
A Low Complexity Multi-level Sphere Decoder for MIMO Systems with QAM signals

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ABSTRACT

In this paper, we present a low complexity modified multi-level sphere decoder (SD) for multiple-input multiple-output (MIMO) systems employing quadrature amplitude modulation (QAM) signals. The proposed decoder, exploiting the multi-level structure of the QAM signal scheme, first decomposes the high-level constellation into low-level 4-QAM constellations, so-called sub-constellations. Then, it deploys SD in the sub-constellations in parallel. In addition, in the searching stage, it uses the optimal low-complexity sort method. Computer simulation results show that the proposed decoder can provide near optimal maximum-likelihood (ML) performance while it significantly reduces the computational load.

Keywords

Multiple-input multiple-output (MIMO), Multi-level stage, Sphere decoder, VBLAST

1. Introduction

Recently, research on the application of multiple transmit and receive antennas, i.e., multiple-input multiple-output (MIMO) systems, to wireless communication systems has been an important issue because MIMO systems are theoretically shown to significantly improve spectral efficiencies [1]. To achieve high spectral efficiencies, a MIMO technique, known as the Vertical Bell Laboratories Layered Space Time (V-BLAST) architecture [2], has been reported. In order to achieve an optimal system performance, maximum likelihood (ML) decoder is required for the V-BLAST. However, brute-force maximum likelihood (BFML) detection, so-called MLD, is not a practical approach due to its complexity, which is exponential with the number of transmit antennas and the level of modulation.

For achieving ML performance at reduced complexity, a class of detection, collectively referred to as sphere decoders (SDs), has been developed [3]-[4]. Both computer simulation and theoretical analysis showed that the average complexity of SDs was remarkably smaller than that of the MLD in many practical scenarios.

The soul of most the SDs was integer lattice theory. Thanks to the optimal ordering exploitation, SDs have been well developed for both real-valued and complex-valued systems.

The SDs, however, suffer from that fact that their complexity is a random quantity, which depends on channel and signal-to-noise ratio (SNR). The variation of complexity of SDs makes them difficult in realizing in hardware implementation. Possible solutions are the usages of a approach based on the Gaussian approximation principle, as known as probabilistic data association (PDA) [5], or based on M-algorithm [5]. The sequential Gaussian approximation (SGA), a combination of PDA principle and M-algorithm, seemed to be a promising approach providing near optimal performance with fixed complexity and memory requirement.

On the other hands, both SD and SGA face the inefficient implementation for MIMO systems employing high-level modulation scheme. Fortunately, constellation of large square M -ary quadrature amplitude modulation (M-QAM), $M = 2^{2k}$, can be decomposed into $L = \log_4(M)$ sub-level constellations thanks to

the intrinsic multi-level structure of the constellation. The multi-level SGA (MSGSA) presented by Jia et al. [5], by exploiting multi-level structure and optimal tree-searching techniques Depth-first-search (DFS) and Breadth-first-search (BFS), showed significant reduction on computational load. However, MSGSA, based on PDA principle, can not be able to employ the optimal order of candidates, which has been shown to significantly improve the tree-search. As a result, possible reduction of the complexity can still be made and motivate use on this work.

In our work, rather than using PDA principle, we employ SD with optimal ordering to work on the multi-level structure to further reduce the complexity. The remaining of this paper is organized as follows. Section II presents the system model that is used for our work. The overview on multi-level structure, MSGSA and the proposed decoder are presented in Section III. To verify the proposed decoder, computer simulation has been implemented. Simulation results are given in Section IV accompanied by discussion. Section V concludes this work.

II. System model

We consider an uncoded V-BLAST configuration with n_T transmit and n_R receive antennas, denoted as (n_T, n_R) system.

At the transmitter, the input data sequence is partitioned into n_T sub-streams (layers), each of which is then modulated by an M-QAM modulation scheme, $M = 2^{2k}$ for some positive integer k , and transmitted from each different transmit antenna. For the sake of simplicity, we investigate one-time-slot complex baseband signal model, where each symbol period a $n_T \times 1$ transmit signal vector \mathbf{s} consisting of n_T symbols, s_i , $i = 1, \dots, n_T$, is sent through n_T transmit antennas. Under the assumptions that the signals are narrow-band and the channel is quasi-static, i.e., it remains constant during some block of arbitrary length and independently changes from one block to another, the relationship between transmitted and received signals can be expressed in the following form:

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{w} \quad (1)$$

where $\mathbf{r} = [r_1 \dots r_{n_R}]^T$ is the $n_R \times 1$ received

signal vector, $(\cdot)^T$ denotes the transpose operator, $\mathbf{w} = [w_1 \dots w_{n_R}]^T$ represents the noise samples at n_R receive antennas, which are modelled as independent samples of a zero-mean complex Gaussian random variable with noise variance σ_n^2 , \mathbf{H} is the $n_R \times n_T$ channel matrix, whose entries are the path gains between transmit and receive antennas modelled as the samples of zero-mean complex Gaussian random variables with equal variance of 0.5 per real dimension. Besides, we can assume that the signals transmitted from individual antenna has equal powers of P/n_T , i.e., $E\{\mathbf{s}\mathbf{s}^H\} = P/n_T \mathbf{I}_{n_T}$, where $(\cdot)^H$ denotes the Hermitian transpose operator, \mathbf{I}_{n_T} indicates the $n_T \times n_T$ identity matrix, and $E\{\cdot\}$ denotes the expectation operator.

Under the assumption that \mathbf{H} is perfectly known at the receiver, the signal can be detected by using MLD according to:

$$\hat{\mathbf{s}} = \arg \min_{\mathbf{s} \in \Omega} \|\mathbf{r} - \mathbf{H}\mathbf{s}\|^2 \quad (2)$$

where Ω is the transmission constellation, $\|\mathbf{A}\|$ denotes the Euclidean norm of matrix \mathbf{A} defined by $\|\mathbf{A}\| = \text{tr}(\mathbf{A}\mathbf{A}^H)$, $\text{tr}(\mathbf{A})$ denotes the trace of matrix \mathbf{A} .

III. Multi-level Sphere Decoder

In this part, for sake of clarity, before present the proposed detection approach, we will present the overview on the multi-level structure of the M-QAM, the principle of the multi-level SGA.

A. Multi-level structure of the N-QAM modulation constellation

The natural multi-level structure of a square M-QAM constellation is that for any M-QAM constellation, it can be constructed by a number of low-level constellation 4-QAMs. Fig.1 illustrated the case on 16-QAM. From Fig.1, the 16 dots represent the full constellation of 16-QAM. However, 16 dots can be divided into 4 areas, each with 4 dots corresponding to 4-QAM. It is worth noting that group of 4-dot 4-QAM can be represented by a pseudo point resulted from their gravity mean. Those pseudo points represented by squares form a pseudo 4-QAM. Therefore, we can construct a searching-tree with fixed number of branches of 4, so-called quad-tree.

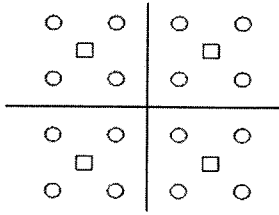


Fig. 1 : Illustration of multi-level structure of square 16-QAM constellation.

B. Basic Principle of Multi-level SGA detectors

The set of pseudo-symbols at the level l^{th} is defined as A_l , $l = 1, \dots, L$, where $L = \log_4(M)$. It can be easy to see that A_l consists of $N_l = M4^{1-l}$ signal points, and the lowest level where $l=1$ is exact the original constellation taken into consideration. Let $a_{l,s}$ is the pseudo symbol in the parent set A_l , $a_{l,s}$ can be calculated by gravity mean of 4 elements of the child set A_{l-1}^s , which is a sub-set of the lower level A_{l-1} as:

$$a_{l,s} = 0.25 \sum_{a_{l-1,k} \in A_{l-1}^s} a_{l-1,k} \quad (3)$$

Note that A_{l-1}^s are non-intersection sub-set of A_{l-1} . With the tree structure, a specific symbol $a_k \in A$ is only coupled to its ancestor $a_{l,s}$ at the l -th level. The joint probability is defined as:

$$p(a_k, a_{l,s}) = p(a_k)I(a_k, a_{l,s}) \quad (4)$$

where $I(a_k, a_{l,s})$ is an indicator function that a_k is a descendant of $a_{l,s}$. Thus, the marginal probability for pseudo symbol $a_{l,s}$ is given as:

$$p(a_{l,s}) = \sum_{a_k \in A} p(a_k)I(a_k, a_{l,s}) \quad (5)$$

Now, suppose that we have detected upto $(j-1)^{th}$ antennas. The MSGA detects for the j^{th} antenna as follows. It examines and choose the most N significant symbol combinations on the top level, l is largest. With N most significant combinations, it moves to lower level, and also keeps only N most significant combinations and continues until reaches to the lowest layer. Herein, the most N significant combinations are chosen based on the approximation moment-match Gaussian

distribution.

C. Proposed Multi-level Sphere decoder

In the proposed multi-level sphere decoder, instead of employing the PDA principle, we apply the Schnorr-Euchner-SD using DFS. In addition, in order to accelerate the search processing leading to complexity reduction, we employ the simply optimal sort techniques [] to arrange candidate list. The principle of the proposed decoder can be stated as follows. Assume that we have examined and got the best solution for the layer j , now we are going to examine for the layer $j-1$. Herein, we start examining in backward fashion from the layer n_T . Instead of examining and sorting all symbols in the constellation, which may be large for high-level modulation scheme, the proposed decoder first examines the top-sub-constellation of pseudo symbols, i.e., the pseudo 4-QAM. The most promising pseudo solution is easily achieved thanking the optimal sort technique. Then, the decoder continues with only the corresponding lower sub-constellation which has gravity mean of the most promising pseudo solution mentioned above. This process continues until all points have been examined or there is no point satisfying the searching condition any more. The illustration of quad-tree examination is shown in Fig.2.

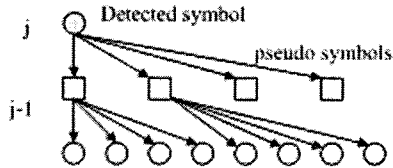


Fig. 2 : Quad-tree search of the proposed decoder.

It is worth noting that, at certain level, by using the pseudo symbol to test the search condition, we might reduce the testing operation much more than using the real original constellation. As a result, we can further to reduce the complexity of the search. In addition, although the number of pseudo sub-layers increase, each layer has only fix length of 4. Therefore, the proposed decoder can solve the problem of memory requirement variation facing in SDs.

IV. Computer simulation results and Discussion

In order to verify the performance of the proposed algorithm, we implement the computer simulation. In the simulation, we apply the proposed decoder into V-BLAST system of (4,4) employing 16-QAM. We evaluate the complexity by examining the average number of addition, multiplication and comparison operations required by the decoder. For clearly showing the benchmark of the posed decoder, we compare its performance to those of the original MSGA employing DFS with $N=20$ [5] and MLD. Due to the exponential complexity of MLD, we do not show in our comparison.

Fig.3 shows the comparison of performances of the proposed decoder and its counter parts in term of bit-error-rate (BER). As can be seen from the figure, both the proposed decoder and MSGA suffer from BER degradation in comparison with MLD. The BER degradation can be explained as that the reduction in number of candidates to be examined by using the pseudo symbol somewhat reduce the performance. However, the proposed decoder shows nearly 2dB improvement in comparison with MSGA.

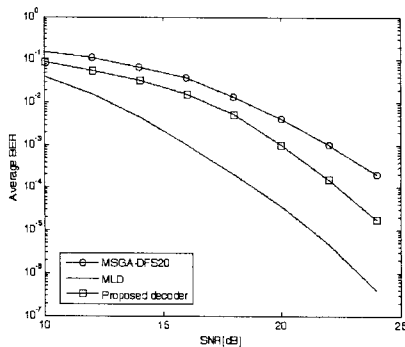


Fig. 3 : Performance of the proposed decoder and its counterparts in V-BLAST system of (4,4) employing 16-QAM.

The comparison of complexities of the proposed decoder and MSGA is presented in Table 1. It is easy to see that the proposed decoder can offer the lower complexity reduction than MSGA does. This is because that in the proposed decoder, the simply optimal sort technique used for preparing candidates has been exploited. As a result, not

only the complexity required for the search reduces but also the number of operations of searching lowers.

	MSGA-DFS20	Proposed
ADD	6050	2537
MUL	8587	4827
COMP	1143	2417
TOTAL	15780	9781

Table 1 : Comparison of complexities of the proposed decoder and MSGA-DFS20

V. Conclusion

In this paper, a low complexity decoder based on ordering SDs has been proposed. The proposed decoder, by exploiting the multi-level structure of square M-QAM, has shown significant reduction in complexity as well as nearly 2dB BER lower than its counterpart.

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