A Modified Shooting Method Technique for the Analysis of the Limited Slip Capacity of UHPFRC-NC Composite Structure

Sang-Mook Han[*] , Xiangguo Wu[**], Sung-Wook Kim[***], Su-Tae Kang[****]

ABSTRACT

Shear connectors have a finite slip capacity because of the mechanism by which they transfer the shear between UHPFRC and NC elements. At high degree of shear connection, non-linear analysis techniques are required to allow for compressive plasticity and tensile cracking behaviour of the elements. As with all non-linear problems, a closed form solution is difficult to find. A Modified Shooting Method Technique is developed here for non-linear analysis of UHPFRC/concrete composite. The initial effective moment is derived according to the prestressing force. The composite structure is divided into small segments which length is much less than the length of the structure and it can be assumed that the forces and displacements within each segment are constant. An equivalent analysis in composite girders would be to fix the slip strain in each segment and develop a moment curvature relationship for this slip strain in each segment. Additive forces and moment analysis on each section of the segments are analyzed by MSMT. Finally the ultimate slippage of the interface can be evaluated by the MSMT model. This paper presents a nonlinear analysis method for limited slip capacity of UHPFRC-NC interface.

1 Introduction

Ultra high performance fiber reinforcement concrete (UHPFRC) and Normal Concrete (NC) composite design is a new generation of composite structure. The saint-Pierre-La-Cour bridge and the Glenmore footbridge is its first application in the world [1]. It is necessary in the design procedure to ensure that the connectors do not fracture through excessive slip before one of the required limit states. Endeavoring to ensure that mechanical shear connectors do not fracture prematurely has been one of the most intractable problems in composite construction [2][3]. Unlike both rigid plastic analysis and linear analysis that deal primarily with the behavior at a section of a beam, connector fracture deals with the whole length of the beam where the section properties could range from fully plastic to full elastic.

When the composite girder is first loaded, the five materials, i.e. NC, reinforcement steel bar, shear connectors, UHPFRC and prestressed tendon, have linear elastic properties and hence linear elastic partial interaction theory can be applied to determine the slip distribution. On further loading and particularly with low degrees of shear connection, the slips tend to uniformity. At high degree of shear connection, nonlinear techniques are required to allow for compressive plasticity and tensile cracking behaviour of UHPFRC and NC elements. As with all non-linear problems, a closed form solution is difficult to find. A modified Shooting Method Technique (MSMT) is developed here for non-linear analysis of UHPFRC-NC composite based on parametric study. Finally the ultimate slippage of the interface can be evaluated by the MSMT model. This paper presents a nonlinear analysis method for limited slip capacity of UHPFRC-NC interface.

[*] member, Kumoh National Institute of Technology, e-mail: smhan@kumoh.ac.kr

[**] member, Kumoh National Institute of Technology, E-mail: wxg_heu@hotmail.com

[***] member, Korea Institute of Construction and Technology, e-mail: swkim@kict.re.kr

[***] member, Korea Institute of Construction and Technology, E-mail: alphard@kict.re.kr

2 MSMT non-linear analysis techniques

The composite structure is divided into small segments with length x as shown in Fig.1, where the segment length is much less than the length of the structure, so that it can be assumed that the forces and displacements within each segment are constant. An equivalent analysis in composite girders would be to fix the slip strain in each segment and develop a moment curvature relationship for this slip strain in each segment.

For a given curvature, the position of the neutral axis can be varied until the axial compressive force above the neutral axis is equal to the axial tensile force below the neutral axis. Once this situation is reached, the moment M for the curvature κ can be derived from the axial forces. This procedure will be repeated for other curvatures to generate a moment curvature relationship that is applicable throughout the length.

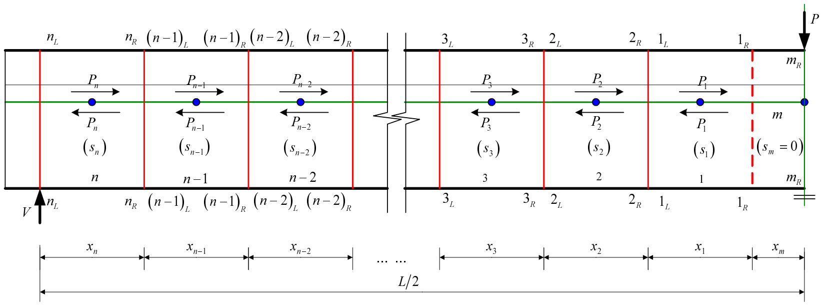
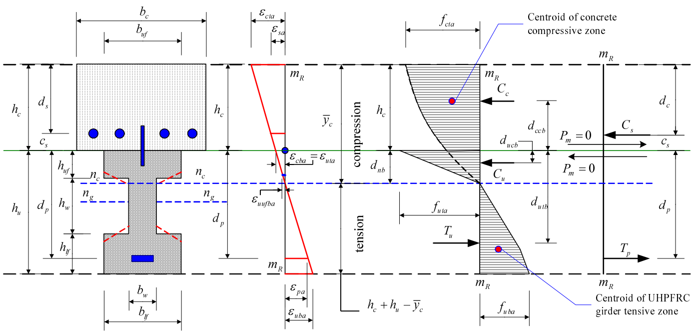


Fig.1 Non-linear analysis of UHPFRC/Concrete composite

2.1 Behavior analysis of mid-span section $m_R - m_R$

The analysis is started at the mid-span section $m_R - m_R$ where concrete and UHPFRC elements are in limit state or the prestressed tendon is in yield state. The mid-span resultant moment based on elements yield state is the boundary condition. To carry out the non-linear longitudinal analysis, the moment analysis at the section $m_R - m_R$ should be carried out firstly.



(a) section dimension (b) additive strain distribution (c) additive stress distribution

Fig.2 m_R - m_R section limit flexural behavior analysis

After NC is poured on UHPFRC girder, the shear connector will connect the two elements as a composite structure. Under external loading, the composite will work as a integrity. Since the slip on the interface boundary of mid-span section $m_R - m_R$ is zero, the two element additive strain should be same as shown in Fig.2 (b) in which $n_c - n_c$ is the new neutral axis of the composite structure. The bottom fiber additive strain of UHPFRC girder is 0.003. Therefore the neutral axis is selected as the original neutral axis position $n_c - n_c$. The limit state is defined based on the tendon yielding state. The final resultant force and moment can be obtained as

$$F_R = T_{ue} - C_{ue} + F_{pe} - C_c - C_s - C_u + T_{uw} + T_{ulf1} + T_{ulf2} + F_{pa} \quad (+) \qquad (1)$$

$$M_R = M_{ute} - M_{uce} + M_{pe} + M_c + M_s - M_{uc} + M_{uw} + M_{ulf1} + M_{ulf2} + M_{pa} \quad (+) \qquad (2)$$

2.2 The first transformation from element 1 to element 2

With the balance of the section $m_R - m_R$, the forces should be in equilibrium i.e.

$$T_{ue} - C_{ue} + F_{pe} - C_c - C_s - C_u + T_{uw} + T_{ulf1} + T_{ulf2} + F_{pa} = 0 \quad (3)$$

The composite neutral axis should be adjusted if the forces equilibrium cannot be satisfied automatically. Based on the initial evaluation of the bottom fiber of UHPFRC girder, the strain is lower than the post cracking equivalent strain. The moment of Eq.2 should be balanced with the support vertical force multiplied by the arm. The vertical support force can be obtained as

$$V_s = 2\left(M_{ute} - M_{uce} + M_{pe} + M_c + M_s - M_{uc} + M_{uw} + M_{ulf1} + M_{ulf2} + M_{pa}\right)/L \text{ (up)} \quad (4)$$

Since element $1_R - 1_R - m_R - m_R$ is located at the mid-span of the composite as shown from Fig.1, the slip is assumed to be undertaken by the adjacent element *1* and to be zero. The slip s_1 is initially guessed as s_{ig} and uniformly distributed in the element *1*. Then the interfacial shear force can be determined as $P_1 = q \cdot x_1 = K_{si} s_1$. Now consider the section $1_L - 1_L$ of the element 1, the applied moment M_1 is known as $M_1 = V_s\left(L/2 - x_m - x_1\right)$. The axial force in NC and UHPFRC of element 2 in Fig.3 (a) is the element 1 shear connectors force. Therefore it is necessary to find a strain distribution in the Fig.3 (a) and hence the material properties a stress distribution (c) which is in equilibrium with both M_1 and P_1 as shown in (d). We therefore have to determine both the slop of the strain profiles κ_2 and the position of the strain ε_{c2} and ε_{u2} as shown in (b), that resultant in a section that is in equilibrium with M_1 and P_1. In order to do this, it is necessary to first guess a curvature of the element 2, κ_{2g} shown in Fig.3 (b). Starting with NC element, the position of the strain profile is moved by changing ε_{c2} until the resultant force in NC element is in compressive and equal to P_1, i.e. $C_{c2} + C_{s2} = P_1$.

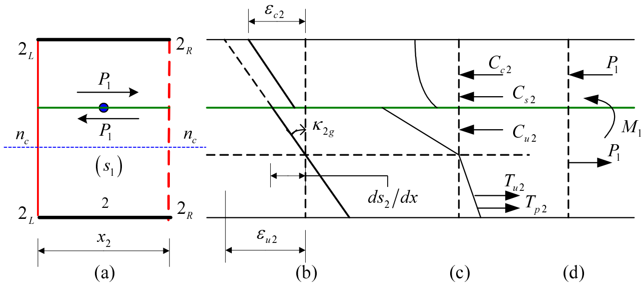


Fig.3 Non-linear analysis

Similarly for UHPFRC element, the position of the strain profile is changed by altering ε_{u2} until in the equivalent stress profile (c) there is a resultant tensile force P_1 in UHPFRC element as shown in (d), i.e. $T_{u2} - C_{u2} + T_{p2} = P_1$. The section $2_R - 2_R$ is now in equilibrium with the axially applied loads at the strain profile defined by κ_{2g}, ε_{c2} and ε_{u2}. Moment of the axial forces in (c) can now be taken to determine the inner moment. And this should be equal to the applied moment M_1, i.e. $\sum M\left(C_{c2}, C_{s2}, C_{u2}, T_{u2}, T_{p2}\right) = M_1$. Otherwise, it will be necessary to try a new curvature κ_{2g} and keep repeating until equilibrium is attained at the strain profile κ_{2g}, ε_{c2} and ε_{u2}. Once the strain profile is known, the slip strain ds_2/dx can be determined as shown in (b). Integrating the slip strain over the length x_2 gives the increase in the slip Δs_1 in element 2 over that of element 1 of s_1 and hence the slip segment 2 is $s_2 = s_1 + \Delta s_1$. The force in the connectors within the element 2 can be determined as $P_2 = K_{si} s_2$.

2.3 The (*n*-1)th transformation from element (*n*-1) to element *n*

The axial force in NC and UHPFRC elements in element *n* will be the force in the shear connectors in the shear span to the right of the section $n_R - n_R$ and the left boundary of the element *n*. The strain profile κ_{ng}, ε_{cn} and ε_{un}, and the slip strain can be achieved using the procedure described in the section 2.2 section. The boundary force is the prestressed tendon anchorage force P_{ane} which is named as anchorage effective prestressing force.

$$P_{ane} = P_{py} - \Delta f_{PA} \cdot A_{ps} \tag{5}$$

Therefore the axial force in NC element is $C_{cn} + C_{sn} = \sum_{j=1}^{n-1} P_j$. The axial force in UHPFRC element is given by $T_{un} - C_{un} + T_{pn} = \sum_{j=1}^{n-1} P_j - \left(P_{py} - \Delta f_{PA} \cdot A_{ps}\right)$.

The applied moment can be written as $M_n = V_s \left(L/2 - x_m - \sum_{j=1}^{n-1} x_j \right) = V_s \cdot x_n$. If the moment boundary condition can be satisfied, the analysis procedure will be ended. Otherwise, a new value of initial slip s_{ig} should be guessed. Once the strain profile is known, the slip strain ds_n/dx can be determined. Integrating the slip strain over the length x_n gives in the slip Δs_n in element *n* over s_{n-1} as $s_n = s_{n-1} + \Delta s_n = s_1 + \sum_{i=1}^{n} \Delta s_i$. The force in the connectors within the element *n* can be determined as $P_n = K_{si} s_n$.

3. Fracture evaluation criterion

The connector fracture safety criterion can be written as $s_n < s_{ult}$. This non-linear analysis procedure can be realized by numerical computation.

4. Discussions

Among the above analysis, one remain problem is the simplification of concrete and UHPFRC constitutive relation in the region of the two elements according to the neutral axis position. This is different depending on the flexural failure or shear failure and the prestressed ratio. For the composite structure made in this thesis, UHPFRC tensile and compressive constitutive relation can be assumed to be linear distribution since the yielding strength of tendon is defined as the ultimate limit state. This simplification is utilized in the numerical program writing. This technology numerical program and corresponding push test are under proceeding now.

5. References

[1] Behloul, M.. HPFRCC field of applications: Ductal recent experience, 5th High Performance Fiber Reinforced Cement Composite (HPFRCC5), 213-222. Mainz, Germany, July 2007.

[2] Johnson, R. P., and Molenstra, N. (1991). "Partial shear connection in composite beams for buildings."*Proc.*, Instn. of Civ. Engrs., London, England, Part 2, 91(Dec.), 679–704.

[3] Oehlers, D. J., and Coughlan, C. G. (1986). "The shear stiffness of stud shear connectors in composite beams."*J. Constr. Steel Res.*, 6(Oct.), 273–284.

[4] Hognestad E., “A study of combined bending and axial load in reinforced concrete members”, University of Illinois Engineering Experimental Station, Bulletin Series No. 399, November 1951, pp.128.

[5] Park R. and Paulay T.(1933). Reinforced concrete structure. A Wiley-Interscience Publication, New York, U.S.A., pp. 11-18.