Normalized Mean Square Covariance and Ergodic MIMO Channel Capacity over Rayleigh Fading Channels

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Abstract

In this paper, we define the normalized mean square covariance (MIMO) of multiple input multiple output (MIMO) channels and study the relationship between ergodic MIMO channel capacity and the channel NMSV value.

I. Introduction

In this paper, we study the relationship between normalized mean square covariance and ergodic multiple input multiple output (MIMO) channel capacity. In [1] and [2], the concept of normalized mean square covariance (NMSV) was introduced to effectively characterize wide-sense stationary uncorrelated scattering (WSSUS) channels. In this paper, we apply the NMSV concept to characterize MIMO channels and study its relationship with the ergodic channel capacity.

One of the most important performance degrading factors in wireless communications is multipath fading. To combat the effect of fading, various techniques have been developed to exploit the diversity in multipath fading stochastic processes such as spread spectrum techniques. It is natural to expect that the effectiveness of such techniques depends on the amount of diversity in the channel or the amount of the correlation among the channel fading levels. To characterize the overall correlation among a group of random variables, the concept of normalized mean square covariance was introduced. It was shown in [1] and [2] that the performance of various communication systems is closely related to the NMSV values of the channels under consideration. For this reason, it is meaningful to use the NMSV value as an indicator of the the channel quality.

Although the usefulness of the NMSV concepts has been verified with various communication systems, it was only for single input single output (SISO) systems. In this paper, we extend the previous research by applying the NMSV concept to MIMO systems. We desire to establish the relationship between MIMO system performance and the NMSV value of MIMO channels. Since there are so many variations in the system configurations depending on the knowledge at transmitter and receiver, it will not be a simple to task consider all possible MIMO systems. As an initial application of the NMSV concept to MIMO systems, we only consider a system in which the channel state information is fully available at both transmitter and receiver. We desire to show that the channel capacity of the system is closely related to the NMSV value of the channel under consideration.

II. System Description

In this paper, we shall consider a MIMO system in which the channel fading level between each transmitting and receiving antenna pair is flat Rayleigh faded and constant over each modulation symbol duration. We shall assume that the numbers of transmitting and receiving antennas are the same and is denoted by N. The input and output symbols shall be denoted by N dimensional complex valued column matrices $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively. We shall assume that additive white Gaussian noise is added at the receiver so that the input and output symbols are related by

$$\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}$$

where $\boldsymbol{n}$ is the zero mean circularly symmetric complex Gaussian random vector with covariance matrix $N_0 \boldsymbol{I_N}$ and $\boldsymbol{H}$ denotes the NxN complex fading level matrix between the transmitting and receiving antennas. We shall assume that the elements H_{ij}'s of $\boldsymbol{H}$ are zero mean complex random variables that are not necessarily independent. However, we assume that H_{ij}'s are independent of the noise random vector $\boldsymbol{n}$. For normalization, we assume that

$$\sum_{i=1}^{N}\sum_{j=1}^{N} E\left[|H_{ij}|^2\right] = N$$

We assume further that the transmitter has perfect knowledge about the channel realization $\boldsymbol{H}$ and adapts to it to achieve the capacity. In this case, as shown in [3], the channel capacity $C(\boldsymbol{H})$ given the channel realization $\boldsymbol{H}$ with the input power constraint $\sum_{n=1}^{N} E\left[|x_n|^2\right] \le E_s$ is

$$C(\boldsymbol{H}) = \max \log_2 \det\left[1 + \rho \boldsymbol{HQH}^{\boldsymbol{H}}\right]$$

where ρ and the maximization is over all NxN positive semi-definite matrix $\boldsymbol{Q}$ with $\boldsymbol{Tr}[\boldsymbol{Q}] = 1$. Again, as shown in [3],

$$C(\boldsymbol{H}) = \max \sum_{n=1}^{N} \log_2(1 + \rho p_n \lambda_n)$$

where λ_n's are the eigenvalues of the matrix $\boldsymbol{HH}^{\boldsymbol{H}}$ and the maximization is over all nonnegative real numbers p_n's with $\sum_{n=1}^{N} p_n = 1$. It is well known that the maximization is obtained by the water-filling algorithm [3, 4]. The ergodic capacity $\boldsymbol{C_E}$ of this MIMO system is defined by

$$\boldsymbol{C_E} = E[\,C(\boldsymbol{H})\,]$$

where the expectation is over complex gaussian random matrix $\boldsymbol{H}$ with given covariance tensor.

Ⅲ. Normalized Mean Square Covariance and Ergodic MIMO Capacity

The main objective of this paper is to apply the concept of NMSV to MIMO channels and then to establish its relationship with the ergodic channel capacity. To this end, we should first define the NMSV of a given channel. We define the NMSV $\boldsymbol{V}$ of the Rayleigh fading channel in Section II

$$\boldsymbol{V} = \frac{\sum_{p=1}^{N}\sum_{q=1}^{N}\sum_{r=1}^{N}\sum_{s=1}^{N}\left|E\left[H_{pq}H_{rs}^{*}\right]\right|^2}{\left[\sum_{p=1}^{N}\sum_{q=1}^{N}E\left[|H_{pq}|^2\right]\right]^2} \qquad (1)$$

It readily follows from the Cauchy-Schwarz inequality that $0 \le \boldsymbol{V} \le 1$. We note that $\boldsymbol{V}$ is closer to 1 with the more correlated channels. In particular, $\boldsymbol{V} = 1$ for perfectly correlated channels.

To establish the relationship between the ergodic MIMO system channel capacity and the NMSV value of the channel under consideration, we performed extensive simulations. For the simulations, we have chosen N=3 and have randomly generated 350 covariance matrices to describe the statistics of the fading levels H_{ij}'s. We note that the covariance matrices are 9 dimensional square matrices since i and j in H_{ij} ranges from 1 o 3 Each 9-dimensional covariance matrix $\boldsymbol{K}$ corresponds to a single MIMO channel model and the NMSV value of the channel is assigned by (1). Forr each of these 350 channel models, we computed the ergodic channel capacity $\boldsymbol{C_E}$ defined in Section II. For the value of signal to noise ratio (SNR) E_s/N_0 we have chosen 25dB. The simulation results are depicted in Figure 1.

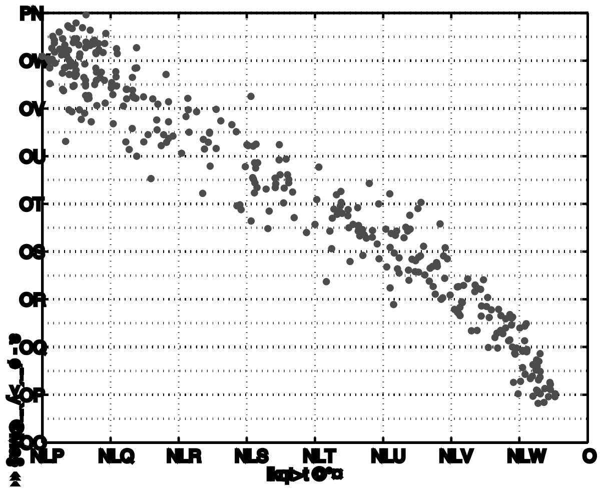

Figure 1. Ergodic channel capacity over MIMO Rayleigh fading channels plotted as a function of the channel NMSV values at E_s/N_0 = 25dB.

In the figure, each dot corresponds to each channel model and the 350 dots shows the relationship between the NMSV value and the corresponding ergodic channel capacity. We clearly see that there exists some compelling monotonic relationship between the NMSV value and the ergodic channel capacity despite certain amount of deviations. From this figure, we notice that the concept of NMSV may potentially be useful in assessing the quality of MIMO channels as well as SISO channels.

Ⅳ. Conclusion

In this paper, we defined the normalized mean square covariance (MIMO) of multiple input multiple output (MIMO) channels and showed that there exists a compelling monotonic relationship between ergodic MIMO channel capacity and the NMSV value of the channel under consideration. However, we should note that the present analysis is only with a very limited type of MIMO systems. Consequently, ensuing research should follow to study the true relationship between NMSV and MIMO channel capacity.

References

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