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Trust Region 기법을 이용한 공력 형상 최적설계

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The Aerodynamic Shape Optimization with Trust Region Methods

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Abstract

In this paper the trust region method is studied and applied in aerodynamic shape optimization. The trust region method is a gradient-based optimization method, but it is not as popular as other methods in engineering computations. Its theory will be explained for unconstrained optimization problems and a trust region subproblem will be solved with the dogleg method. After verifying the trust region method with analytical test problems, it is applied to aerodynamic shape design optimization and the performance of airfoil is improved successfully.

1. Introduction

Unconstrained optimization methods have been used in the aerodynamic shape optimization combined with an adjoint sensitivity method to improve the performance of an airfoil or a wing[1]. By adding penalty terms to an objective function, we can realize constraints in the unconstrained optimization. Most researchers have used line search methods. Trust region methods have shown good performance in the unconstrained optimization[2-3]. In this paper, the trust region method is studied and its capability in the unconstrained optimization will be shown for analytical test problems and the aerodynamic shape optimization of an airfoil will be performed.

2. Trust Region Method

A line search method finds the step size and direction successively, whereas the trust region method conducts concurrent search of step size and direction. Therefore, there can be less function calling in the trust region method if both methods need same optimization iterations. The general theory of trust region methods will be explained in this study. In trust region methods, a direction and step are sought simultaneously within a trust region radius Δ_k to give the best improvement to an objective function *f*. The improvement is determined by calculating a model function m_k. One of popular model functions is shown in Eq. (1).

$$m_k(x_k + p) = f_k + (\nabla f_k)^T p + \frac{1}{2} p^T B_k p \tag{1}$$

In Eq. (1), *p* is a direction vector at the kth iteration, f_k is a gradient vector and B_k is a Hessian matrix. In trust region methods, a step is found by minimizing the model function within the trust region radius. The trust region is increased when the found step gives a sufficient decrease of the objective function and the trust region is decreased when the step gives an insufficient decrease of the objective function. In determining the size of the trust region radius, the following ratio is used.

$$\rho_k = \frac{f(x_k) - f(x_k + p_k)}{m_k(0) - m_k(p_k)} \tag{2}$$

Eq. (2) defines a ratio of the actual reduction to the predicted reduction and ρ is called a reduction ratio. As this ratio approaches to 1, it means that the model function approximates the objective function closely, then the trust region is increased. When the ratio is smaller than 1, the trust region is decreased.

With Eq. (1) and the constraint of the trust region radius, the following trust region subproblem is constructed and it is solved iteratively to find the step *p*.

$$\begin{aligned} &\min\ m(p) \overset{def}{=} f + (\nabla f)^T p + \frac{1}{2} p^T B p \\ &s.t.\ \|p\| \le \Delta \end{aligned} \tag{3}$$

In Eq. (3), Δ is a trust region radius. Eq. (3) can be solved with various trust region methods. The dogleg method, Steihaug's method, and Moré-Sorensen method are popular in solving the trust region subproblem. The dogleg method and Steihaug's method need low cost and the convergence is robust. Moré-Sorensen method uses nearly exact solutions of the subproblem, but the calculation cost is higher than other methods[4]. In this study, the dogleg method will be used.

2.1 Dogleg Method

The dogleg method finds an approximate solution of the trust region subproblem by replacing optimal trajectory with a path whose shape is similar to dog's leg. In Fig. 1, a solid line represents the dogleg path. It is the combination of a Newton step and an unconstraint steepest minimizer.

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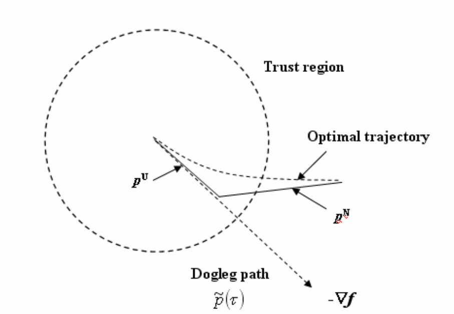

Fig. 1 Shape of dogleg path

The Newton step is computed with the following formula

$$p^N = -B^{-1}\nabla f \tag{4}$$

The unconstrained steepest descent minimizer for the model function is computed as followings:

$$p^U = -\frac{(\nabla f)^T \nabla f}{(\nabla f)^T B \nabla f}\nabla f \tag{5}$$

Finally the dogleg path is defined as

$$\tilde{p}(\tau) = \begin{cases} \tau p^U & , 0 \le \tau \le 1 \\ p^U + (\tau - 1)\left(p^B - p^U\right) & , 1 \le \tau \le 2 \end{cases} \tag{6}$$

The dogleg path intersects the trust region boundary at least once, and this point is set as a new step if it gives sufficient decrease of the objective.

3. Numerical Tests with Analytical Problems

To verify the performance of the trust region method, analytical problems are tested. Hock and Schittkowski(HS) collected analytical test problems from the literature and their problems have been used in verifying and comparing of mathematical programming methods[5,6]. This problem set is also included in the CUTE[7]. The number of design variable is at most 100, so this problem set is for testing small-scale problems. To compare the performance of trust region methods, BFGS is used. Moreover, Eq. (3) is solved with a usual constrained optimization algorithm, that is, the modified method of feasible direction(MMFD).

Among 395 problems, 91 unconstrained problems are tested. The BFGS method found 25 optimum values, the MMFD found 12 optimum values, and the dogleg method found 59 optimum values. Because the BFGS is one of popular unconstrained optimization algorithms, we can conclude that the trust region method shows good performance. In realizing the trust region method, it seems inefficient solving the trust region subproblem with a constrained optimization algorithm. As shown in this numerical test, solving Eq. (3) with the MMFD showed worse performance than with the dogleg method. Figs. 2-6 show the comparison of analytical optimum values f* and calculated optimum values with optimization methods.

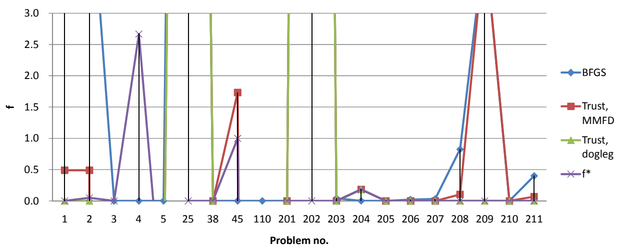

Fig. 2 Comparison of optimum values of test problem 1-211

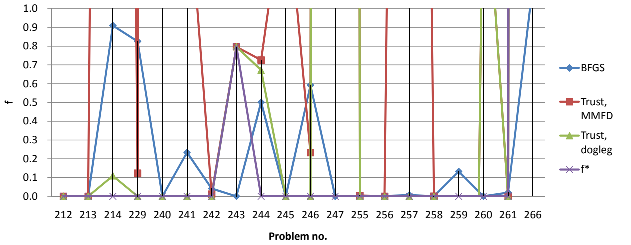

Fig. 3 Comparison of optimum values of test problem 212-266

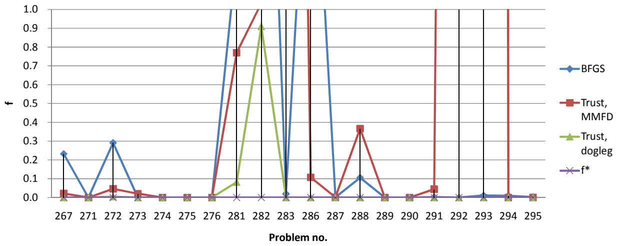

Fig. 4 Comparison of optimum values of test problem 267-295

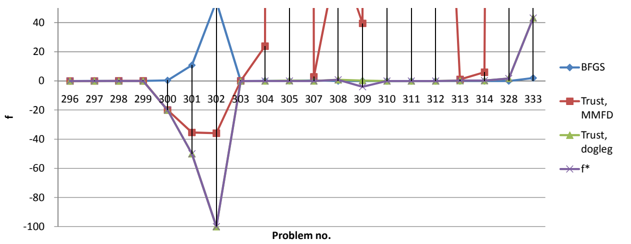

Fig. 5 Comparison of optimum values of test problem 296-333

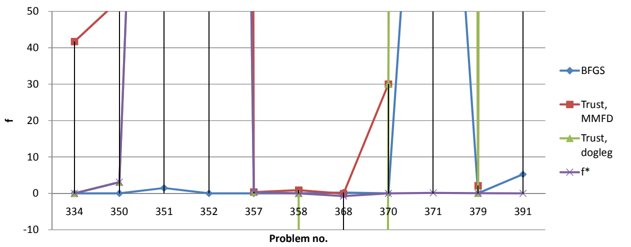

Fig. 6 Comparison of optimum values of test problem 334-391

4. Aerodynamic Shape Optimization

An unconstrained aerodynamic shape optimization is performed with the trust region method. The objective is to minimize the drag while maintaining the desired lift. For such purpose, an objective can be constructed as follows:

$$\min \frac{1}{2}(c_l - c_{l0})^2 + \frac{10}{2}c_d^2 \tag{7}$$

Five Hicks-Henne functions are used to deform the upper surface of airfoil and a number of design variable is five[8]. The sensitivity is computed with a forward finite difference method. The performance of the dogleg method is compared with the BFGS method and MMFD as the numerical tests with analytical problems. The initial airfoil shape is RAE2822 and the Euler equations are solved for the aerodynamic analysis[9]. In the analysis, Mach number is 0.73, the angle of attack is 2.78°, and c_{l0} is 0.8911.

Table 1 and Figs. 7-8 show the summary of optimization results. Results of all three methods are similar, but the number of optimizer iteration is higher for the dogleg method than other methods. The dogleg method shows the lowest objective and the lowest constraint violation. Figs. 7-8 show designed shapes of RAE2822 airfoil and pressure distributions and they accord well each other. Figs. 9-12 show variations of the trust region radius, the reduction ratio, the drag and lift coefficients for the trust region methods. With the dogleg method, the trust region radius changes slightly after the 8th iteration. Similar trends are observed in Figs. 10-12. It seems that the dogleg method reached around the optimum value at the 8th iteration and then it moved around the optimum value a little.

Table 1. Results of aerodynamic optimization of RAE2822

	Objective	c_l	c_d	Iteration
BFGS	0.61075E-03	0.88994	0.11046E-01	9
Trust, MMFD	0.61158E-03	0.88987	0.11053E-01	9
Trust, Dogleg	0.60946E-03	0.89040	0.11039E-01	16

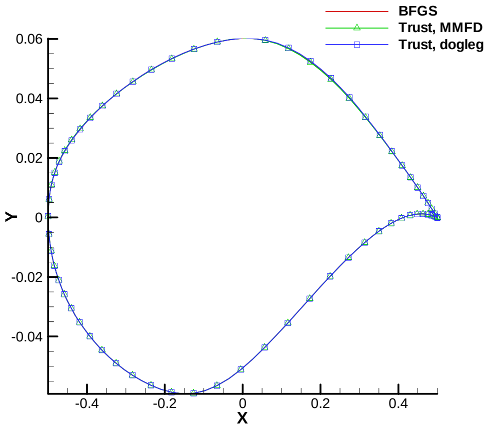

Fig. 7 Designed shapes of RAE2822

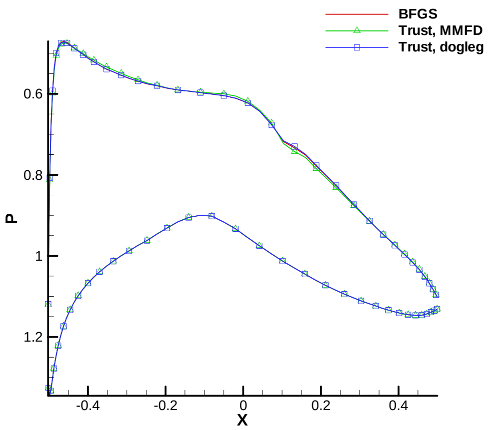

Fig. 8 Pressure distributions of RAE2822

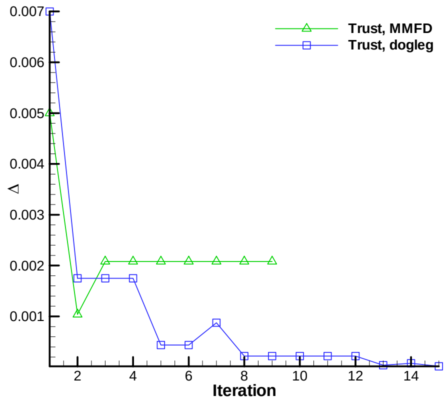

Fig. 9 The history of the trust region radius

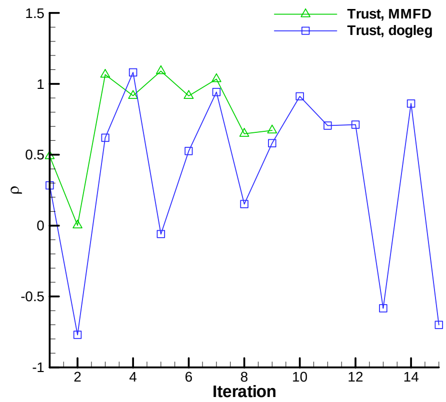

Fig. 10 The history of the reduction ratio

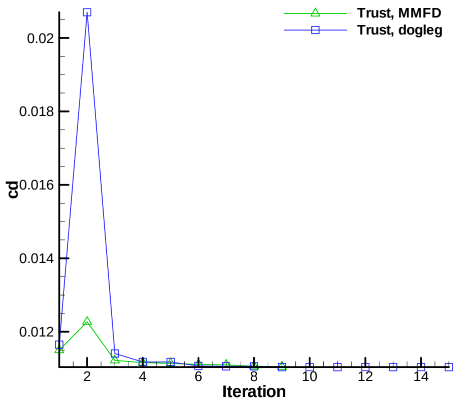

Fig. 11 The history of the drag coefficient

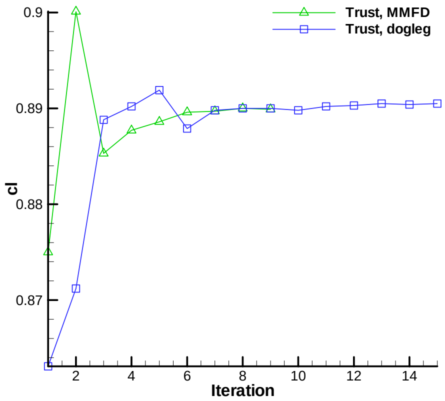

Fig. 12 The history of the lift coefficient

5. Conclusion

In this study, the trust region method is studied and applied in the aerodynamic shape optimization of an airfoil. The trust region subproblem is solved with the dogleg method. Also, the subproblem is solved with the MMFD and the performance of two trust region methods are compared with the BFGS method. In the analytical test problems, the dogleg method showed the best performance. In the aerodynamic shape optimization, however, all three methods successfully weakened the shock strength of the RAE2822 airfoil. The MMFD and the BFGS method exhibited low calculation cost, but the dogleg method exhibited a little lower objective and lower constraint violation. It can be concluded that the performance of the trust region methods are competitive with other methods and its future is promising.

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