_2, λ_3, and λ_4 are substituted for the expression of 4-potential method, and it calculates by each combination.

Next, the above-mentioned result is substituted for the expression, and $\varphi_1(\lambda_1)$, $\varphi_2(\lambda_2)$, $\varphi_3(\lambda_3)$ and $\varphi_4(\lambda_4)$ is solved.

In addition, $\varphi_1(\lambda_1)$, $\varphi_2(\lambda_2)$, $\varphi_3(\lambda_3)$ and $\varphi_4(\lambda_4)$ is applied to the expression, and a, b, c, and d is calculated.

$$a = \frac{\phi_1(\lambda_1)}{\phi_1(\lambda_1)+\phi_2(\lambda_2)+\phi_3(\lambda_3)+\phi_4(\lambda_4)} \quad \cdots(21)$$

$$b = \frac{\phi_2(\lambda_2)}{\phi_1(\lambda_1)+\phi_2(\lambda_2)+\phi_3(\lambda_3)+\phi_4(\lambda_4)} \quad \cdots(22)$$

$$c = \frac{\phi_3(\lambda_3)}{\phi_1(\lambda_1)+\phi_2(\lambda_2)+\phi_3(\lambda_3)+\phi_4(\lambda_4)} \quad \cdots(23)$$

$$d = \frac{\phi_4(\lambda_4)}{\phi_1(\lambda_1)+\phi_2(\lambda_2)+\phi_3(\lambda_3)+\phi_4(\lambda_4)} \quad \cdots(24)$$

$$R = \frac{\phi_1(\lambda_1)R_1+\phi_2(\lambda_2)R_2+\phi_3(\lambda_3)R_3+\phi_4(\lambda_4)R_4}{\phi_1(\lambda_1)+\phi_2(\lambda_2)+\phi_3(\lambda_3)+\phi_4(\lambda_4)}$$

$$= a \cdot R_1 + b \cdot R_2 + c \cdot R_3 + d \cdot R_4 \quad \cdots(25)$$

3. SIMULATION

(1)Process of simulation

The examination in this section is correction of measured earthing resistance by 4-potential method by numerical simulation. Table 1 shows the condition of simulation.

The earth resistance was calculate with eqn.(26). When, Current electrode is hemisphere, and that size is considered in eqn.(26). The correction by 4-potential method used the resistance carve that is given by it.

$$R(\lambda)=\frac{\rho}{2\pi}\left[\left(\frac{1}{r_1}-\frac{1}{P}-\frac{1}{C-Y}+\frac{1}{C-P}\right)+\left|\frac{-1}{C-Y}+\frac{1}{C-r_1}\right|\right] \quad \cdots(26)$$

Where, $Y = C - r_2$, $\lambda = \dfrac{P-r_1}{C-r_1}$

Table 1.Condition of the simulation

resistivity of soil ρ	100[Ω・m]
current I	1[A]
radius of earth electrode r_1	10,20,30,40,50,60,70[m]
radius of current electrode r_2	0.001[m]
distance of current electrode from center of earth electrode C	100[m]
λ	0.05〜0.95

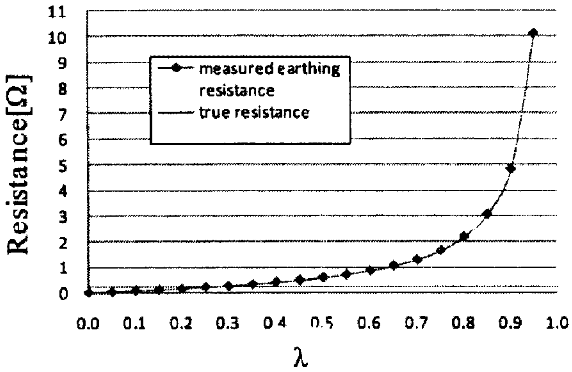

Fig.2 Earth resistance curve(r_1=70cm)

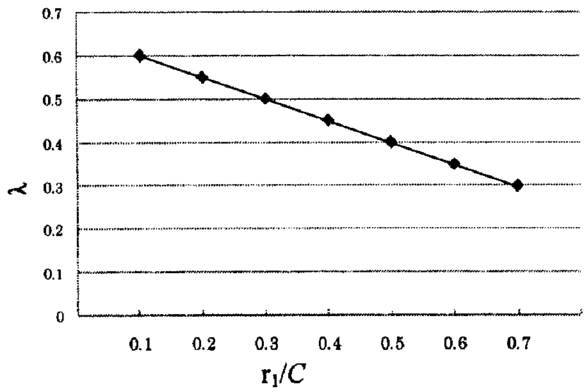

Fig.3 λ which is close to the true value (C=100m)

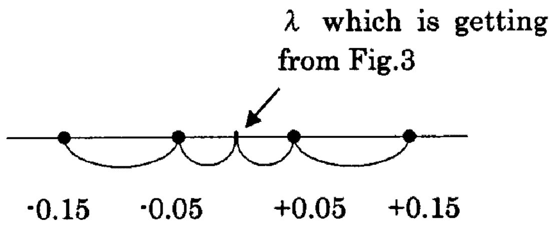

Fig.4 Determination of λ combinations

(2)Simulation result and consideration

Fig.2 is earth resistance carve provided by eqn.(26). It was represents true resistance of hemispherical electrode. That curve and true resistance line intersect at λ=0.3. And, All cases were analyzed in the same way (r1=10,20,30,40,50,60[m]). r_1/C with variation of λ shows Fig.3.

Table 2 is correction result by 4-potential method that earth resistance was given by simulation. And, the true resistance also shows in it the lowest row.

The combination of λ has a rule. It can find from Fig.3 and Table 2. Fig.4 showed that rule. λ_1, λ_2, λ_3, and λ_4 combine according to Fig.4, correction value approximate to the true resistance.

In the measurement of the earthing resistance, when earth electrode size (r_1) and distance of earth electrode for current electrode (C) are known. The measured resistance were able to correction by 4-potential method that combination decide λ according to r_1/C with λ correspond to it (Fig.3 and 4).

Table 2. Correction resistance by 4-potential method

Combination of λ	Calculation resistance[Ω]						
	r_1=10	r_1=20	r_1=30	r_1=40	r_1=50	r_1=60	r_1=70
0.05,0.1,0.15,0.2	1.20	0.38	0.17	0.09	0.06	0.04	0.04
0.05,0.15,0.25,0.35	1.40	0.54	0.28	0.17	0.12	0.10	0.10
0.1,0.15,0.2,0.25	1.39	0.52	0.26	0.15	0.10	0.08	0.08
0.1,0.2,0.3,0.4	1.49	0.63	0.35	0.23	0.17	0.15	0.16
0.15,0.2,0.25,0.3	1.48	0.61	0.33	0.21	0.15	0.13	0.13
0.15,0.25,0.35,0.45	1.53	0.69	0.42	0.29	0.23	0.21	0.23
0.2,0.25,0.3,0.35	1.52	0.68	0.39	0.26	0.20	0.18	0.19
0.2,0.3,0.4,0.5	1.56	0.74	0.47	0.34	0.29	0.28	0.31
0.25,0.3,0.35,0.4	1.55	0.72	0.44	0.32	0.26	0.24	0.27
0.25,0.35,0.45,0.55	1.57	0.77	0.50	0.39	0.34	0.34	0.39
0.3,0.35,0.4,0.45	1.57	0.75	0.49	0.37	0.31	0.31	0.34
0.3,0.4,0.5,0.6	1.58	0.78	0.53	0.43	0.39	0.40	0.47
0.35,0.4,0.45,0.5	1.58	0.78	0.52	0.40	0.36	0.36	0.42
0.35,0.45,0.55,0.65	1.58	0.80	0.55	0.45	0.42	0.44	0.52
0.4,0.45,0.5,0.55	1.58	0.79	0.54	0.43	0.40	0.41	0.48
0.4,0.5,0.6,0.7	1.59	0.80	0.56	0.46	0.43	0.45	0.54
0.45,0.5,0.55,0.6	1.58	0.80	0.55	0.45	0.42	0.43	0.52
0.45,0.55,0.65,0.75	1.58	0.80	0.55	0.45	0.42	0.45	0.53
0.5,0.55,0.6,0.65	1.58	0.80	0.55	0.45	0.42	0.44	0.52
0.5,0.6,0.7,0.8	1.58	0.79	0.54	0.44	0.40	0.42	0.49
0.55,0.6,0.65,0.7	1.58	0.79	0.55	0.44	0.41	0.42	0.50
true resistance[Ω]	1.59	0.80	0.53	0.40	0.32	0.27	0.23

4. CONCLUSIONS

When building foundation size and distance of earth electrode for current electrode (C) are known, that simulation result apply to combination of λ. When this method was established, earthing resistance of building foundation is able to knowing in limitation measurement area.

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5. REFERENCES

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