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Ghost Junction Method for Flow Network System Analyses

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유동망 시스템 해석을 위한 유령 정션 기법

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Abstract

Numerical predictions on flow phenomena in pipe network systems have been considered as playing an important role in both designing and operating various facilities of piping or duct systems, such as water supply, tunnel or mine ventilation, hydraulic systems of automobile or aircraft, and etc. Traditionally, coupling conditions between junction and connected branches are assumed to satisfy conservation law of mass and to share an equal pressure at junction node. However, the conventional methodology cannot reflect momentum interactions between pipes sufficiently. Thus, a new finite volume junction treatment is proposed both to reflect the interchanges of linear momentums between neighbor branches at junction and to include the effect of wall at junction in present work.

1. Introduction

In the event of dealing with flow networks numerically, a junction treatment is a prerequisite technique to arrive at theoretical solutions of the flow fields. Traditionally, over a few decades, coupling conditions based on mass conservation law at junction-node have been applied to give boundary conditions for connected pipe flow analyses(see[1-4]).

A node-based approach like this is proved to be well-posed mathematically(see[5]) and convenient to implement because it requires no additional information beside the following two conditions, which are sharing a pressure and satisfying mass conservation law at the singular junction node. However, conventional method cannot account for momentum interactions without incorporating loss coefficients, since the node-based treatment does not consider momentum balances at junction, i.e. the interchange of momentums among neighbor branches and the crucial effect of the reaction force at wall-region of junction are exclusive. Of course, the loss coefficients are sometimes available from experimental database and could be specified dynamically according to geometry and flow conditions. Despite the coefficients for a large amount of junction-branch configurations can be found in the famous reference [6], these coefficients are almost inaccessible when connected branches are equal or more than four. Moreover, the minor losses due to geometry and three dimensional flow physics are likely to become non-negligible especially for short pipe systems while they are negligible in many cases of long pipe systems.

Consequently, GJM can be an alternative method in case that the loss coefficients along streamlines between neighbor branches are available from neither experimental databases nor theoretical analyses.

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2. Ghost Junction Method

2.1 Methodology

A structured FVM of one-dimensional governing equations is applied for each pipe and three-dimensional unstructured form is for junction.

An integral form of Euler equations is expressed by (1) and an additional equation of state is required for compressible flow analyses, where Q is vector of conservative variables, F is flux vector and $\mathcal{H}$ is source vector per unit volume.

$$\frac{d}{dt}\int_{V(t)} Q\,dV + \oint_{s(t)} F\,ds = \int_{V(t)} \mathcal{H} dV \qquad (1)$$

Herein, a ghost junction cell is surrounded by interfaces with neighbor branches and wall surfaces (i.e. of course, the other type of surfaces, such as windows, could be existent) and the numerical flux at interfaces should be calculated in three-dimensional space for junction volume.

And then, the flux can be reused as boundary flux when calculating one-dimensional pipe flows so that GJM should obey the following three conditions to satisfy consistencies of flux at interfaces.

$$a)\ F_{1d}^{\rho} = F_{3d}^{\rho}\quad b)\ F_{1d}^{\rho e^t} = F_{3d}^{\rho e^t}\quad c)\ F_{1d}^{\rho U} = n_x F_{3d}^{\rho u} + n_y F_{3d}^{\rho v} + n_z F_{3d}^{\rho w} \qquad (2)$$

For ghost junction

Assuming all source vectors are zero for a ghost junction cell, a semi-discretized form of governing equations can be expressed as follows, where Q is vector of conservative variables, F is flux vector, U is outgoing velocity normal to surfaces and N is unit vector normal to surfaces.

$$V(t)\frac{\mathrm{Q}^{\mathrm{n+1}} - \mathrm{Q}^{\mathrm{n}}}{\Delta t} = -\sum_{i=1} F_i s(t)_i - Q\frac{\Delta V(t)}{\Delta t} \qquad (3)$$

where

$$Q = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \rho w \\ \rho e^t \end{pmatrix}, \qquad F = \begin{pmatrix} \rho U \\ \rho u U + n_x p \\ \rho v U + n_y p \\ \rho w U + n_z p \\ (\rho e_t + p) U \end{pmatrix},$$

$$N = (n_x, n_y, n_z), \qquad U = n_x u + n_y v + n_z w$$

Supposing a representative surface for this region of wall, numerical flux on the representative wall can be obtained from a simple mathematical manipulation. The integrant of unit normal vector *N* over surfaces of the imaginary control volume vanishes mathematically.

$$\int_s N ds = 0 \tag{4}$$

The effect of walls can be simplified by use of a representative wall and the resultant is expressed in (5), where pseudo-walls include all surfaces except real-walls, N and s represent connected angle and section area respectively.

$$pN_R s_R = \underbrace{\sum_{i=m+1}^{n} pN_i s_i}_{real\ wall} = -\underbrace{\sum_{i=1}^{n} pN_i s_i}_{pseudo\ wall} \tag{5}$$

Herein, the volume-size and shape of ghost junction cell are still ambiguous. In addition to a quasi-steady assumption, a ghost junction cell is supposed to have imaginary dimensions rather than real ones (i.e. this is the reason we call it a “ghost”). With these assumptions, neither size nor the exact shape of the control volume gives meaningful impact on the numerical flow field in junction.

For a pipe

A semi-discretized form of Euler equations for j^{th} cell in a pipe can be written as (6), where ρ is density, *U* is velocity, *p* is pressure, e_t is total energy, *g* is gravity acceleration, *Z* is relative height from reference position, *V* is a volume of j^{th} cell, *R* is a residual vector, H_V is a main source vector, D_V, S_V are the auxiliary source vectors and s is a cross-section of pipe.

$$V\frac{Q^{n+1} - Q^n}{\Delta t} = -\left(s_{j+\frac{1}{2}}F_{j+\frac{1}{2}} - s_{j-\frac{1}{2}}F_{j-\frac{1}{2}}\right) + H_V = -R(Q) \tag{6}$$

where $Q = \begin{pmatrix} \rho \\ \rho U \\ \rho e_t \end{pmatrix}, \quad F = \begin{pmatrix} \rho U \\ \rho U^2 + p \\ (\rho e_t + p) U \end{pmatrix}, \quad e_t = e + \frac{1}{2}U^2 + gZ$

The main source vector H_V is associated with pressure force due to different cross sections, wall friction and gravity force, where D_H is hydraulic diameter, *Pr* is perimeter of the cross-section, g_x is x component of gravity acceleration vector, $\tau_{w,S}$ is steady part of wall shear stresses, $\tau_{w,U}$ is unsteady part of wall shear stresses, C_f is skin friction, $\dot{U}$ is instantaneous acceleration, *m* is mass of fluid in j^{th} cell and Δx is grid-size.

$$H_V = \begin{bmatrix} 0 \\ (\tau_{w,S} + \tau_{w,U})Pr\Delta x + p(s_{j+1/2} - s_{j-1/2}) + mg_x \\ 0 \end{bmatrix} \tag{7}$$

where $\tau_{w,S} = C_{f,S}\,\frac{1}{2}\rho U|U|, \qquad \tau_{w,U} = C_{f,U}\,\frac{1}{4}\rho D_H \dot{U}$

The proposed GJM is expected to reflect the effect of a linear momentum interaction for arbitrary combinations of junction and branches. Additionally, non-iterative GJM is also expected to consume less computing times than the conventional iterative methods. However, take notice that GJM may not be able to represent non-linear effects by itself and the only way to account for the nonlinearities is considered as to incorporate loss coefficients.

Finally, to overcome difficulties from heterogeneity of spatial dimension between junction(3D) and branches(1D), numerical analyses are conducted using Roe’s FDS and then a scaling function is designed to control the problematic source produced at interfaces between junction and branches. The abrupt change of pressure at interfaces due to this source is generally expressed as (8).

$$\Delta p = -\rho a \Delta U_n - \rho a \frac{1}{n_x + n_y + n_z}\left(\frac{M + |M|}{1 + M}\right)\Delta U_t \tag{8}$$

2.2 Scaling function

From analogy to change of linear momentum and large amount of numerical tests, a non-dimensional scaling function is designed as (8). In subsonic region, the function has a positive value of magnitude from zero to one.

$$G = \frac{|\Delta U_n|}{a}, \qquad 0 < G < 1 \tag{9}$$

Replacing ΔU_n with $G\Delta U_n$, equation (8) can be written as.

$$\Delta p + \rho|\Delta U_n|\Delta U_n = -\rho a \frac{1}{n_x + n_y + n_z}\left(\frac{M + |M|}{1 + M}\right)\Delta U_t \tag{10}$$

See the left hand side of (10) is analogous to change of linear momentum in agreement with our goal and this implies that linear momentum interaction is strongly affected by ΔU_t and pipe- direction vector (n_x, n_y, n_z). Moreover, the relation (10) justifies that the condition C3 in (2) should be applied to generally account for the effect of branch angle.

Herein, the scaling process can be implemented through a projection method and among all primitive variables in the ghost junction cell, only the velocity components should be projected by (11), where the interface, the left cell and the right cell of the interface are denoted by the subscript j+1/2, j and j+1 respectively.

$$\begin{cases} u_{j+1/2}^L = u_{j+1}^n - G\Delta u^n + u_j^t \\ u_{j+1/2}^R = u_{j+1} \end{cases} \tag{11}$$

3. Numerical Tests

General features of GJM are investigated numerically. Test configurations and the results are shown in Fig.1~2 and in Fig.3~12.

Theoretical and experimental energy loss coefficients are referred in [6-8]. Also, the empirical correlations are referred in [8].

3.1 Test Configurations

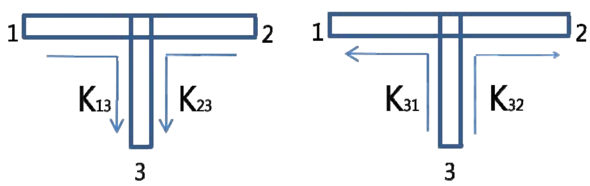

Fig.1 Counter combinig and Counter dividing

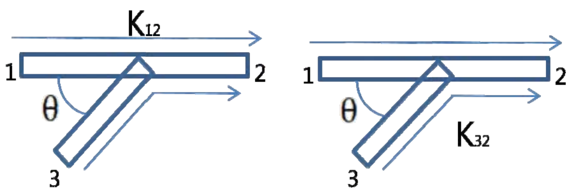

Fig.2 Straight combinig and Branch combining

3.2 Test Results

(1)Counter combining at low Mach ($A_r = \frac{A_1}{A_3} = 1,\ Q_r = \frac{\dot{m}_1}{\dot{m}_3}$)

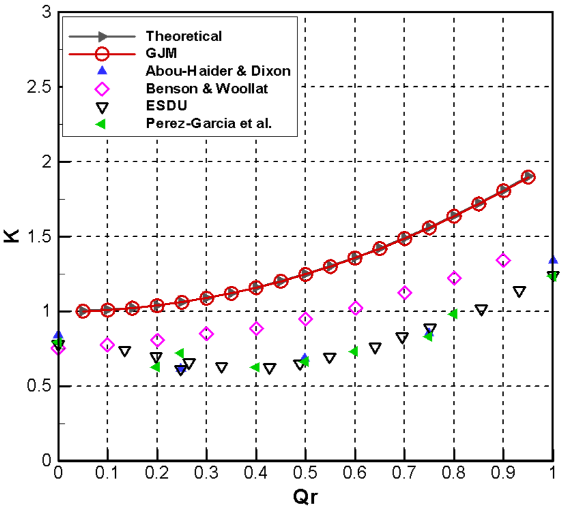

Fig.3 K_{13} - Counter combinig ($A_r = 1,\ \theta = 90^o$)

(2)Counter combining at high Mach ($A_r = \frac{A_1}{A_3} = 1,\ Q_r = \frac{\dot{m}_1}{\dot{m}_3}$)

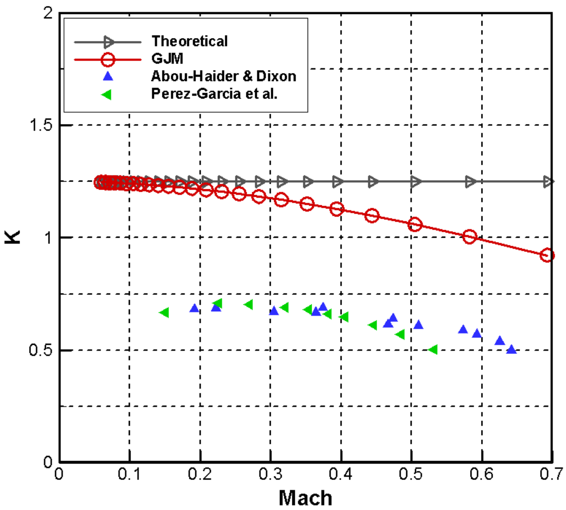

Fig.4 K_{13} - Counter combinig ($Q_r = 0.5$)

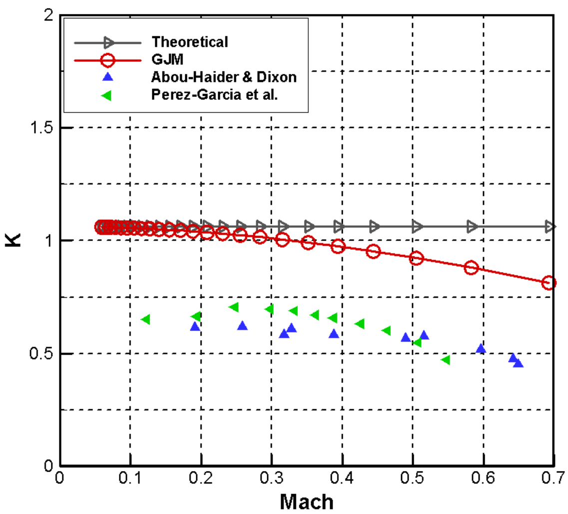

Fig.5 K_{13}- Counter combinig ($Q_r = 0.25$)

(3)Counter dividing at low Mach ($A_r = \frac{A_1}{A_3} = 1,\ Q_r = \frac{\dot{m}_1}{\dot{m}_3}$)

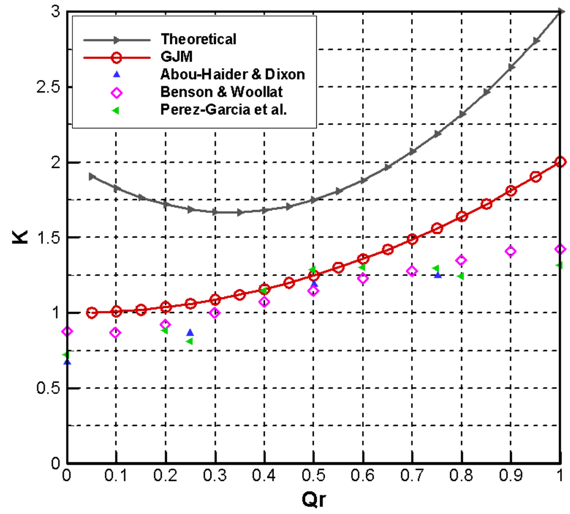

Fig.6 K_{31}- Counter dividinig ($A_r = 1,\ \theta = 90^o$)

(4)Counter dividing at high Mach ($A_r = \frac{A_1}{A_3} = 1,\ Q_r = \frac{\dot{m}_1}{\dot{m}_3}$)

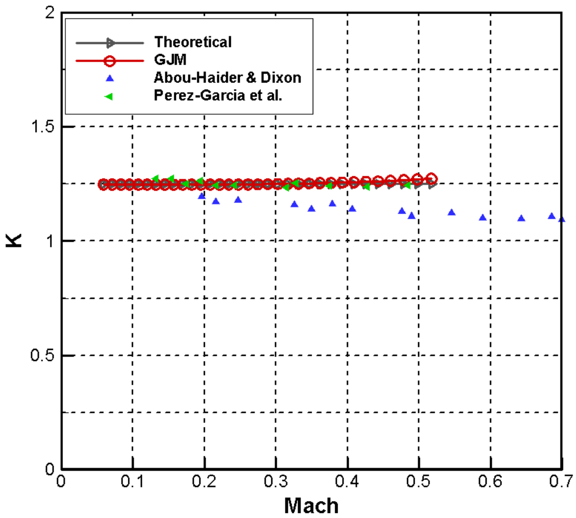

Fig.7 K_{31}- Counter dividing ($Q_r = 0.5$)

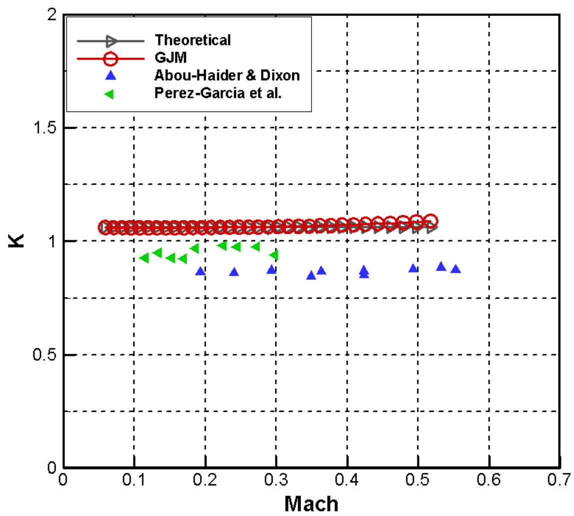

Fig.8 K_{31}- Counter dividing ($Q_r = 0.25$)

(4)Straight combining ($A_r = \frac{A_2}{A_3} = 1,\ Q_r = \frac{\dot{m}_3}{\dot{m}_2}$)

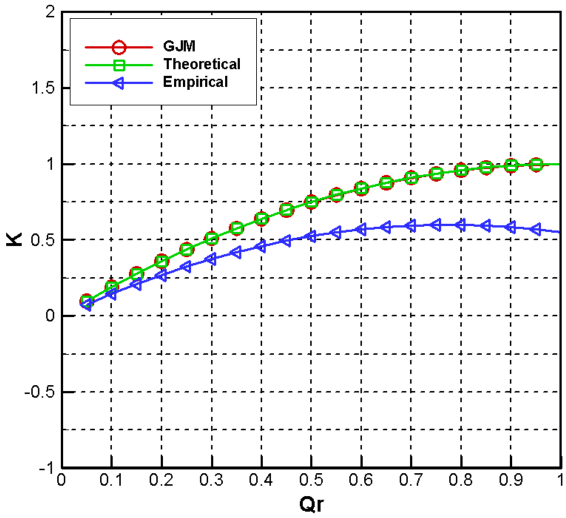

Fig.9 K_{12}-Straight combinig ($A_r = 1,\ \theta = 90^o$)

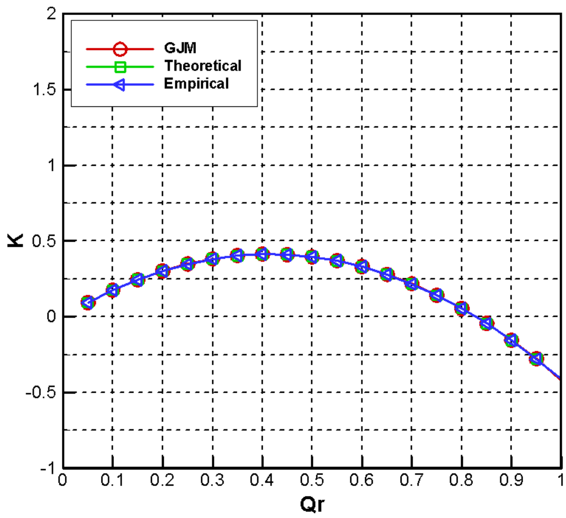

Fig.10 K_{12}- Straight combinig ($A_r = 1,\ \theta = 45^o$)

(5)Branch combining ($A_r = \frac{A_2}{A_3} = 1,\ Q_r = \frac{\dot{m}_3}{\dot{m}_2}$)

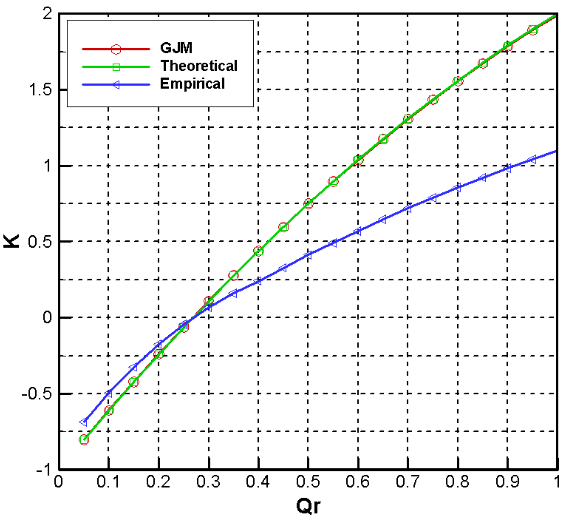

Fig.11 K_{32}- Branch combinig ($A_r = 1,\ \theta = 90^o$)

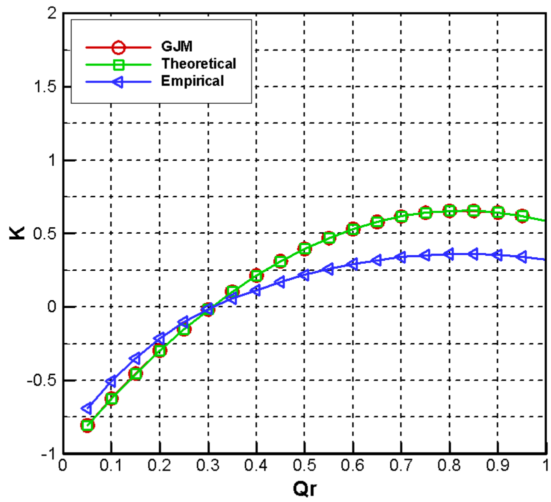

Fig.12 K_{32}- Branch combinig ($A_r = 1,\ \theta = 45^o$)

4. Discussion and Conclusions

A new finite volume junction treatment GJM is proposed and tested numerically at T-type junction configurations. The GJM shows a good agreement with theoretical linear momentum analyses. Moreover, in the case of high Mach flows as shown in Fig.4~5, it predicted loss coefficients to be intermediate value between experimental coefficients and theoretical one. Consequently, GJM is expected to be an alternative method which can predict minor losses at junction without empirical correlations because it is designed to generally reflect linear momentum interactions for arbitrary junction-branch configurations. Additionally, non-iterative GJM is also expected to have an advantage of fast computations over the conventional iterative methods.

REFERENCE

[1] **V. Guinot**, Boundary Condition Treatment in 2x2 Systems of Propagation Equations, International Journal for Numerical Methods in Engineering, 42(1998), pp.647-666.

[2] **T. Kiuchi**, An implicit method for transient gas flows in pipe networks, International Journal of Heat and Fluid Flow, 15(1994), pp.378-383.

[3] **J. J. Guy**, Computation of unsteady gas flow in pipe networks, Institute of Chemical Engineers 1967, London, Symposium Series No. 23, pp.139-145.

[4] **A. Osiadacz**, Simulation of Transient Gas Flows in Networks, International Journal for Numerical Methods in Fluids, 4(1984), pp.13-24.

[5] **M. Herty and M. Seaïd**, Simulation of transient gas flow at pipe-to-pipe intersections, International Journal for Numerical Methods in Fluids, 56(2007), pp. 485-506

[6] **E. Fried and I. E. Idelchik**, Flow Resistance: A Design Guide for Engineers, Taylor&Francis, Philadelphia, 1989.

[7] **K. Oka and H. Ito**, Energy Losses at Tees With Large Area Ratios, Journal of Fluid Engineering , 127(2005), pp.110-116.

[8] **J. Perez-Garcia *et al.***, Numerical and experimental investigations on internal compressible flow at T-type junctions, Experimental Thermal and Fluid Science, 31(2006), pp.61-74.