Proceedings of KIIS Spring Conference 2008 Vol. 18, No. 1.

On fuzzy preinvex mappings associated with interval-valued Choquet integrals

* Lee Chae Jang, ** Hyun Mee Kim,
* Dept. of Mathematics and Computer Science, Konkuk University
** Dept. of Mathematics Kyunghee University

Abstract

In this paper, we consider define fuzzy invex sets and fuzzy preinvex functions on the class of Choquet integrable functions, and interval-valued fuzzy invex sets and interval-valued fuzzy preinvex functions on the class of interval-valued Choquet integrals. And also we prove some properties of them.

Key words : fuzzy invex set, fuzzy preinvex mapping, interval-valued Choquet integrals, comonotonically additive.

1. Introduction

Jang et al. have been studied interval-valued Choquet integrals with respect to fuzzy measures [3,4,5]. In this paper, we introduce the concepts of fuzzy invex sets and fuzzy preinvex functions on the class of Choquet integrable functions, and interval-valued fuzzy invex sets and interval-valued fuzzy preinvex functions on the class of interval-valued Choquet integrals. We also discuss fuzzy preinvex mapping ϕ_c^* defined by Choquet integrals.

2. results

Throughout this paper, we assume that X is a locally compact Hausdorff space, Ω is a $\sigma-$algebra of X, M is the class of measurable functions of X, M^+ is the class of non-negative measurable functions in M, and O is the class of open subsets of X, K is the class of continuous functions on X with compact support, and K^+ is the class of non-negative functions in K.

Definition 2.1 [3,4,5] A closed set-valued function F is said to be measurable if for each open set $O \subset R^+$,

$$F^{-1}(O) = \{x|F(x) \cap O\} \neq \varnothing \in \Omega.$$

Definition 2.2 [3,4,5] Let F be a closed set-valued function. A measurable function $f: X \to R^+$ satisfying $f(x) \in F(x)$ for all $x \in X$ is called a measurable selection of F.

Definition 2.3 [3,4,5] (1) Let F be a closed set-valued function and $A \in \Omega$. The set-valued Choquet integral of F on A is defined by

$$(C)\int_A F d\mu = \{(C)\int_A f d\mu | f \in S(F)\},$$

where $S(F)$ is the family of $\mu - a.e.$ measurable selections of F.

Theorem 2.4 ([3,4,5]). A closed set-valued function F is measurable if and only if there exists a sequence of measurable selections $\{f_n\}$ of F such that

$$F(x) = cl\{f_n(x)\} \text{ for all } x \in X.$$

Theorem 2.5 ([3.4.5]). If F is a closed set-valued function and Choquet integrably bounded and if we define $f^+(x) = sup\{r| r \in F(x)\}$ and $f^-(x) = inf\{r| r \in F(x)\}$ for all $x \in X$, then f^+ and f^- are Choquet integrable selections of F.

Theorem 2.6 [3,4,5] Let $K, F, G \in \mathbb{T}$. Then we have

(1) $F \sim F$,

(2) $F \sim G \to G \sim F$,

(3) $F \sim A$ for all $A \in I(R^+)$,

(4) $F \sim G$ and $F \sim K \to F \sim (G+K)$.

Let M^+ be the set of continuous

non-negative functions $f: X \to R^+$ with compact support. We consider the following class of interval-valued functions;

$\mathbb{T} = \{F \mid F: X \to I(R^+)$ is measurable and Choquet integrable bounded$\}$.

Definition 2.7 A subset $\mathbb{T}_0$ of $\mathbb{T}$ is said to be a fuzzy invex at G with respect to H, if for each $F \in \mathbb{T}_0$, $G + tH(F,G) \in \mathbb{T}_0$ for all $t \in [0,1]$ where $H: \mathbb{T}_0 \times \mathbb{T}_0 \to \mathbb{T}$ is an interval-valued mapping.

We consider the following class of interval-valued functions with continuous selections; for each $g \in \mathbb{T}$,

$$\mathbb{T}_G = \{F \in \mathbb{T} \mid F \sim G, S(G) \subset M^+\}.$$

Theorem 2.8 If $G \in \mathbb{T}$ and $H(F,G)$ is comonotonic to G for all $F \in \mathbb{T}$, then $\mathbb{T}_G$ is an interval-valued fuzzy invex set at G with respect to the mapping H, where $H : \mathbb{T}_G \times \mathbb{T}_G \to \mathbb{T}$ is defined by

$$H(F,G) = H([f^-,f^+],[g^-,g^+]) = [\eta(f^-,g^-), \eta(f^+,g^+)].$$

Definition 2.9 Let $\mathbb{K}$ be a non-empty invex subset of $\mathbb{T}$. A mapping $\Phi : \mathbb{K} \to I(R^+)$ is said to be interval-valued fuzzy preinvex at G with respect to H, if

$$\Phi(G + tH(F,G)) \le (1-t)\Phi(G) + t\Phi(F)$$

for all $t \in [0,1]$ and $F \in \mathbb{K}$.

Lemma 2.10 If $F, G \in \mathbb{T}$ with $F \ge G$ and $F - G \sim G$, then we have

$$(C)\int F - G\, d\mu = (C)\int F d\mu - (C)\int G\, d\mu.$$

Now, we denote the following class; for each $G \in \mathbb{T}$, $\mathbb{T}_G^* = \{F \in \mathbb{T}_G \mid F \ge G\}$ and then it is clearly fuzzy invex subset of $\mathbb{T}$.

Theorem 2.11 Let $\mathbb{T}_G^*$ be as the same above set. Assume that for all $F \in \mathbb{T}_G^*$

(i) $H^*(F,G)$ is comonotonic to G,

(ii) $H^*(F,G) \le F - G$, and

(iii) $F - G \sim G$.

If we define $\Phi_c^* : \mathbb{T}_G^* \to I(R^+)$ is defined by $\Phi_c^*(F) = (C)\int F d\mu$, then Φ_c^* is fuzzy preinvex at G with respect to H^*.

Theorem 2.12 Let $\phi_c^* : \mathbb{F}_g^* \to R^+$ is defined by $\phi_c^*(f) = (C)\int f d\mu$ and $\mathbb{T}_G^*$ be as the same above set. If $\phi_c^* : \mathbb{F}_g^* \to R^+$ is a fuzzy preinvex at g with respect to the mapping η, then $\Phi_c^* : \mathbb{T}_G^* \to I(R^+)$ is an interval-valued fuzzy preinvex at G with respect to the mapping $H^* : \mathbb{T}_G^* \times \mathbb{T}_G^* \to \mathbb{T}$, where H^* is defined by $H^*(F,G) = H^*([f^-,f^+],[g^-,g^+]) = [\eta(f^-,g^-), \eta(f^+,g^+)]$

3. References

[1] J. Aubin, "Set-valued analysis", Birkauser Boston, 1990.

[2] R .J. Aumann, "Integrals of set-valued functions", J. Math. Anal. Appl. Vol. 12, pp 1-12, 1965.

[3] Lee-Chae Jang. T. Kim, and Jong Duek Jeon, "On comonotonically additive interval-valued Choquet integrals(II)", KFIS, Vol. 14, No. 1, pp.33-38, 2004.

[4] Lee-Chae Jang. T. Kim, and Jong Duek Jeon, "On set-valued Choquet integrals and convergence theorem (II) ", Bull. Kerean Math.Soc., Vol. 20, No. 1, pp.139-147, 2003.

[5] Lee-Chae Jang, "Interval-valued Choquet integrals and their applications",Journal of Applied Mathematics and Computing, Vol. 16, No.1-2, pp.429-443, 2004.

[6] Lee-Chae Jang and H. M. Kim, "On fuzzy preinvexity in Choquet integrals",

[7] M. A. Noor, "Fuzzy preinvex functions", Fuzzy Sets and Systems, Vol. 64, pp.95-104, 1994.