

# 독립적 적응을 기반으로 한 업데이트 간격이 다른 Affine Projection 알고리즘 필터의 조합

김 광훈, 최 영석, \*송 우진  
포항공과대학교 전자전기공학과

E-mail : dawnight@postech.ac.kr, ychoi23@jhu.edu, wjsong@postech.ac.kr

## Combination of Two Affine Projection Algorithm Filters with Different Update Interval based on Independent Adaptation

Kwang-Hoon Kim, Young-Seok Choi, \*Woo-Jin Song

Dept. Electronic and Electrical Engineering

Pohang University of Science and Technology (POSTECH)

### Abstract

We propose a adaptive combination of affine projection algorithm (APA) filters with different update interval. Two APA filters with different update interval are adapted independently in order to keep the advantages of both component filters. This novel scheme provides improvement of performance in term of the convergence rate and the steady-state error. Experimental results show good properties of the proposed algorithm.

### I. Introduction

The affine projection algorithm (APA) is a useful adaptive filter whose main purpose is to speed up the convergence of the LMS-type filters [1]. In the APA, the update interval influences both the convergence rate and the steady-state error. To solve the conflicting trade-off between the fast convergence rate and the low steady-state error, we incorporate convex combination of two individual filters as shown in Fig.1 [2]. The combination is executed in a manner that the overall filter keep the advantage of both component filters.

In this paper, we present an improved APA with an adaptive combination of one fast (i.e., with a

small update interval) and one slow (with a large update interval) AP filter. It is effective for combining fast convergence and low steady-state error.

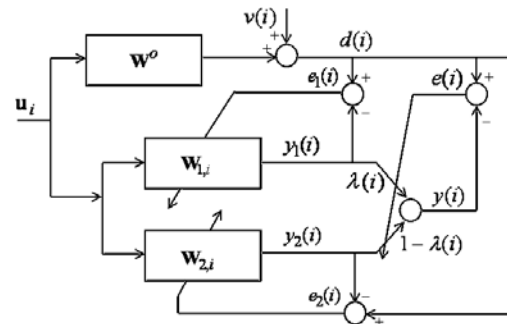


Fig.1. Adaptive Combination of two adaptive filters

### II. Proposed Algorithm

Let  $w_i$  be an estimate for  $w^o$  at time  $i$ , and we obtain the APA update equation such that

$$w_i = w_{i-1} + \mu U_i^* (U_i U_i^*)^{-1} e_i, \quad (1)$$

where  $e_i = d_i - U_i w_{i-1}$  and  $\mu$  is step-size [1].

This work was supported by the Brain Korea (BK) 21 Program funded by the Ministry of Education and HY-SDR Research Center at Hanyang University under the ITRC program of MIC, Korea

$$U_i = \begin{bmatrix} \mathbf{u}_i \\ \mathbf{u}_{i-1} \\ \vdots \\ \mathbf{u}_{i-K+1} \end{bmatrix}, \mathbf{d}_i = \begin{bmatrix} d(i) \\ d(i-1) \\ \vdots \\ d(i-K+1) \end{bmatrix}$$

Various properties of the APA have been exploited. We also have found that the effect of update interval on the convergence performance. The smaller the update interval is, the faster the convergence rate is, while resulting in the high steady-state error. On the other hand, a large update interval yields a lower steady-state error, but leads to the slow convergence rate.

Based on the above property of the APA, we can achieve our goal by using an adaptive convex combination of two APA filters: the first being a fast filter and the second a slow filter.

The output of the overall filter written as

$$y(i) = \lambda(i)y_1(i) + [1 - \lambda(i)]y_2(i), \quad (2)$$

where  $y_k(i) = \mathbf{u}_i \mathbf{w}_{k,i-1}$  with  $\mathbf{w}_{k,i-1}$  being the adaptive filter coefficient vectors of the  $k$ -th filter and  $\mathbf{u}_i$  is input vector, and  $\lambda(i) \in [0, 1]$  is a mixing parameter. The mixing parameter  $\lambda(i)$  is adapted using stochastic gradient scheme so that  $J(i) = e^2(i)$  is minimized, where  $e_i = d(i) - \mathbf{u}_i \mathbf{w}_{i-1}$ . The update equation for the mixing parameter written as

$$a(i+1) = a(i) + \mu_a e(i)[y_1(i) - y_2(i)] \cdot \lambda(i)[1 - \lambda(i)]. \quad (3)$$

By choosing appropriate mixing parameter  $\lambda(i)$  at each time, the combination scheme would result in the respective merits of both APA filters. As the result, overall filter is obtained by

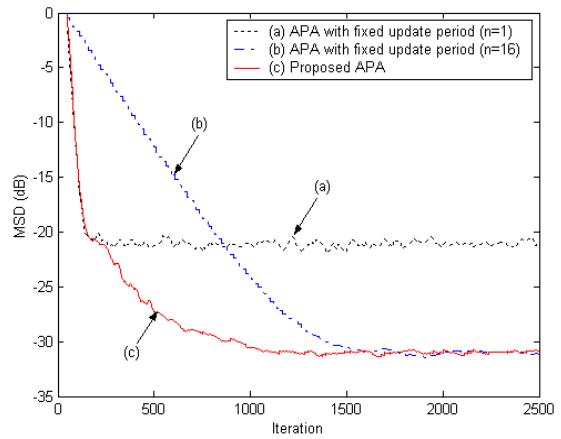
$$\mathbf{w}_{k,i} = \begin{cases} \mathbf{w}_{k,i-1} + \mu U_i^* (U_i U_i^*)^{-1} \mathbf{e}_{k,i} & \text{if } \text{mod } n_k = 0 \\ \mathbf{w}_{k,i-1} & \text{otherwise} \end{cases}$$

$$\mathbf{w}_i = \lambda(i)\mathbf{w}_{1,i} + [1 - \lambda(i)]\mathbf{w}_{2,i} \quad (4)$$

where  $\mathbf{e}_{k,i} = \mathbf{d}_i - U_i \mathbf{w}_{k,i-1}$

### III. Experimental Results

We illustrate the performance of the proposed algorithm by carrying out the computer simulations in a system identification scenario. The unknown system has 32 taps and is randomly generated. The adaptive filter and unknown system are assumed to have the same number of taps. The input signal is obtained by filtering a white, zero-mean, gaussian random sequence through a first-order system  $G(z) = 1/(1 - 0.9z^{-1})$ . The SNR is calculated by  $\text{SNR} = 10 \log_{10}(E[y^2(i)]/E[v^2(i)])$ ,  $y(i) = \mathbf{u}_i \mathbf{w}^o$ . The measurement noise  $v(i)$  is added to  $y(i)$  such that  $\text{SNR} = 30$  dB. The mean square deviation (MSD),  $E \|\mathbf{w}^o - \mathbf{w}_i\|^2$ , is taken and averaged over 100 independent trails. The order of APA is set to  $K = 16$ . The step-size is  $\mu = 0.1$  and  $\mu_a = 100$ ,



$\rho = 0.9$ .

Fig.2. MSD curves for the APA with fixed update interval and the proposed APA

### IV. Conclusion

The proposed scheme is able to retain the best properties of each component filter, that is, fast convergence and small steady-state error.

### References

[1] K. Ozeki and T. Umeda, "An adaptive algorithm using an orthogonal projection to an affine subspace and its properties," *Electron. Commun. Jpn.*, 1984.  
 [2] J. Arenas-García, V. Gómez-Verdejo, and A. R. Figueiras-Vidal, "New algorithms for improved adaptive convex combination of LMS transversal filters", *IEEE Trans. Instrum. Meas.*, vol. 54, pp. 2239-2249, Dec. 2005