Noise PDF Analysis of Nonlinear Image Sensor Model with Application: Iterative Radiometric Calibration Method

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Abstract— **The paper presents the advanced radiometric calibration method, called the IRCM (Iterative Radiometric Calibration Method), in order to avoid an operational constraint (solar source) for calibration. The IRCM assumes that an optical instrument is equipped with a filter assembly which consists of same band filters with different transmission ratios. Given all the noise sources (including the artificial one caused by the filters) of an image sensor, the noncentral χ^2 distribution of the output result is induced by the approach of a noise PDF (Power Density Function). Finally, the radiometric calibration problem is transformed into equating two independent relations for the image sensor gains through the specified distribution.**

Key Words: Noise PDF, Nonlinear Image Sensor Model, Iterative, Radiometric Calibration

I. Introduction

The previous research works [1][2] about the radiometric calibration have focused on combination of the Sun and an extra source as the Moon, a star, a lake, and an ocean. They mostly monitor the Sun for a short-term radiometric calibration and the extra sources for a long-term radiometric calibration. The data measured by the extra sources are used to compensate for the calibration error mainly due to the degradation of the solar diffuser. As the approach to mitigate the operational constraint such as a solar source, some instruments [3][4] performs the on-board calibration with a black body. Since the black body as a calibration source is built-in, the calibration operation can be promptly implemented whenever required. Despite the improved operation constraint, however, the black-body calibration is applicable to the infrared channels only. Healy [5] has derived a noise distribution of a linear image sensor model and extracted the linear sensor gain as a noise parameter estimation. And Matsushita [6] has used an asymmetric feature of a noise distribution which comes from nonlinearity of a image sensor, instead of the noise distribution itself. The asymmetric features obtained by the multiple intensity levels are used to optimize the nonlinear radiometric response function. While the noise observation methods have shown the successful results with the video camera, it is the basic drawback when they are applied in the space that the noise distribution of the space instrument is normally too small to be detected even around high intensities. Moreover, a nonlinear image sensor model is taken into account in the paper, differently from Healey.

The IRCM conceptually adopts same band filters with different transmission ratios, as an artificial noise source of an input radiance value. The filter varies input electrons at a detector level as noise sources do. The results aggregated at an image processor level shows a kind of a noise PDF (Power Density Function) which is similar to the noncentral χ^2 distribution. Such a statistical feature enables an unknown input radiance to be estimated, which easily leads to gain estimations of an image sensor. Since the proposed IRCM requires only one consistent scene, it is advantageous that there is no operation constraint such as a solar source.

Section II induces the noise PDF applying the artificial noise sources to a nonlinear image sensor model. In section III, the gain estimation method is presented based the results in the previous section. Finally, the concluding remark is given in Section IV.

II. Derivation of Noise PDF

Considering the nonlinear image sensor model and the noise sources, the relation between the input-radiance (L) and the output-digital value (D) is

$$D = g_l(g_s g_o TL + N_{DSR}) - g_n(g_s g_o TL + N_{DSR})^2 + N_{EQ}, \tag{1}$$

in which g_l, g_n, g_s, and g_o are the linear gain,the nonlinear gain, the pixel sensitivity, and the optical gain, respectively. As the noise sources, N_{DSR} and N_{EQ} are defined by

$$N_{DSR} = TO_D + N_S + N_R, \tag{2}$$

and

$$N_{EQ} = O_E + N_Q, \tag{3}$$

where T is an integration time; O_D and O_E are the offset values due to the dark signal and the electrical components, respectively; N_S (shot noise), N_R (read-out noise), and N_Q (quantization noise) are the random noise variables. As the artificial noise source, the input radiance through the filters is represented by

$$L = \overline{L} + \Delta L, \tag{4}$$

in which $\overline{L}$ is the mean value of L and ΔL is a random variable whose mean is zero. Then, we obtain the averaged value of D

$$\overline{D} = g_{lso}T\Big(\overline{L}+\frac{O_D}{g_{so}}\Big) - g_{nso}T^2\Big(\overline{L}+\frac{O_D}{g_{so}}\Big)^2 + O_E + \overline{D}_N, \tag{5}$$

and its stochastic value

$$\begin{aligned} D_N &= (g_{lso} - 2\mu g_{nso})\widehat{N}_{LSR} - g_{nso}\widehat{N}^2_{LSR} + N_Q \\ &= \hat{\rho}\widehat{N}_{LSR} - g_{nso}\widehat{N}^2_{LSR} + N_Q \\ &= \widehat{N}_{LI} + N_Q, \end{aligned} \tag{6}$$

where

$$g_{so} = g_s g_o, \quad g_{lso} = g_l g_{so}, \quad \mu = T\Big(\overline{L} + \frac{O_D}{g_{so}}\Big), \tag{7}$$

and

$$\widehat{N}_{LSR} = T\Delta L + N_{SR}/g_{so} = T\Delta L + \widehat{N}_{SR}. \tag{8}$$

Three relations are sequentially induced based upon the above equations as follows:

$$\mu = \frac{g_{lso} - \sqrt{g^2_{lso} - 4g_{nso}(\overline{D} - O_E - \overline{D}_N)}}{2g_{nso}}, \tag{9}$$

$$g^2_{lso} \geq 4g_{nso}(\overline{D} - O_E - \overline{D}_N), \tag{10}$$

and

$$\hat{\rho} = \sqrt{g^2_{lso} - 4g_{nso}(\overline{D} - O_E - \overline{D}_N)} \geq 0. \tag{11}$$

Since the PDF of $\widehat{N}_{LSR}$ is formulated in

$$f_{\widehat{N}_{LSR}}(x) = e^{-x^2/2\varphi^2}/\sqrt{2\pi\varphi^2} \tag{12}$$

according to the previous result [7], we find the PDF of D_N,

$$\begin{aligned} f_{D_N}(x) = \frac{1}{2q}\Bigg(&\mathrm{erf}\Big(\frac{\widehat{\psi}_1(x-q/2)}{\sqrt{2}\varphi}\Big) - \mathrm{erf}\Big(\frac{\widehat{\psi}_1(x+q/2)}{\sqrt{2}\varphi}\Big) \\ &-\mathrm{erf}\Big(\frac{\widehat{\psi}_2(x-q/2)}{\sqrt{2}\varphi}\Big) + \mathrm{erf}\Big(\frac{\widehat{\psi}_2(x+q/2)}{\sqrt{2}\varphi}\Big)\Bigg), \end{aligned} \tag{13}$$

with the condition of

$$x \leq \frac{\hat{\rho}^2}{4g_{nso}} - \frac{q}{2}, \quad q \neq 0, \tag{14}$$

in which q is the quantization step,

$$\widehat{\psi}_1(x) = \frac{\hat{\rho} + \sqrt{\hat{\rho}^2 - 4g_{no}x}}{2g_{no}}, \tag{15}$$

$$\widehat{\psi}_2(x) = \frac{\hat{\rho} - \sqrt{\hat{\rho}^2 - 4g_{no}x}}{2g_{no}}, \tag{16}$$

and

$$\varphi^2 = T^2\mathbf{V}(\Delta L) + \mathbf{V}(\widehat{N}_{SR}). \tag{17}$$

The summing and the averaging procedures are used to increase a mean and a variance of a noise PDF in a low intensity region and to make the overall range of the noise PDF satisfy (14) with the decreased variance, respectively. In accordance with (6), $\widehat{N}_{LI}$ is reformulated by

$$\begin{aligned} \widehat{N}_{LI} &= -g_{nso}\varphi^2\Big(\frac{\widehat{N}_{LSR} - \hat{\rho}/2g_{no}}{\varphi}\Big)^2 + \frac{\hat{\rho}^2}{4g_{nso}} \\ &= -g_{nso}\varphi^2\widehat{N}'^2_{LSR} + \frac{\hat{\rho}^2}{4g_{nso}}, \end{aligned} \tag{18}$$

in which $\widehat{N}'_{LSR}$ is the Gaussian distribution whose mean and variance are $-\hat{\rho}/2g_{nso}\varphi$ and 1, respectively. Then, define

$$\begin{aligned} X_{LI}(k,n) &= \frac{1}{n}\sum_{l=1}^{k}\widehat{N}_{LI,l} \\ &= -\frac{g_{nso}\varphi^2}{n}\sum_{l=1}^{k}\widehat{N}'^2_{LSR,l} + \frac{k\hat{\rho}^2}{4ng_{nso}}, \end{aligned} \tag{19}$$

where k is the number of the added random variables, n is 1 or k, $\widehat{N}_{LI,l}$ and $\widehat{N}'_{LSR,l}$ are the l-th measured random variables, and $X_{LI}(k,1)$ and $X_{LI}(k,k)$ indicate the summing and the averaging procedures, respectively. Since $\sum_{l=1}^{k}\widehat{N}'^2_{LSR,l}$ is a noncentral χ^2 distribution in k degree of freedom, we immediately have the PDF of $X_{LI}(k,n)$ as follows:

$$\begin{aligned} &f_{X_{LI}(k,n)}(x) \\ &= \frac{n}{2g_{nso}\varphi^2}e^{-(z(x)+\nu)/2}\Big(\frac{z(x)}{\nu}\Big)^{k/4-1/2}B_{k/2-1}(\sqrt{\nu z(x)}), \end{aligned} \tag{20}$$

in which

$$z(x) = \frac{n}{g_{nso}\varphi^2}\Big(\frac{k\hat{\rho}^2}{4ng_{nso}} - x\Big), \qquad x < \frac{k\hat{\rho}^2}{4ng_{nso}}, \tag{21}$$

$$\nu = \frac{k\hat{\rho}^2}{4g^2_{nso}\varphi^2}, \tag{22}$$

$$B_{\frac{k}{2}-1}(\sqrt{\nu z(x)}) = \Big(\frac{\sqrt{\nu z(x)}}{2}\Big)^{k/2-1}\sum_{m=0}^{\infty}\frac{(\nu z(x)/4)^m}{m!\Gamma(\frac{k}{2}+m)}, \tag{23}$$

and $\Gamma(k/2+q)$ is the Gamma function. Here, it is noted that

$$f_{D_{N,\Delta T}}(x) = \int_{-\infty}^{\infty} f_{N_Q}(\zeta)f_{X_{LI}(1,1)}(x-\zeta)d\zeta. \tag{24}$$

Meanwhile, if $k \geq 2$, the summing or averaging of the uniform distribution (N_Q) can be approximately considered the Gaussian distribution, $G_Q(k,n)$, whose mean and variance are zero and $kq^2/12n^2$, respectively. Therefore, defining

$$\begin{aligned} X_{LIQ}(k,n) &\approx X_{LI}(k,n) + \frac{1}{n}\sum_{l=1}^{k}N_{Q,l} \\ &= X_{LI}(k,n) + G_Q(k,n), \end{aligned} \tag{25}$$

we find the characteristic function of $X_{LIQ}(k,n)$, in case of $k \geq 2$,

$$\begin{aligned}
&\Phi_{X_{LIQ}(k,n)}(\omega) \\
&\approx \Phi_{X_{LI}(k,n)}(\omega)\Phi_{G_Q(k,n)}(\omega) \\
&= \Phi_{X_{LI}(k,n)}e^{-kq^2\omega^2/12n^2} \\
&= \Phi_{X_{LI}(k,n)}\left(1 - \frac{kq^2}{12n^2}\omega^2 + \frac{1}{2!}\left(\frac{kq^2}{12n^2}\right)^2\omega^4 - \cdots\right) \\
&= \sum_{m=0}^{\infty}\frac{1}{m!}\left(\frac{kq^2}{12n^2}\right)^m(-j\omega)^{2m}\Phi_{X_{LI}(k,n)}. \qquad (26)
\end{aligned}$$

Consequently, the PDF of $X_{LIQ}(k,n)$ becomes, if $k = n = 1$,

$$f_{X_{LIQ}(1,1)}(x) = f_{D_{N,\Delta T}}(x), \qquad x \leq \frac{\hat{\rho}^2}{4g_{nso}} - \frac{q}{2}, \qquad (27)$$

and if $k \geq 2$,

$$\begin{aligned}
&f_{X_{LIQ}(k,n)}(x) \\
&\approx f_{X_{LI}(k,n)}(x) + \sum_{m=1}^{\infty}\frac{1}{m!}\left(\frac{kq^2}{12n^2}\right)^m\frac{d^{2m}}{dx^{2m}}f_{X_{LI}(k,n)}(x), \qquad (28)
\end{aligned}$$

$$x \leq \frac{k\hat{\rho}^2}{4ng_{no}} - \frac{kq}{2}. \qquad (29)$$

Using the concept of a noncentral χ^2 distribution, the mean of the $X_{LIQ}(k,n)$ is given by

$$\begin{aligned}
\overline{X}_{LIQ}(k,n) &= \mathbf{E}(X_{LIQ}(k,n)) \\
&= \mathbf{E}(X_{LI}(k,n)) + \frac{k}{n}\mathbf{E}(N_Q) \\
&= -\frac{g_{nso}\varphi^2}{n}(k+\nu) + \frac{k\hat{\rho}^2}{4ng_{nso}} \\
&= -\frac{k}{n}g_{nso}\varphi^2 \\
&= \frac{k}{n}\overline{D}_N, \qquad (30)
\end{aligned}$$

which is always negative. And the variance of $X_{LIQ}(k,n)$ is obtained by, in accordance with the variance of a noncentral χ^2 distribution,

$$\begin{aligned}
&\mathbf{V}[X_{LIQ}(k,n)] \\
&= \mathbf{V}[X_{LI}(k,n)] + \frac{k}{n^2}\mathbf{V}[N_Q] \\
&= \frac{g_{nso}^2\varphi^4}{n^2}(2k+4\nu) + \frac{kq^2}{12n^2} \\
&= \frac{kg_{nso}^2\varphi^4}{n^2}\left(2 + \frac{\hat{\rho}^2}{g_{nso}^2\varphi^2}\right) + \frac{kq^2}{12n^2} \\
&= \frac{2}{k}\overline{X}_{LIQ}^2(k,n) - \frac{\hat{\rho}^2}{ng_{nso}}\overline{X}_{LIQ}(k,n) + \frac{kq^2}{12n^2}. \qquad (31)
\end{aligned}$$

Since $\overline{X}_{LIQ}(k,n) \leq 0$, we find in (31) that the variance of $X_{LIQ}(k,n)$ increases as $|\overline{X}_{LIQ}(k,n)|$ does.

III. Iterative Radiometric Calibration

For $\mathbf{V}(\Delta L)$, the normal noise sources (shot and read-out) and the scaled transformation simply give

$$\mathbf{V}(\widehat{N}_{SR}) = \frac{\mu}{g_{so}} + \frac{\tau^2}{g_{so}^2} = \frac{T}{g_{so}}\left(\overline{L} + \frac{O_D}{g_{so}}\right) + \frac{\tau^2}{g_{so}^2}, \qquad (32)$$

where τ^2 is the variance value of N_R. Assuming the normal imaging mode with two different integration times (T_1, T_2), we can obtain two linear equations:

$$\overline{X}_{LIQ,1}(k,n) = -T_1\frac{kg_{nso}}{n}\left(\overline{L} + \frac{O_D}{g_{so}}\right) - \frac{kg_{nso}\tau^2}{ng_{so}^2}, \qquad (33)$$

and

$$\overline{X}_{LIQ,2}(k,n) = -T_2\frac{kg_{nso}}{n}\left(\overline{L} + \frac{O_D}{g_{so}}\right) - \frac{kg_{nso}\tau^2}{ng_{so}^2}. \qquad (34)$$

Given T_1 and T_2, (33) and (34) naturally lead to the identification of

$$\frac{kg_{nso}}{n}\left(\overline{L} + \frac{O_D}{g_{so}}\right) \quad \text{and} \quad \frac{kg_{nso}\tau^2}{ng_{so}^2}, \qquad (35)$$

which also allows the definition of $F_1(T)$,

$$F'(T) = -T\frac{kg_{nso}}{n}\left(\overline{L} + \frac{O_D}{g_{so}}\right) - \frac{kg_{nso}\tau^2}{ng_{so}^2}. \qquad (36)$$

Moreover, by (7) and (9), TO_D/g_{so} becomes a function of g_{iso} and g_{nso}, assuming $\overline{L} = 0$ during the dark signal acquisition mode,

$$F''(g_{iso}, g_{nso}) = \frac{g_{iso} - \sqrt{g_{iso}^2 - 4g_{nso}(\overline{D}_d - O_E - \overline{D}_{N,d})}}{2g_{nso}}. \qquad (37)$$

In the calibration process, the same band filters with the different transmission ratios are used to obtain the earth images, assuming that the acquired image number of each filter follow the Gaussian distribution whose mean and variance are $\overline{L}$ and $\overline{L}^2$, respectively. Then, according to (17) and (30), we obtain

$$\overline{X}_{LIQ,3} = -\frac{kg_{nso}}{n}(T\overline{L})^2 - \frac{kg_{nso}}{n}F'(T), \qquad (38)$$

which is also rephrased in

$$T\overline{L} = G(T, g_{nso}) = \sqrt{\left|\frac{n}{kg_{nso}}\overline{X}_{LIQ,3} + F'(T)\right|}. \qquad (39)$$

And combining (9), (37), and (39) gives the relation of g_{iso} and g_{nso},

$$\begin{aligned}
&G(T, g_{nso}) + F''(g_{iso}, g_{nso}) \\
&= \frac{g_{iso} - \sqrt{g_{iso}^2 - 4g_{nso}(\overline{D}_3 - O_E - \overline{D}_{N,3})}}{2g_{nso}}. \qquad (40)
\end{aligned}$$

Meanwhile, by replacing $\overline{L}^2$ with $\overline{L}$ for $\mathbf{V}(\Delta L)$, (38) and (39) are reformulated into

$$\overline{X}_{LIQ,4} = -\frac{kg_{nso}}{n}(T\overline{L}) - \frac{kg_{nso}}{n}F'(T), \qquad (41)$$

and

$$T\overline{L} = G(T, g_{nso}) = -\frac{n}{kg_{nso}}\overline{X}_{LIQ,4} - F'(T). \qquad (42)$$

While $\overline{X}_{LIQ,3}$ can be calculated without knowing the input radiance value, $\overline{L}$, due to the characteristic of the Gaussian distribution with the variance of $\overline{L}^2$, the input radiance value is necessary to obtain $\overline{X}_{LIQ,4}$ based upon the variance of $\overline{L}$. For the reason, we cannot induce g_{nso} just by equating (39) and (42). Instead, the following iterative approach is taken to perform the radiometric calibration:

1) calculate $\overline{X}_{LIQ,3}$ with the filer data,
2) select the candidate of g_{nso},
3) obtain $T\overline{L}$ in (39),
4) calculate $\overline{X}_{LIQ,4}$ by the value of $\overline{L}$ given in Step 4),
5) check (42),
6) if no problem is found in Step 5), find g_{lso} in (40), otherwise, go to Step 2).

In particular, the non-linear gain in Step 2) needs to be selected in a way to decrease the error in Step 5) for the solution convergence.

In case that $F^{'}(T)$ and $F^{''}(g_{lso}, g_{nso})$ are relatively very small, (39) becomes

$$G(T, g_{nso}) \approx \sqrt{\left|\frac{n}{kg_{nso}}\overline{X}_{LIQ,3}\right|}, \quad (43)$$

which results in , by (5),

$$g_{nso} = c_1 g_{lso}^2 \quad (44)$$

and

$$c_1 = -\frac{nkX_{LIQ,3}}{(\overline{D} - O_E - \overline{D}_N - nX_{LIQ,3})^2} \geq 0. \quad (45)$$

In the similar way, applying the above assumption to the case of $\mathbf{V}(\Delta L) = \overline{L}$ also leads to

$$g_{nso} = c_2 g_{lso} + c_3, \quad (46)$$

in which

$$c_2 = -\frac{nX_{LIQ,4}}{k(\overline{D} - O_E - \overline{D}_N)} \geq 0, \quad (47)$$

and

$$c_3 = -\frac{n^2 X_{LIQ,4}^2}{k^2(\overline{D} - O_E - \overline{D}_N)} \leq 0. \quad (48)$$

Two independent equations, (44) and (46), are induced as a function of g_{lso} and g_{nso}. By equating them in terms of each gain, we can obtain at most two pairs of solutions. In practice, however, one of two solutions is identical to the real gain values and the other meaningless. Accordingly, we learn that, through the proposed IRCM, the real gain values can be recognized as the iteration goes on, if the gain candidate is selected within the convergence bound of the right solution. As a matter of fact, the first nonlinear gain candidate, which is actually the previously calibrated gain, is likely to be selected around the real value, since the frequent radiometric calibration in a real orbit guarantees a gain difference under a convergence bound. The example is given in Fig.1, assuming that the larger gain values are the meaningful solutions.

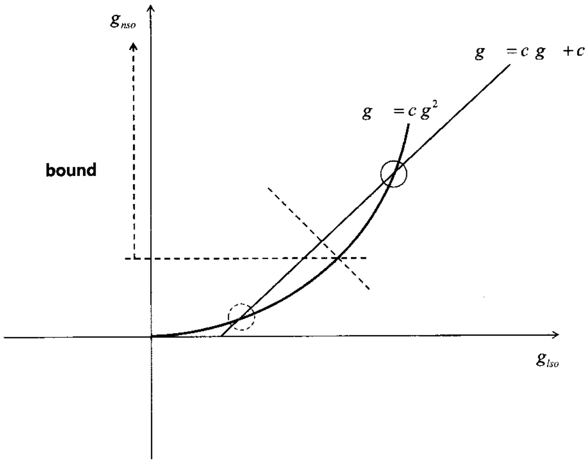

Fig. 1. Convergence Bound Example with (42) and (46)

IV. CONCLUDING REMARK

The IRCM method, using the two different input noise variances, is proposed to identify the image sensor gains without any knowledge of an input radiance. The independence of the solar radiance, in the radiometric calibration process, is expected to alleviate the operation constraint and to minimize the sun exposure during the hot case.

For the further study, generalization of the convergence proof and bound will be addressed to show the IRCM's applicability to many different types of sensors.

REFERENCES

[1] H. H. Kieffer and R. L. Wildey, 1996. Establishing the Moon as a Spectral Radiance Standard, *Journal of Atmos. Oceanic Tech.*, Vol. 13, pp. 360-375.

[2] Howard S. Bowen, 2002, Absolute Radiometric Calibration of the IKNOS sensor Using Radiometrically Characterized Stellar Sources, *FIEOS Conference Proceedings.*

[3] R. Schuster *et al.*, 2002, Design and Calibration of the BIRD Payload Platform, *FIEOS Conference Proceedings.*

[4] E. M. Twarog *et al.*, 2006, WindSat On-orbit Warm Load Calibration, *IEEE Transaction on Geoscience and Remote Sensing*, Vol. 44, No. 3, pp. 516-529.

[5] Glenn E. Healey and Raghava Kondepudy, 1994, Radiometric CCD Camera Calibration and Noise Estimation. *IEEE Trans. on Pattern Analysis and Machine Intelligence*, Vol. 16, No. 3, pp. 267-276.

[6] Y. Matsushita and S. Lin, 2007, Radiometric Calibration from Noise Distribution, *Computer Vision and Pattern Recognition Conference Proceeding.*

[7] H.C. Myung and H.S. Youn, 2007, Noise PDF Analysis of Nonlinear Image Sensor Model: GOCI Case, *International Symposium on Remote Sensing 2008.*