

## Asymmetry in Directional Distribution due to Refraction of Real Sea Waves 실 해역에서 굴절에 의한 방향분산함수의 비대칭성

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### 1. INTRODUCTION

For many engineering applications measured directional spectra are not available and designers have only bulk estimates of wave height, period, and direction. For these applications analytic models for wave directional spectra can be used to generate representative wave conditions. It is important that these analytic models accurately represent the wave directional spreading in coastal areas, since damage predictions for coastal structures are sensitive to the directional characteristics of the design wave field, as shown by Suh et al. (2002). Van Dongeren et al. (2003) also demonstrated the importance of wave directional spreading in the generation of long waves in the surf zone. It should be noted that the imposition of directional symmetry will degrade their accuracy in areas where wave refraction is important. Relatively longer waves begin refracting in relatively deeper water and upon reaching shallow water their directions are more shore-normal than shorter waves (Lee et al., 2003). Goda and Suzuki (1975) showed that the directional spreading function becomes narrower as waves propagate over a planar slope. Their directional spreading function was based on directional symmetry. However, multi-directional random wave spectra will become more directionally asymmetric in shallower water due to the difference in refraction of directionally symmetric components. Recently, Lee et al. (2007) developed a directional spreading function that

considers directional asymmetry as well as symmetry. They found, for waves over a planar slope, asymmetry of directional distribution exists and their function is better than the previous Goda and Suzuki's (1975) function.

In this study, for nearshore waves that have undergone refraction, it is found that directional asymmetry is common and the error in fitting the present function to measured data is reduced when considering asymmetry.

### 2. APPLICATION OF DIRECTIONAL FUNCTION TO FIELD DATA

#### 2.1 Directional Spreading Function

Multi-directional random waves can be expressed by the directional spectrum  $S(f, \theta)$  given by  $S(f, \theta) = S(f)G(f; \theta)$  where  $S(f)$  is the frequency spectrum and  $G(f; \theta)$  is the directional spreading function. Lee et al. (2007) suggested a directional spreading function that considers directional asymmetry as well as symmetry. The suggested function is given by

$$G(f; \theta) = G_0 \cos^{2s} \left( \frac{\theta - \theta_p}{2} \xi \right) \quad (1)$$

where

$$\xi = \begin{cases} \exp(-\mu), & \theta \geq \theta_p \\ \exp(+\mu), & \theta \leq \theta_p \end{cases} \quad (2)$$

$$G_0 = \left[ \int_{\theta_{\min}}^{\theta_{\max}} \cos^{2s} \left( \frac{\theta - \theta_p}{2} \xi \right) d\theta \right]^{-1} \quad (3)$$

and  $s$  is Goda and Suzuki's (1975) spreading

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parameter given as

$$s = \begin{cases} s_{\max}(f/f_p)^5, & f \leq f_p \\ s_{\max}(f/f_p)^{-2.5}, & f \geq f_p \end{cases} \quad (4)$$

The asymmetry parameter  $\mu$  is positive when the left-side components (i.e., positive directional components relative to the peak from the peak wave direction  $\theta_p$ ) are more broadly distributed than the right-side components. When the directional spreading function is symmetric, the asymmetry parameter  $\mu$  is zero and the present function given by Eqs. (1) to (4) is the same as that of Longuet-Higgins et al.'s (1961) function. We use only one parameter  $\mu$  to describe both right- and left-side components of asymmetric distribution, which is quite convenient in expressing real sea spectra.

## 2.2 Comparison of Suggested Directional Spectra to Field Data

As a test of the new asymmetric directional spreading function, we compare the function to directional spectra measured at the US Army Corps of Engineers Field Research Facility at Duck, NC. Measured spectra were calculated using data from a long duration, bi-directional array of 15 bottom-mounted pressure sensors deployed in a water depth of 8 m (the "8-meter Array", see Long and Oltman-Shay, 1991). These data are unique in that the measurements of the wave directional distributions are much more highly resolved than those obtained by wave buoys or smaller arrays. Approximately six months of archived spectra, collected every three hours from January through June 2006 for a total of 1,383 spectral records, were used for the comparison.

The range of wave conditions contained in these data is shown in Fig. 1 where  $H_{mo}$  is root-mean square wave height and  $\theta_m$  is the mean wave direction. The wave conditions consist of some long period swell as well as higher-frequency wind seas with a wide range of incident wave angles. The data also include some very high angle, high-frequency conditions. Wave angles represent directions of wave approach between  $-90^\circ$  and  $90^\circ$  and increasing counterclockwise from shore normal

( $0^\circ$ ). All archived spectra had a resolution of 0.0098 Hz and  $2^\circ$ , and the resolved frequency range was  $0.04 < f < 0.32$  Hz.

Differences between the mean direction and the peak direction,  $\theta_m - \theta_p$ , are indicative of the skewness of the directional distribution. The time series shown in Fig. 1(d) suggests that skewed distributions and, hence, directional asymmetries are common at this coastal site and that the asymmetry is caused by wave refraction. The influence of refraction is evident by the fact that the median value of  $\theta_m - \theta_p$  for  $\theta_p > 0$  is  $-5.2^\circ$  and  $+5.1^\circ$  for  $\theta_p < 0$ . Since refraction will skew the distributions toward shore normal, refraction-induced skewness causes  $\theta_m - \theta_p$  to be oppositely signed from  $\theta_p$ .

Next, we find the directional spreading function that best fits the nearshore directional spectra. The percent error in the best-fit spectra is denoted  $E_p$  and defined as

$$E_p = \frac{\sum_{i=1}^{i_{\max}} \sum_{j=1}^{j_{\max}} |En_{i,j} - Ee_{i,j}|}{\sum_{i=1}^{i_{\max}} \sum_{j=1}^{j_{\max}} Ee_{i,j}} \times 100(\%) \quad (5)$$

where  $Ee$  is the exact wave energy which is calculated by using the Snell's law for each wave energy component,  $En$  is the wave energy of the best-fit to the exact spectrum, and the subscripts  $i$  and  $j$  denote the  $i$ -th frequency and  $j$ -th directional component, respectively.

Time series of the best-fit  $s_{\max}$  and  $\mu$  values along with the calculated function-data errors are shown in Fig. 2 for both the symmetric and asymmetric functions. The time series is limited to thirty days for plotting clarity. The figure clearly shows that the functional error is reduced when asymmetry is considered. The mean errors for the asymmetric and symmetric functions, averaged over the six-month data set, were 50 and 58 percent, respectively. For completeness, the mean error for using the symmetric function without the peak direction correction was also calculated, and it only made a minor difference for the symmetric function (59 %). It is also evident

from Fig. 2 that the asymmetric function tends to increase the optimal  $s_{\max}$  parameter, which means waves are more narrowly focused to the peak direction.

Finally,  $\mu$  is expected to be correlated with  $\theta_m - \theta_p$  and also show a variation with  $s_{\max}$ . Fig. 3 demonstrates that the best-fit  $\mu$  values from the present directional spreading function show the appropriate variation with  $s_{\max}$  and the measured  $\theta_m - \theta_p$ . For this figure the best-fit parameters for the asymmetric function were segregated into two groups based on their  $s_{\max}$  values. The figure shows that the slope of the measured  $\theta_m - \theta_p$  vs. the best-fit  $\mu$  values is positive as expected, and also that as the spreading parameter (best-fit  $s_{\max}$ ) increases the slope decreases. The average slopes of  $\theta_m - \theta_p$  vs. the best-fit  $\mu$  from these segregated data are listed in Fig. 4 and correspond well to the overall slopes derived from the fits to idealized spectra shown in Lee et al. (2007).

## 5. CONCLUSION

In the present study we developed a more general directional spreading function for multi-directional random waves, which allows for directional distributions of arbitrary asymmetry. Directional asymmetry can be found in real seas where waves are shadowed by islands or man-made structures or in coastal areas where wave refraction is important. This new directional spreading function will allow for more realistic design wave conditions to be used in the modeling of coastal structures both numerically and in laboratory experiments.

As a test of the new asymmetric directional spreading function, we compared the function to directional spectra measured at the US Army Corps of Engineers Field Research Facility at Duck, NC. The measured data showed that directional asymmetry is common at this coastal site and the asymmetry is caused by wave refraction. Also, time series of the best-fit  $s_{\max}$  and  $\mu$  values along with the calculated function-data errors show that the functional error is reduced when asymmetry

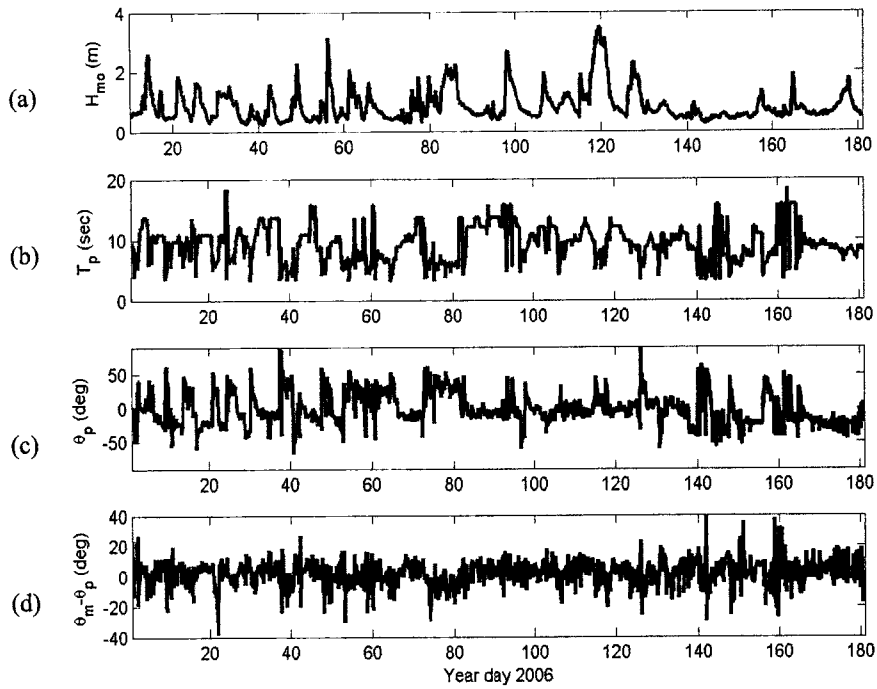
and refractive effects on the peak wave direction are considered.

## ACKNOWLEDGEMENTS

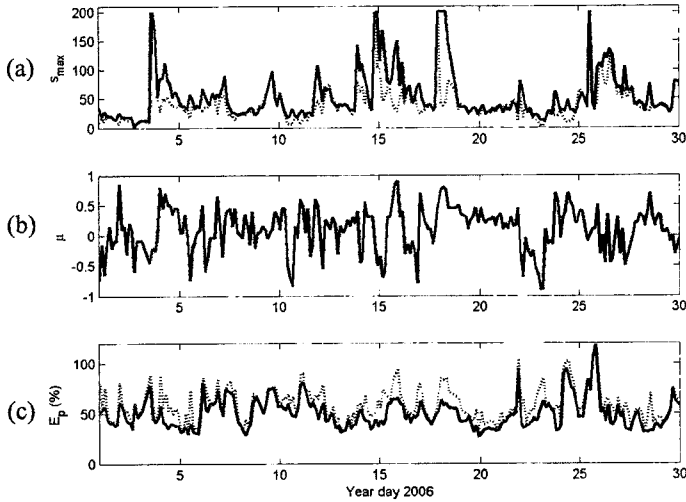
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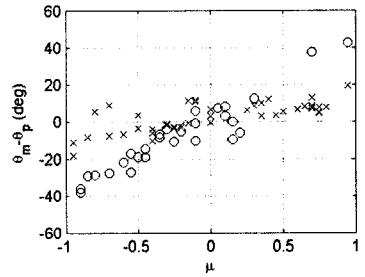
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**Fig. 1.** Six months of wave conditions measured at the FRF 8-m array: (a)  $H_{m0}$ , (b) peak period, (c) peak direction (i.e. peak direction at the peak period), (d) difference between the mean direction and peak direction at the peak period.



**Fig. 2.** (a) Time series of best-fit  $s_{max}$ , (b) best-fit  $\mu$ , (c) percent error; dashed line = symmetric function, solid line = asymmetric function.



**Fig. 3.** Measured  $\theta_m - \theta_p$  versus best-fit asymmetry parameter,  $\mu$ . Symbols:  $7 < s_{max} < 13$  (o) and  $s_{max} > 150$  (x). Mean slopes are 0.39, 0.23, 0.15, 0.10 for  $s_{max} \approx 10, 25, 75, >100$ , respectively.