준정상 이론에 의한 교량 플러터의 간략식

A Simplified Formula of Bridge Deck Flutter Based on the Quasi-Steady

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유체내에 잠겨있는 물체의 진동은 공기력을 유발시키며 이러한 공기력에 의해 발생되는 진동을 물체의 거동에 의해 발생되는 가진이라 한다. 또한 물체에 작용하는 외부 공기력이 없이도 물체의 주기적인 움직임에 의해 발생되는 에너지로부터 공기력을 생성시킨다. 이러한 메커니즘에 의해 생성되는 공기력을 공기자발력(self-excited force) 이라 하며 교량의 내풍안정성과 관련이 있다. 본 논문에서는 MIE 메커니즘에 의해 발생되는 플루터 현상을 수학적으로 살펴보고, 단일모드에 대한 플러터계수를 이용한 플러터 발생풍속 산정식을 유도하였다. 또한 준정상 이론을 적용하여 단일모드에 대한 플러터 발생 예측식을 간략화하였다. 제안된 식의 플러터 발생풍속을 구조물의 진동수비가 서로 다른 3개의 π형 단면에 대해 검토하였다.

핵심 용어 : 플러터, 간략식, 준정상, 공탄성

1. INTRODUCTION

Wind-induced phenomena have been treated by a variety of engineering disciplines, each having its particular terminology. At a critical wind speed, commonly referred to as the negative damping threshold, this type of mechanism may eventually lead to destructive forces on the bridge, as was the case in the collapse of Tacoma Narrows Bridge in 1940. The existing bridge state-of-art aeroelastic response methodology owes its origin to the studies made earlier on airfoil or thin plate theory. Classical theories for the analysis of airfoils and thin plates were developed to better understand the response characteristics of fixed-wing aircraft. The basic ideas behind their formulations are still being used by researchers and form the basis of current bridge aerodynamcis analyses. In this paper, aerodynamic stability based on quasi-steady assumption are briefly described and a comparison researches on flutter derivatives are introduced from the point of aerodynamic force coefficients related with the aerodynamic derivatives.

2. BACKGROUND THEORY

The aerodynamic forces as shown in Fig. 1 are separated into aerodynamic force and buffeting components. Movement-induced excitation is due to the aerodynamic force that arise from movements of the vibration body oscillator. Even without an external exciting aerodynamic force, a body oscillator may undergo sustained vibration if there is an energy source from which the oscillator can extract energy during each cycle of free movement. This type of Vibration is called self excited and is related to aerodynamic stability (flutter).

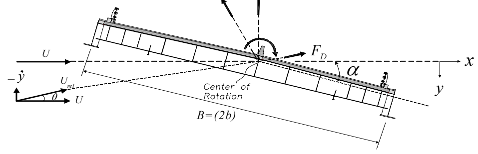

Fig. 1 Aerodynamic forces on bridge deck

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The full multi-mode system of equations can be expressed in matrix notation as

$$\hat{\mathbf{M}}^s \ddot{\boldsymbol{\eta}}(t) + \hat{\mathbf{C}}^s \dot{\boldsymbol{\eta}}(t) + \hat{\mathbf{K}}^s \boldsymbol{\eta}(t) = \hat{\mathbf{F}}^{ae}(x,t,\eta_n,\dot{\eta}_n,\ddot{\eta}_n) \tag{1}$$

where η = generalized coordinate vector, **C** and **D** = the modal damping and stiffness matrices of the system, respectively, $\hat{\mathbf{F}}^{ae}(x,t,\eta_n,\dot{\eta}_n,\ddot{\eta}_n)$ = the aeroelastic force.

Taking the Fourier transform on either side of Eq.(1) and then Eq.(2) is modal equilibrium equation in frequency domain. For massive long bluff bodies in air flow, such as bridge decks, inertial components can be neglected.

$$\left(-\hat{\mathbf{M}}^{\mathbf{s}}\omega^2 + i\hat{\mathbf{C}}^{\mathbf{s}}\omega + \hat{\mathbf{K}}^{\mathbf{s}}\right)\cdot \hat{\mathbf{X}}_\eta(\omega) = \left(i\hat{\mathbf{C}}^{\mathbf{ae}}\omega + \hat{\mathbf{K}}^{\mathbf{ae}}\right)\cdot \hat{\mathbf{X}}_\eta \tag{2}$$

where

$$\mathbf{C}_{ae} = \begin{bmatrix} H_1^* & BH_2^* \\ BA_1^* & B^2A_2^* \end{bmatrix}, \qquad \mathbf{K}_{ae} = \begin{bmatrix} H_4^* & BH_3^* \\ BA_4^* & B^2A_3^* \end{bmatrix} \tag{3}$$

$\hat{\mathbf{C}}^{\mathbf{ae}}$ and $\hat{\mathbf{K}}^{\mathbf{ae}}$ = modal aeroelastic damping matrix and adroelastic stiffness matrix to be defined in terms of the aerodynamic derivatives, respectively. i = imaginary unit. Substituting Eq.(3) into Eq.(2), the compliance matrix (non-dimensional frequency response matrix, $\hat{\mathbf{H}}_\eta(\omega)$) is obtained as Eq. (4).

$$\left[-\left(\hat{\mathbf{M}}^s\right)\omega^2 + i\left(\hat{\mathbf{C}}^s - \hat{\mathbf{C}}^{ae}\right)\omega + \left(\hat{\mathbf{K}}^s - \hat{\mathbf{K}}^{ae}\right)\right]\cdot \hat{\mathbf{X}}_{\boldsymbol{\eta}}(\omega) = \mathbf{Q}_{\tilde{\mathbf{F}}}(\omega) \tag{3}$$

$$\hat{\mathbf{H}}_\eta(\omega) = \left[-\left(\hat{\mathbf{M}}^s\right)\omega^2 + i\left(\hat{\mathbf{C}}^s - \hat{\mathbf{C}}^{ae}\right)\omega + \left(\hat{\mathbf{K}}^s - \hat{\mathbf{K}}^{ae}\right)\right]^{-1} \tag{4}$$

The impedance matrix (dynamic stiffness) is then

$$\hat{\mathbf{S}}_\eta(\omega_r,u_{cr}) = \left[\mathbf{I} - \boldsymbol{\kappa}_{jk} - diag\left[\frac{1}{\omega_j^2}\right]\omega_r^2 + 2i\cdot\omega_r\cdot diag\left[\frac{1}{\omega_j}\right]\left(diag\left[\zeta_j\right] - \boldsymbol{\zeta}_{jk}\right)\right] \tag{5}$$

where ω_r and u_{cr} = the corresponding in-wind preference or resonance frequency and critical velocity.

$$\boldsymbol{\kappa}_{jk} = \frac{1}{\omega_j^2\tilde{m}_j}\hat{\mathbf{K}}_{jk}^{ae}, \qquad \boldsymbol{\xi}_{jk} = \frac{\omega_j}{2}\cdot\frac{1}{\omega_j^2\tilde{m}_j}\hat{\mathbf{C}}_{jk}^{ae} = \frac{1}{2\omega_j\tilde{m}_j}\hat{\mathbf{C}}_{jk}^{ae} \tag{6}$$

It has been considered convenient to normalized aerodynamic damping and stiffness with $\rho B^2\omega_r/2$ and $\rho B^2\omega_r^2/2$ in Eq.(6), where ω_r is the in-wind resonance frequency.

$$\kappa_{11} = \frac{\rho B^2}{2\tilde{m}_1}\left(\frac{\omega_r}{\omega_1}\right)^2 H_4^*, \quad \kappa_{12} = \frac{\rho B^3}{2m_1}\left(\frac{\omega_r}{\omega_1}\right)^2 H_3^*, \quad \kappa_{21} = \frac{\rho B^3}{2m_1}\left(\frac{\omega_r}{\omega_2}\right)^2 A_4^*, \quad \kappa_{22} = \frac{\rho B^4}{2m_2}\left(\frac{\omega_r}{\omega_2}\right)^2 A_3^* \tag{7}$$

$$\zeta_{11} = \frac{\rho B^2}{4m_1}\left(\frac{\omega_r}{\omega_1}\right)H_1^*, \quad \zeta_{12} = \frac{\rho B^3}{4m_1}\left(\frac{\omega_r}{\omega_1}\right)H_2^*, \quad \zeta_{21} = \frac{\rho B^3}{4m_2}\left(\frac{\omega_r}{\omega_2}\right)A_1^*, \quad \zeta_{22} = \frac{\rho B^4}{4m_2}\left(\frac{\omega_r}{\omega_2}\right)A_2^* \tag{8}$$

Unstable behavioris caused by the effects of $\boldsymbol{\kappa}_{jk}$ and $\boldsymbol{\xi}_{jk}$. The effects of $\boldsymbol{\xi}_{jk}$ is to change the damping properties of the combined structure and flow system, while the effects of $\boldsymbol{\kappa}_{jk}$ is to change the stiffness properties. The bridge deck extracts energy from the flow that may result in a continuously growing response if this energy exceeds the energy dissipated, and in the limit state the structural displacement response will become infinitely large if the absolute value of the determinant to the non-dimensional impedance matrix is zero. The unstable behavior contains a combined motion in degree of freedom, in which case the instability limit may be identified form Eq.(5). Otherwise, A purely single mode unstable behaviorcontains motion either in the vertical direction or in torsion. Such an instability limit may then be identified from the first of the second row of the matrices in Eq. (5) and is expressed as simplified formula. Based on the quasi-steady, the aerodynamic derivatives are expressed in terms of static force coefficients as and non-dimensional velocity (Chen et al. 2002, Cho et al. 2007).Matsumoto (1996) proposed the relationships between the aerodynamic derivatives as follows (Matsumoto et al. 1996)

$$H_1^* = kH_3^*,\quad H_4^* = -kH_2^*,\quad A_1^* = kA_3^*,\quad A_4^* = -kA_2^* \qquad (9)$$

Adopting the relationship, Eq. (9), aerodynamic derivatives can be represented

$$\kappa_{11} = \frac{\rho B^2}{2\tilde{m}_1}\frac{U^2}{B^2\omega_1^2}(C_l' + C_d),\quad \kappa_{12} = \frac{\rho B^3}{2m_1}\frac{U^2}{B^2\omega_1^2}C_l',\quad \kappa_{21} = \frac{\rho B^3}{2m_1}\frac{U^2}{B^2\omega_1^2}C_m',\quad \kappa_{22} = \frac{\rho B^4}{2m_2}\frac{U^2}{B^2\omega_2^2}C_m' \qquad (10)$$

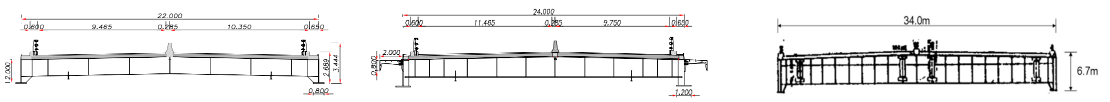
$$\zeta_{11} = \frac{\rho B^2}{4m_1}\frac{U}{B\omega_1}\left(C_l' + C_d\right),\quad \zeta_{12} = \frac{\rho B^3}{4m_1}\frac{U}{B\omega_1}\left(C_l' + C_d\right),\quad \zeta_{21} = \frac{\rho B^3}{4m_2}\frac{U}{B\omega_2}C_m',\quad \zeta_{22} = \frac{\rho B^4}{4m_2}\frac{U}{B\omega_2}C_m' \qquad (11)$$

The bridge deck extracts energy from the flow that may result in a continuously growing response if this energy exceeds the energy dissipated, and in the limit state the structural displacement response will become infinitely large if the absolute value of the determinant to the non-dimensional impedance matrix is zero. The unstable behavior contains a combined motion in d.o.f, in which case the instability limit may be identified form Eq.(5). Otherwise, A purely single mode unstable behavior contains motion either in the vertical direction or in torsion. Such an instability limit may then be identified from the first of the second row of the matrices in Eq. (5) and is expressed as simplified formula. Dynamic stability limit in vertical and torsional direction are defined by the following mean wind velocity.

$$V_{cr} = B\cdot\omega_1\cdot\frac{4\tilde{m}_1}{\rho B^2}\cdot\frac{\zeta_1}{-\left(C_L' - C_D\cdot D/B\right)} \quad \text{(in vertical dir.) (12)} \qquad V_{cr} = B\cdot\omega_2\cdot\left[\frac{2\tilde{m}_2}{\rho B^4}\cdot\frac{1}{C_M'}\right]^{1/2} \quad \text{(in torsional dir.) (13)}$$

3. NUMERICAL EXAMPLE

Three different 2-edge girder models were made to investigate flutter derivatives. Sec-1, Sec-2 were constructed at a geometric scale of 1:50 (tested in Univ. of Western Ontario) and Sec-3 model was test in Korea Univ., respectively (Cho et al.1, 2006). Structural frequency ratio(f_h / f_α) is given as follow : Sec-1 = 3.0, Sec-2 = 2.59 and Sec-3 = 1.84.



(a) Sec-1 (b) Sec-2 (c) Sec-3

Fig. 2 Bridge deck sections

The conversion of the flutter derivatives to an equivalent aeroelastic damping as a ratio to critical is through the following relationship (Eq. 14 and 15) for the vertical and torsional responses respectively. Therefore, a positive value of the aerodynamic derivativeis indicative of negative aerodynamic damping.

$$\zeta_h^{ae} = \left(\frac{\omega_1}{\omega_r}\right)\zeta_h - \frac{\rho B^2}{4m}H_1^*\left(\frac{U}{\omega_r B}\right) \qquad (14)$$

$$\zeta_\alpha^{ae} = \left(\frac{\omega_1}{\omega_r}\right)\zeta_\alpha - \frac{\rho B^4}{4I}A_2^*\left(\frac{U}{\omega_r B}\right) \qquad (15)$$

In Fig.3 the reduced wind speed is plotted against the in-wind frequency and damping ratio for the three reference sets. As the onset of flutter is initiated when the net damping ratio becomes zero, the critical flutter speed can be accurately evaluated based on the numerical results from Fig. 3. The in-wind damping in torsional motion represents the negative sign beginning at reduced wind speeds of about 4.68, 7.23 and 3.58.

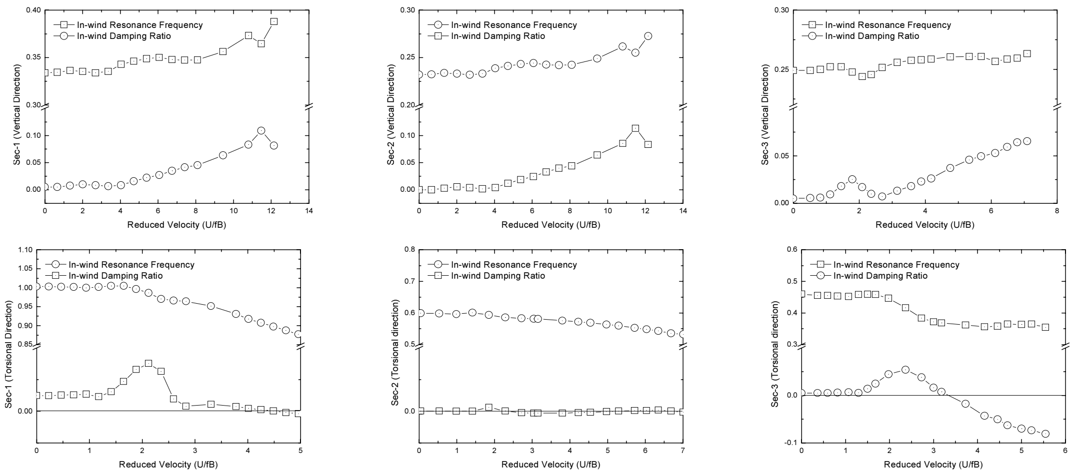


Fig. 3 Equivalent in-wind resonance frequency and damping ratio

Table 1 illustrates the resulting real critical flutter speeds (V_{cr}) which were evaluated by wind tunnel test, Eq. (18) and Eq. (20). A simplified flutter formula(Eq.13) based on the quasi-steady theory have critical flutter wind velocities which are approximately 6%~18% higher than Eq. 15. Compared to the cases 1~3, 2-edge girder section, the critical flutter wind velocity increases with related to the structural frequency ratio and bridge mass moment of inertia.

Table 1. Results for the case studies

Case	Wind Tunnel Test	Equation (13)	Equation (15)	**Difference**
1	Not Observed	**121.5 m/s**	103.3 m/s	18 %
2	Not Observed	**121.8 m/s**	104.1 m/s	17 %
3	52.4 m/s	**59.24 m/s**	55.9 m/s	6 %

4. CONCLUSION REMARK

In this paper an approximate method to calculate flutter critical wind speed and damping ratiois presented. The proposed simplified formula based on the quasi-steady theory is only applicable to cases in which the bending-torsional modes are uncoupled. The proposed simplified formula have critical flutter wind velocities which are approximately 6%~18% higher than purely s.d.o.f equivalent aeroelastic damping equations with flutter derivatives (Eq. 14-15). Nevertheless, it helps to better understand the motion induced vibration (MIE), flutter mechanism and represents the first step toward a simple engineering tool is capable of estimating the prediction of the critical wind speed without flutter derivatives. Finally, the flutter velocity by similar sections with 2-edge girder section is investigated. In particular, it is shown that, the critical flutter wind velocity increases with related to the structural frequency ratio and bridge mass moment of inertia.

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