Unstructured Quadrilateral Surface Grid Generation and Cell Size Control

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Abstract: In this paper grid generation of unstructured quadrilateral surface grids is described. The current approach uses conventional Advancing Front Method which is used to generate unstructured triangular grids. Grid cell size control is done by using closeness-based global interpolation method controlled by pre-described control elements. Algorithm and procedure for quadrilateral grid generation using AFM method and cell size control method are described. Examples of quadrilateral grid generation are shown, and difficulties and problems related to the current approach are also discussed.

Keywords: CFD, FEM, Unstructured Quadrilateral Grids, Advancing Front Method, Hybrid Grid System.

1. INTRODUCTION

Numerical problem solving techniques such as CFD and FEM method are being widely used in various fields of engineering such as fluid mechanics and structural analysis. These techniques can be categorized into finite difference method, finite volume method, and finite element method, and others. Each of these numerical techniques eventually solves a set of algebraic equations which are obtained by approximately applying the governing partial differential equations on a set of simpler shaped cells which replace the whole domain of interest including boundaries.

It is well-known that the quality and the number of those cells have strong influence on the convergence and accuracy of solutions obtained by those numerical techniques, and furthermore grid generation is usually considered as a bottleneck for a routine application of numerical techniques in the engineering process because of its labor-intensive and error-prone nature. The grid system can be categorized into two types: structured grids and unstructured grids. The structured grid system is characterized by its orderly fashion of node connectivity among neighboring nodes, while the unstructured grids don't require any orderliness of node connectivity. Generating structured quadrilateral meshes for general 2-D region and structured hexahedral meshes for 3-D region usually requires user's experience and labor due to its requirement for orderliness of node distribution, while unstructured grids with triangular cells(2-D) or tetrahedral cells(3-D) can be much easily generated even for very complex domains due to its favorable flexibility nature of node connectivity.

On the other hand the shape of grid cell has strong influence on the accuracy and the cost of numerical analysis. Quadrilateral cells(2-D) or hexahedrons(3-D) have several desirable properties compared to the triangles(2-D) or tetrahedrons(3-D). Blacker summarized those properties and those are quoted as follows:[1]

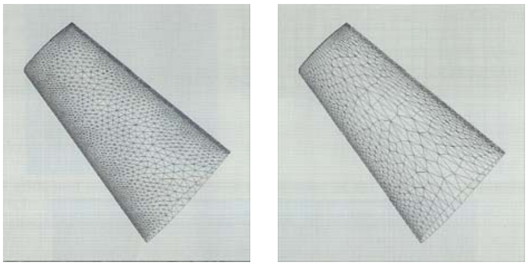
- Hexahedron provides shape functions with additional terms that may increase the accuracy of the solution
- Hexahedron provides directional sizing without losing accuracy. For example, a very thin hexahedron within a boundary layer for fluid flow calculations performs far better than thin tetrahedron.
- A conformal mesh provides the most accurate form of connectivity within the region. Non-conformal interfaces may decrease the meshing complexity but increase the error, at least locally.
- A hexahedra mesh decreases the overall element count. A tetrahedral mesh usually increases the element count 4 to 10 fold over a hexahedral mesh.

In author's lab a research to develop a computer program which allows an automatic grid generation of unstructured grids for general 3-D regions is in progress, and robust surface grid generation method for general-shaped boundary surfaces is required to generate hybrid grids consisting of unstructured triangles and quadrilaterals. This paper describes how surface grid generation of unstructured quadrilateral grids can be accomplished by applying Advancing Front Method(AFM) which are conventionally used to generate triangular cells[2].

2. UNSTRUCTURED GRIDS

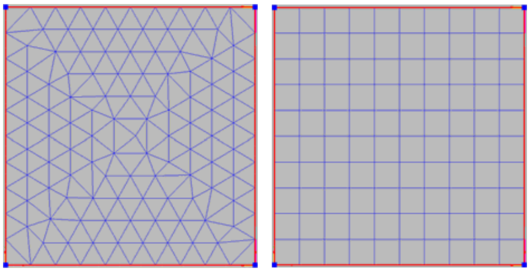
In CFD research field structured multi-block grids have been traditionally the grid topology of more preference in the past compared to the unstructured grids in the structural research area using FEM methodology. Trend is, however, shifting, and unstructured tetrahedral grids are being adopted more widely in the CFD area also due to its relative advantageous features compared to structured approach including flexibility and robustness of grid generation even for very realistic and complicated geometries. The case of Fluent could be example of that kind of trend. This one of major commercial CFD software has changed its underlying grid system from traditional structured grid based approach to unstructured grid based one[3]. Even though structured grid system has its own advantages and strengths, one of major drawbacks of structured grid approach is the requirement of experience and trial-and-error in the grid generation stage while unstructured grid system can be relatively easily constructed.

In recent years many researchers and CFD software users are using unstructured grids to calculate flow fields for very complicated geometries, such as aircrafts with enough details considered. It can be said that due to its major advantage of flexibility, tetrahedral grids are used most often among various grid types in the 3-dimensional unstructured grid approach. For the accurate calculation of flow fields by the flow solvers the grid system should have good quality regardless of grid types. For the case of generating 3-dimensional tetrahedral grids, triangular surface grid generation is required, and the triangular cells can have its best possible quality when they are equilateral. Even though there are other measures for grid quality evaluation such as grid size gradient, grid quality is most influenced by the angular skewness which is directly affected by how much the shape deviates from the equilateral shape for the case of triangles. This is why relatively many triangular cells are distributed near the leading edge of a wing in spite of low flow gradient in the span-wise direction compare to the chord-wise direction, as shown in Fig. 1(a). By using stretched(or anisotropic) triangular cells it is possible to use less number of cells, but the deterioration of flow solution



(a) Isotropic cells (b) Anisotropic cells

Fig. 1 Triangular surface grid of M6 wing



(a) Triangles (224 cells) (b) Quadrilaterals (100 cells)

Fig. 2 Cell count of unit square region

accuracy should be accepted or be compensated by paying additional cost. Fig. 1(b) shows an example of stretched triangular cells for the same wing as Fig. 1(a) which is quoted from the reference[4].

Another possible way to save cell count for the same problem is to use quadrilateral cells in stead of triangles. Assuming unit cell size in terms of distance between neighboring nodes, the area of an equilateral triangle is approximately 0.433 compared to 1.0 of a square which is equilateral rectangle. That means using quadrilateral cells we can theoretically decrease the cell count by factor of about 2.3 covering the same area compared to using triangles. In 3-dimensional cases, the volume of equilateral tetrahedron with unit edge length is about 0.108, while that of hexahedron is 1.0, which means factor of 9 to 10 theoretical saving of cell count by using hexahedron in stead of tetrahedrons. This possible saving is also mentioned by Blacker, as described in the previous chapter. When triangular cells are generated for a unit square area with node distance of 0.1 using our program, the result shows a grid system with 224 triangular cells, while rectangular grid system results in exactly 100 quadrilateral cells, as shown in Fig. 2(a)-(b). The cell count ratio of this example gives 2.24 which is quite close to the theoretical value of 2.3.

Quadrilateral grids and hexahedral grids have several advantageous features including stretched aspect ratio without grid quality deterioration as well as cell count saving. Therefore the way to generate quadrilateral cells by extending the advancing front ability which is already implement into our program is studied.

3. QUADRILATERAL GRID GENERATION

3.1 AFM for Quadrilaterals

In this research Advancing Front Method is used to generate unstructured quadrilateral grids. Our experience in generation of triangular grids along surface given in the STL format and the computer program which is under development for this kind of job in our lab was somewhat directly extended and was used for this quadrilateral grid generation problem.[5]

The procedure of quadrilateral grid generation using AFM method can be summarized as follows:

1) First, the geometry data is imported into the program. The program assumes the geometry definition in the STL format, which describes the surface shape with a collection of triangulated surface segments. Once the geometry data is read, the program figures out the topology of the object by identifying topological edges and corners.
2) When the topological edges are automatically found by checking the angles between neighbor triangular cells, the topology of the body surface such as the number of faces can be identified. When those faces are identified, boundary node points along those face edges are distributed. When calculating node distribution, the same cell size control method used for the surface grid generation is used, and the size control method is briefly described in the next section. Fig. 3(a) shows an example of node distribution along boundary edges of a rectangular surface.
3) A closing loop along the boundary edges of the given surface forms a front and, the initial distribution of points along the edges is used as initial fronts for generating surface unstructured quadrilateral grids for each faces along the input surface by using the well-known Advancing Front Method (AFM). Once the initial fronts are decided, each front is advanced inward cell by cell along the given surface until the front has no element to advance. Figures from Fig. 3(b) to Fig. 3(d) show a sequence of front advance results. For the advance of each front element, new positions of two end nodes of each element is predicted by using cell size calculation method (more about this in the next section) and correction of the position including merging with an old node and distance checking between neighboring nodes are performed. The cells and nodes in those figures are shown as the results of going through the process of prediction and correction of position.
4) As the fronts advance along the given 3-D surface, the un-traveled area surrounded by the fronts gets smaller and smaller, in some cases one front is divided into several fronts because of merging between two nodes along the same front, or in other cases two fronts can be merged into one front due to collision between fronts advancing in the opposite direction. Fig. 3(e) and Fig. 3(f) show the situation just before and after merging of two nodes happens.
5) If the surface of interest is flat, then the advance of fronts occurs along that flat surface, but for the generation of surface grid system, the advance of fronts should be guided by the given surface shapes which are curved in general. If the advance is not guided properly, then the resultant grid would not follow the given surface. In the current method this problem is overcome by projecting newly-generated nodes onto the given STL surface segments after each advance of front element. The test case shown in Fig. 3 is for a flat surface, and the Fig. 3(g) shows the resultant unstructured quadrilateral grids, where each cell is highlighted with a special color which can tell what kind of correction each cell has gone through.
6) Fig. 3(h) shows the final grid system generated by the current AFM method, and the next step is grid quality improvement by applying grid smoothing technique which will be implemented into the program later on.

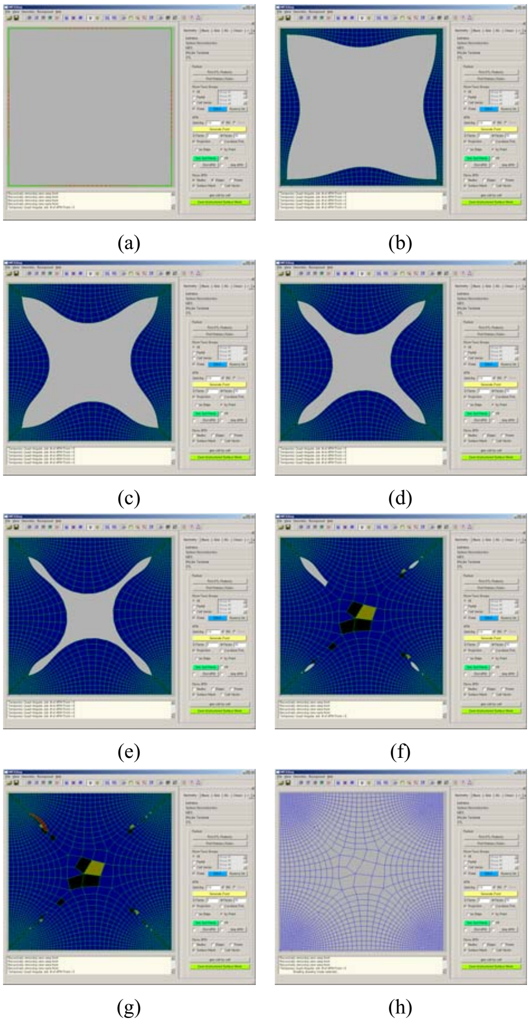


Fig. 3 Procedure of quadrilateral grid generation by AFM

3.2 Cell Size Control

Cell size control is needed not only to improve the quality of grids but also to increase the efficiency and accuracy of flow calculation. In this work cell size control is done by using the method which calculates cell size at a point P by weight-averaging size information prescribed at each of n control points based on the closeness of the point P and each control point, as shown in Fig. 4.

The basic idea can be explained by Eq. (1), which calculates cell size at a point P is a weight-average of n values which are pre-determined(that is user-specified) at n control elements. Here, the weight values are to satisfy the normalization condition, as shown in Eq. (2). There can be many choices for the weighting value, and in this research a weight which is determined by the distance between the point of interest and each of control elements is used, as given in Eq. (3). In the equation parameter α is a control factor which allows users more flexibility to control the clustering between nodes, and $\boldsymbol{c_i}$ is the inverse of the distance between the point, ***P***, and the control element, ***i***, as shown in Eq. (4). More details of the current cell size control method can be found in the reference[5].

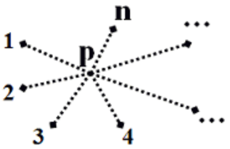


Fig. 4 Cell size interpolation from n control elements

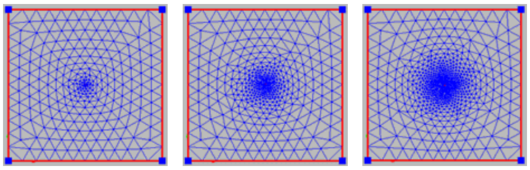

(a) α=1.5 (a) α=2.0 (a) α=2.5

Fig. 5 Effects of α -Parameter

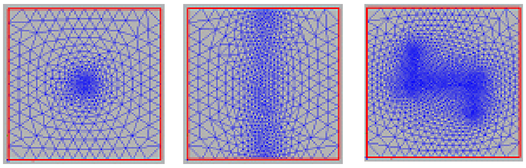

(a) Point (b) Line (c) Polyline

Fig. 6 Types of grid size control elements

$$\delta_P = \sum_{i=1}^{n} \omega_i \delta_i \tag{1}$$

$$\sum_{i=1}^{n} \omega_i = 1 \tag{2}$$

$$\omega_i = \frac{c_i^a}{\sum_{i=1}^{n} c_k^\alpha} \tag{3}$$

$$c_i = \frac{1}{d_i} \tag{4}$$

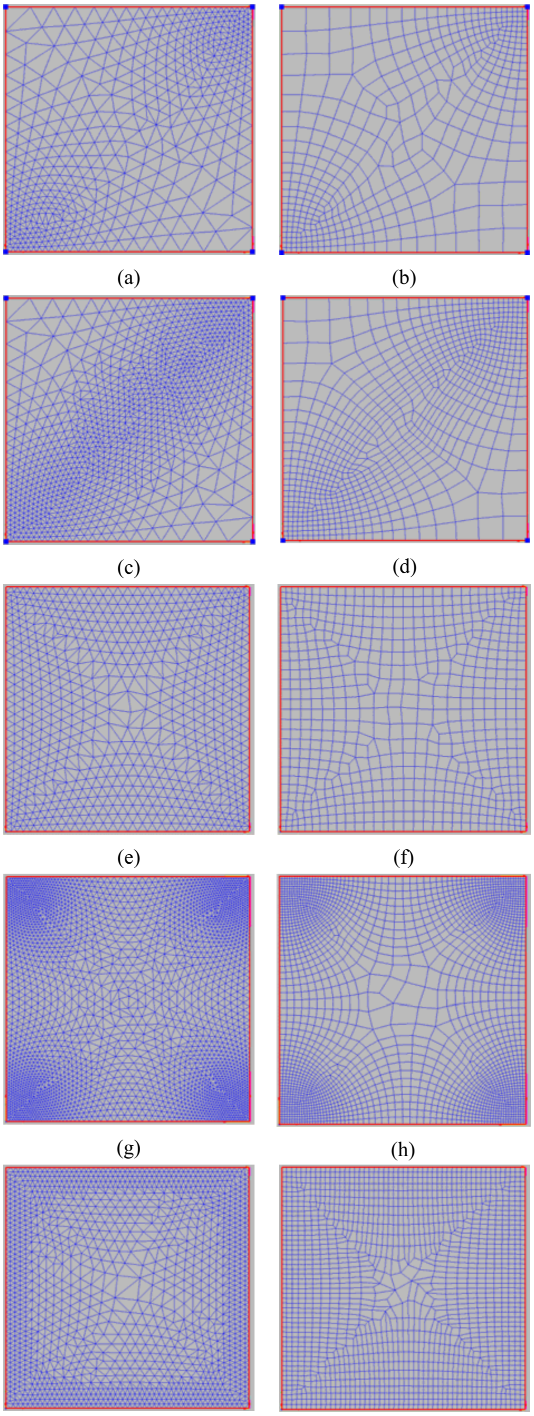
Fig. 5 shows the effect of changing α-parameter, and the parameter determines how far the size information assigned to each control element is effective. The effective distance increases as a-parameter becomes higher.

Fig. 6 shows the results of using different types of control elements: point, line, and polyline. This method enables easy and simple control of grid size without using any background grid which requires additional calculation of identifying the cell enclosing the point of interest among the background cells.

4. EXAMPLES AND DISCUSSION

Current approach of generating quadrilateral grids using AFM method is tried for a simple rectangular region with different grid size control setting, and the results are show in the Fig. 7 together with triangular counter part side by side for the purpose of easy comparison. Left column of Fig. 7 shows the triangular grid generation results and cell sizes of those left results can be used as a barometer for checking the cell size of quadrilateral grid results on the right column of Fig. 7.

Fig. 7(a) and (b) show the case of control points residing at the opposite corners. Fig. 7(c) and (d) are the case with a diagonal line as a control element. Fig. 7(e) and (f) are the case with 4 control points at each corner with smaller



(a) (b) (c) (d) (e) (f) (g) (h) (i) (j)

Fig. 7 Examples of quadrilateral grid generation

α-parameter, while Fig. 7(g) and (h) are the case with higher α-parameter. Fig. 7(i) and (j) show the case with 4 line control element residing at the edges of the rectangular boundary.

From these results we can find that the resultant cell size distribution of quadrilateral grids are not as much controlled as intended compared to triangular cases. This problem might be caused by the fact that quadrilateral cells have more constraints in terms of node positioning, and therefore they are much less flexible than triangular ones. And further problem to point out is that quadrilateral cells are not equilateral, especially near the center area where front advance comes to an end. These are the results obtained from the AFM approach and any of grid smoothing is applied yet. If the grids are improved with proper grid smoothing method, the resultant grids would be better than the current ones.

And in some cases with different size control setting which are not shown here, our program fails to generate meaningful quadrilateral grids in its lease sense. It means that more research to improve robustness of current approach is required.

5. CONCLUSION

In this work advancing front method is used to generate unstructured quadrilateral grids, and the resultant grids are compared with the triangular counterparts. Even though the current result is not as much satisfying as one could expect, it is still encouraging that the AFM method can be reasonably used to generate quadrilateral cells. Furthermore when the current approach is combined with smoothing method which is the next research item in our lab, its flexibility and robustness for generating unstructured quadrilateral grids will be much enhanced.

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