변분법을 이용한 축방향으로 움직이는 보의 스펙트럴 요소 모델링

Dynamics of an Axially Moving Bernoulli-Euler Beam : Variational Method-Based Spectral Element Modeling

최정식* 이우식**

Choi, Jungsik Lee, Usik

ABSTRACT

The spectral element model is known to provide very accurate structural dynamic characteristics, while reducing the number of degree-of-freedom to resolve the computational and cost problems. Thus, the spectral element model with variational method for an axially moving Bernoulli-Euler beam subjected to axial tension is developed in the present paper. The high accuracy of the spectral element model is the verified by comparing its solutions with the conventional finite element solutions and exact analytical solutions. The effects of the moving speed and axial tension the vibration characteristics, wave characteristics, and the static and dynamic stabilities of a moving beam are investigated.

---

1. Introduction

The moving belts used in power transmission are an example of a class of axially moving structures. Axially moving speed may significantly affects the dynamic characteristics of moving structures even at low speed, giving rise to the variation of natural frequencies and complex modes. Above a certain critical moving speed, axially moving structures may experience severe vibrations, static instability, or dynamic instability to result in structural failures. Thus, it is important to accurately predict the dynamic characteristics and instability of such structures in advance for the successful analysis and design of a broad class of technological devices. The literature regarding axially moving structures is quite wide, and an extensive literature overview can be found in Wickert and Mote (1988).

The axially moving beam-like one-dimensional structure with flexural rigidity has been traditionally represented by the Euler-Bernoulli beam (BE-beam) model or Timoshenko beam model. The solutions of the equations of motion for the moving beam models have been obtained by various solution techniques including the Galerkin's method assumed mode method (Lee, 1993), finite element method (FEM), Green's function method, transfer function method (Riedel and Tan, 1998), perturbation method (Öz, 2001), and the Laplace transform method .

In the literature (Doyle, 1997; Lee and Lee, 1998; Lee et. al., 2000, 2001), it has been well recognized that the spectral element method (SEM) is an exact solution method for the dynamic analysis of structures. In SEM, the spectral element matrix (or exact dynamic stiffness matrix) is formulated in frequency-domain by using exact dynamic shape functions. Therefore it does not require any structural discretization to improve the solution accuracy for a uniform beam, regardless of its length. As it is one of element methods, the conventional finite element assembly procedure can be equally applied to formulate the global system dynamic equation of a structure. In SEM, the dynamic responsesin

---

* 인하대학교 기계공학과 석사과정

** 교신저자, 정회원, 인하대학교 기계공학과 교수

E-mail : ulee@inha.ac.kr
TEL : (032)860-7318 FAX : (032)866-1434
frequency- and time-domains are computed very efficiently by using the forward-FFT (simply, FFT) and inverse-FFT (simply, IFFT) algorithms. Recently, Le-Ngoc and McCallion (1999) derived the dynamic stiffness matrix for the axially moving string to obtain exact eigenvalues. However, the spectral element model in terms of exact dynamic stiffness matrix has not been introduced in the literature for axially moving beam structures.

The purposes of the present paper are first to formulate the spectral element model for the transverse vibration of an axially moving BE-beam model subjected to an axial tension, and then to verify its high accuracy by comparing with the solutions by the other solutions methods, and finally to investigate the effects of the moving speed and axial tension on the vibration and stability of the moving beam.

2. Equation of Motion

Consider a BE-beam model of flexural rigidity *EI*, which travels under an applied axial tension *N* with constant transport speed *c*. The equation of motion and relevant boundary conditions can be derived form the extended Hamilton's principle

$$\int_{t1}^{t2}(\delta T - \delta U + \delta W)dt = 0 \tag{1}$$

where *T* and *U* are the kinetic energy and the potential energies, respectively, and δW is the virtual work. The kinetic and potential energies are given by

$$U = \frac{1}{2}\int_0^L (EIw''^2 + Nw'^2)dx \tag{2}$$

$$T = \frac{1}{2}\rho A\int_0^L [c^2 + (\dot{w} + cw')^2]dx \tag{3}$$

Equation of motion can be derived as substituting Eq.(2) and (3) into Eq. (1)

$$EIw'''' - Nw'' + \rho A\left(\ddot{w} + 2c\dot{w}' + c^2 w''\right) = f(x,t)\ . \tag{4}$$

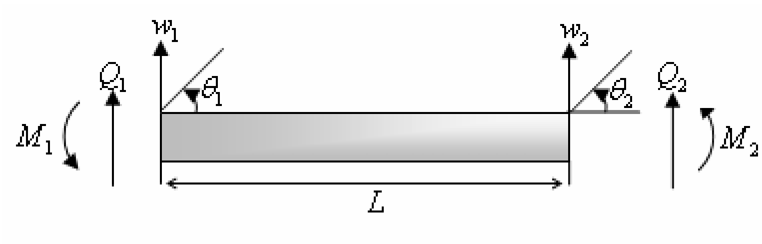

Fig. 1 Sign convention

3. Spectral Element Formulation

The spectral element formulation begins with the governing equations of motion without external forces. The free vibration response of the moving BE-beam model are then represented in the discrete Fourier transform (DFT) forms as (Doyle, 1997; Lee *et. al*., 2000)

$$w(x,t) = \sum_{n=0}^{N-1} W_n(x)e^{i\omega_n t} \tag{5}$$

The weak form of the governing equation can be constructed.

$$\int_0^L [EIW'''' - NW'' + \rho A(-\omega^2 W + i\omega 2cW' + c^2 W'') - F]\delta W\, dx \tag{6}$$

From Eq. (6) Spectral matrix can be derived.

$$\mathbf{S}(\omega)\mathbf{d} = \mathbf{f} \tag{7}$$

where

$$\mathbf{f} = \int_0^L \mathbf{N}^{\mathbf{T}} F dx + \{Q_1 \quad M_1 \quad Q_2 \quad M_2\}^{\mathbf{T}}, \quad \mathbf{d} = \{W_1 \quad \theta_1 \quad W_2 \quad \theta_2\}^{\mathbf{T}}$$

$$\mathbf{S}(\omega) = \mathbf{Y}^{-\mathbf{T}}\{[EI\mathbf{K}^2\,\mathbf{\Phi}\mathbf{K}^2 - (N - \rho A c^2)\mathbf{K}\,\mathbf{\Phi}\mathbf{K} - \rho A\omega^2\,\mathbf{\Phi}] \\ - \omega\rho Ac(\mathbf{K}\,\mathbf{\Phi} - \mathbf{\Phi}\mathbf{K}) + i\rho Ac(\omega\mathbf{I} - c\mathbf{K})\mathbf{\Psi}\}\mathbf{Y}^{-1}$$

4. Numerical example

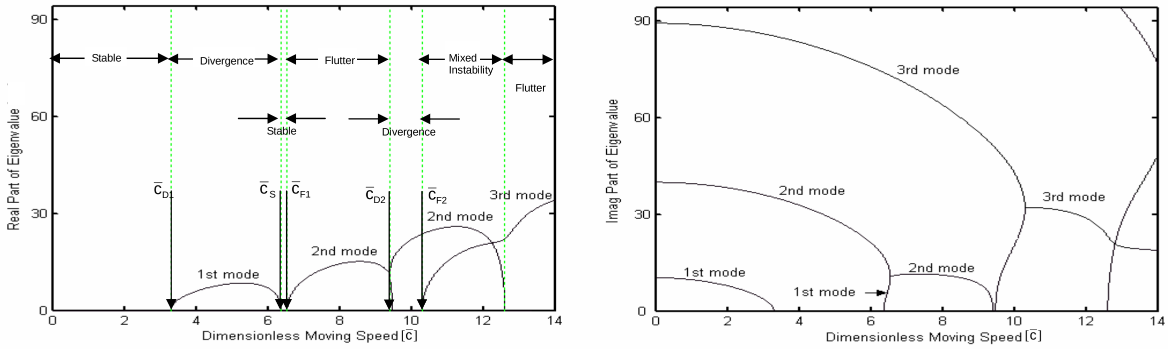

Fig. 2 The dimensionless eigenvalues $\bar{\lambda} = \mathrm{Re}(\bar{\lambda}) + i\,\mathrm{Im}(\bar{\lambda})$ *vs*. the moving speed of beam $\bar{c}$,

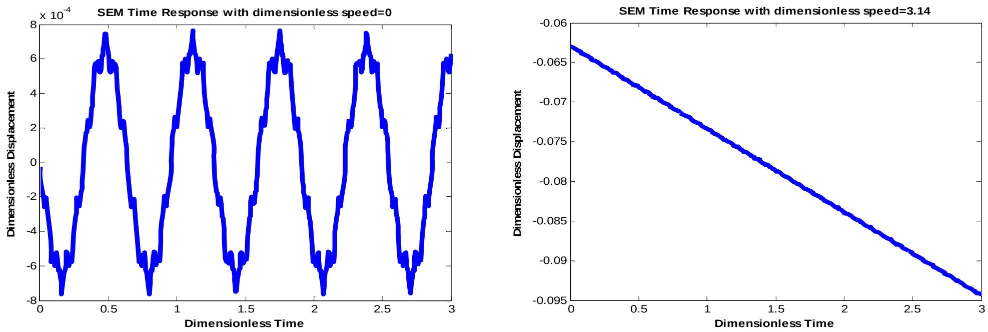

Fig. 3 Dimensionless time responses at various moving speeds of beam

Table 1. Comparison of the natural frequencies obtained by the present SEM, FEM, and analytical method

Dimensionless Moving Speed	Method	N_E (N_{DOF})	Dimensionless Natural Frequency			
			$\mathrm{Im}(\lambda_1)$	$\mathrm{Im}(\lambda_2)$	$\mathrm{Im}(\lambda_3)$	$\mathrm{Im}(\lambda_5)$
0	Analytical	1	9.870	39.478	88.826	246.740
	SEM	1 (2)	9.870	39.478	88.826	246.740
	FEM	10 (20)	9.870	39.482	88.874	247.714
		50 (100)	9.870	39.478	88.827	246.742
$0.5\,\bar{c}_{D1}$	SEM	1 (2)	8.175	38.355	87.856	245.888

	FEM	10 (20)	8.175	38.361	87.910	246.901
		50 (100)	8.175	38.355	87.856	244.889
$\bar{c}_{D1}$	SEM	1 (2)	0	34.811	84.909	243.325
	FEM	10 (20)	0	34.820	84.980	244.456
		50 (100)	0	34.811	84.909	243.325

Note: N_E = number of finite elements, N_{DOF} = number of degrees-of-freedom

5. Conclusions

In this paper, the dynamic equations of motion for the moving BE-beam model subjected to an axial tension are derived and then the spectral element model is formulated by using the exact dynamic shape functions. The high accuracy of the spectral element is then verified by comparing its solutions with the exact analytical solutions and conventional FEM solutions. The critical moving speed at which the divergence instability occurs is analytically derived in a closed form. Through some numerical studies, when the moving speed reaches the lowest divergence speed, the first natural frequency vanishes and the first bending mode disappears, resulting in the divergence.

6. Reference

1. Doyle, J. F., 1997, Wave Propagation in Structures: Spectral Analysis Using Fast Discrete Fourier Transforms, Springer-Verlag, New York.
2. Hwang, S. J. and Perkins, N. C., 1992, “Supercritical Stability of An Axially Moving Beam,” Journal of Sound and Vibration, Vol. 154, No. 3, pp. 381~409.
3. Lee, H. P., 1993, “Dynamics of A Beam Moving Over Multiple Supports,” International Journal of Solids and Structures, Vol. 30, No.2, pp. 199~209.
4. Newland, D. E., 1993, Random Vibrations, Spectral and Wavelet Analysis, 3rd ed., Longman, New York.
5. Öz, H. R., 2001, “On the Vibrations of An Axially Traveling Beam on Fixed Supports with Variable Velocity,” Journal of Sound and Vibration, Vol. 239, No. 3, pp. 556~564.
6. Pellicano, F. and Vestroni, F., 2001, “Non-linear Dynamics and Bifurcations of An Axially Moving Beam,” Journal of Vibration and Acoustics, Vol. 22, pp. 21~30.
7. Riedel, C. H. and Tan, C. A., 1998, “Dynamic Characteristics and Mode Localization of Elastically Constrained Axially Moving Strings and Beams,” Journal of Sound and Vibration, Vol. 215, No. 3, pp. 455~473.
8. Usik, Lee and Hyungmi-Oh, 2004, “Dynamics of an axially moving Bernoulli-Eluer beam : Spectral Element Modeling and Analysis, KSME International journal, Vol. 18, No. 3, pp. 395~406.