A Study of Ordering Sphere Decoder Class for Space-Time Codes

Van-Su Pham, Linh Mai, S.M. Humayun Kabir and Giwan Yoon
Information and Communications University (ICU)
E-mail : {vansu_pham, gwyoon}@icu.ac.kr

ABSTRACT

In this paper, an overview on the ordering sphere decoder (SD) class for space-time codes (STC) will be presented. In SDs, the ordering techniques are considered as promising methods for reducing complexity by exploiting a sorted list of candidates, thus decreasing the number of tested points. First, we will present the current state of art of SD with their advantages and disadvantages. Then, the overview of simply geometrical approaches for ordering is presented to address the question to overcome the disadvantages. The computer simulation results shown that, thanks to the aid of ordering, the ordering SDs can achieve optimal bit-error-rate (BER) performance while requiring the very low complexity, which is comparable to that of linear sub-optimal decoders.

Keywords

Space-Time Code, Multiple-input Multiple-output (MIMO) system, Sphere Decoder, Ordering, Optimal Detection, Wireless Communication

I. Introduction

Multiple-input and multiple-output (MIMO) systems have been promising approaches for enhancing both data transmission rate and transmission quality for the next-generation communication system due to the fact of exploitation of both spatial multiplexing and diversity gain [1]-[2]. In order to fully realize the capacity and diversity potential in the MIMO systems, optimal maximum-likelihood detection (MLD) and other optimal decoding algorithms are crucially desired. MLD, although allows to attain optimal bit-error-rate (BER) performance, its complexity, which is exponentially proportion to the number of transmit antenna and the level of modulation scheme, makes it impractical.

By relaxing the BER performance, a bunch of sup-optimal decoder classes have been proposed with very low complexity. Successive Interference Cancellation (SIC) and Ordering SIC are some names of them [3]-[4]. In spite of the fact that SIC or OSIC decoders reduce detection complexity, their solution is suboptimal and is significantly outperformed by the optimal one. Thus, techniques for achieving the optimal solution at reduced complexity are desirable. One of such techniques is sphere decoding, which allows the optimal solution to be reached at polynomial average complexity [5]-[9]. It is shown that in SD, the tree search is a crucial process to determine the complexity of the decoder algorithms. In other words, the quicker the search is, the lower complexity the decoder algorithm is. Therefore, in order to accelerate the search process, many techniques have been proposed for preparing an optimal set of test-needed candidates. In this study, we will present a review of simply geometric-based ordering techniques for decoders.

The remaining of paper is organized as follows. Section II will present the system model of a MIMO system. The Sphere Decoders will be revised in Section III. The ordering techniques will be given in Section IV. Performance and complexity comparison of ordering sphere decoders and their counterparts are given in Section V. The conclusion of the study is presented in the final section, Section VI.

II. System model

We consider an uncoded V-BLAST MIMO system with n_T transmit and n_R receive antennas ($n_R \geq n_T$). The bit data sequence is first mapped using a certain M-ary modulation scheme to form a transmit symbol vector $s = [s_1\, s_2 \ldots s_{n_T}]^T$, then is transmitted from n_T transmit antennas to the receiver. With assumption that the signals are narrow band, the received signal vector can be formed as follows:

$$Y = Hs + w \qquad (1)$$

Where w represents the noise samples at n_R receive antennas, which are modeled as independent samples of zero-mean and variance σ^2 complex Gaussian random variable, H is the $n_R \times n_T$ channel matrix, whose entries are the path gains between transmit and receive antennas modeled as the samples of zero-mean complex Gaussian random variable with equal variance of 0.5 per real dimension. In addition, we assume that the signals transmitted from individual antenna have equal power of P/n_T.

III. Sphere decoder

Sphere decoders were first proposed to find vectors of shortest length in a given lattice by Fincke and Pohst [5]. It then has been adjusted to solve the so-called integer least-square problem:

$$\arg\min_{u \in Z^n} \|x - Mu\|^2 \qquad (2)$$

Where x is an $m \times 1$ real vector, M is an $m \times n$ real matrix called the lattice-generating matrix, Z^n denotes the n-dimensional integer lattice, and u is an $n \times 1$ vector with integer entries.

It is easy to see that, for an uncoded MIMO system employing M-QAM symbols, after real-decoupling signal model of (1), we can apply (2) for finding ML solution.

$$\hat{u} = \arg\min_{u \in Z^n} \|x - Mu\|^2 \qquad (3)$$

The idea behind SD is to search over only lattice points that lie in a certain hypersphere of radius C_0 centered at the received signal x. It is obvious that the closest lattice point inside the hypersphere is also the closest point in the whole lattice, and is the ML solution.

The main issue to be resolved is how to find the closest lattice point in the hypersphere. A lot of efforts have been put into the search for algorithms achieving ML or near-ML performance with lower complexity.

A) Real-valued Sphere Decoders

Real-valued sphere decoders were proposed to solve the ML decoding problem in (3). A lattice point Mu is in a hypersphere of radius C_0 centered at x if and only if

$$\|x - Mu\|^2 \le C_0 \qquad (4)$$

Let the channel matrix M be QR decomposed as

$$M = [Q_1\ Q_2][R\ \ 0_{(m-n),n}]$$

Where R is an $n \times n$ real, upper triangular matrix, Q is an $m \times m$ real, orthogonal matrix, Q_1 and Q_2 respectively contains the first n and the last m-n columns of Q.

Then the condition (4) can be rewritten as:

$$\|Q_1^H x - Ru\|^2 \le C_0 - \|Q_2 x\|^2 \qquad (5)$$

Defining $y = Q_1^H x$ and $C = C_0 - \|Q_2^H x\|^2$, (5) can be reexpressed as:

$$\sum_{i=1}^{n} (y_i - \sum_{j=i}^{n} r_{i,j} u_j) \le C \qquad (6)$$

where $r_{i,j}$ is the entry at row i and column j of R.

It can be seen that, the necessary condition for Mu to lie inside the sphere is that

$$(y_n - r_{n,n} u_n^2) \le C \qquad (7)$$

Equivalently,

$$LB(u_n) = \frac{-\sqrt{C} + y_n}{r_{n,n}} \le u_n \le \frac{\sqrt{C} + y_n}{r_{n,n}} = UB(u_n) \qquad (8)$$

Continuing in a similar fashion for $u_i\ (i = 1,..,n-1)$ we have $LB(u_i) \le u_i \le UB(u_i)$.

Based on the intervals $[LB(u_i), UB(u_i)]$, so-called natural spanning [5], Pohst considered the enumeration, and u_i takes on the sequence of values: $LB(u_i)$, $LB(u_i)+2, \ldots, UB(u_i)$.

The other variation of the Pohst strategy, where the intervals are spanned in a zigzag order, starting from the middle point [6]:

$$\tilde{y}_i = \lceil \frac{1}{r_{i,i}} (y_i - \sum_{j=i+1}^{n} r_{i,j} u_j) \rfloor \qquad (9)$$

Where $\lceil . \rfloor$ denotes round function.

Therefore, Schnorr-Euchner enumeration will produce at each level i the sequence for u_i as follows:

- If $y_i - \sum_{j=i+1}^{n} r_{i,j} u_j - r_{i,i} \tilde{y}_i \ge 0$ then $u_i \in \{\tilde{u}_i, \tilde{u}_i + 2, \tilde{u}_i - 2, \tilde{u}_i + 4, \ldots\} \cap [LB(u_i), UB(u_i)]$
- If $y_i - \sum_{j=i+1}^{n} r_{i,j} u_j - r_{i,i} \tilde{y}_i \le 0$ then $u_i \in \{\tilde{u}_i, \tilde{u}_i - 2, \tilde{u}_i + 2, \tilde{u}_i - 4, \ldots\} \cap [LB(u_i), UB(u_i)]$

As mentioned above, in order to apply real-valued sphere decoder, a complex MIMO system must be decoupled into its real and imaginary parts so as to form an equivalent real-valued system. Consequently, they are most efficient for lattice-based modulation schemes such as M-QAM or PAM. For other complex constellation, namely, M-PSK, the use of the real-valued SDs is inefficient due to the existence of "invalid" candidates, leading to degradation in system performance. The problem of invalid candidates can be solved by using the method of eliminating "invalid" candidates or the complex sphere decoders [7]-[9].

B) Complex Sphere Decoders

In general, the complex Sphere Decoders follow all steps of the real-valued SD discussed previously. The main difference is that the QR decomposition is applied directly to the complex system. Similarly, we also have a necessary condition as:

$$\|v_n - R_{n,n}x_n\|^2 \leq C \qquad (10)$$

Equivalently

$$\|x_n - v_n/R_{n,n}\|^2 \leq C/R_{n,n}^2 \qquad (11)$$

This inequality limits the search to points of the constellation contained in a complex disk of radius $d_n = \sqrt{C}/R_{n,n}$ centered at $\tilde{x}_n = v_n/R_{n,n}$. Let z_n is a M-PSK symbol, $\theta \in \{0, 2\pi/M, \dots, 2\pi(M-1)/M\}$, the intersection of boundary can be found by:

$$\cos(\theta_n - \tilde{\theta}_n) \geq \eta \qquad (12)$$

It is easy to see that [7]:

- If $\eta > 1$, then the search disk doesn't contain any point of the M-PSK constellation.
- If $\eta < -1$, then the search disk includes the entire constellation.
- If $-1 \leq \eta \leq 1$, the possible angle for searching z_n satisfies (12)

IV. Simply geometrical ordering techniques

As we can see from the discussion above, the crucial point to reduce the complexity of SDs is to construct the most optimal candidate set. The optimization here means that the set of candidates contains as small number of candidates as possible, and the set of candidates need as less number of search to reach the solution as possible. Normally, the set of candidates is sorted based on the increasing of its Euclidean distance. As a result, this leads to a bottleneck of computation if the system is large and/or the high-level modulation scheme is employed. A lot of efforts have been put into dealing with this issue. In this section, we present three popular ordering techniques, namely PAM-oriented ordering, QAM-oriented ordering, and PSK-oriented ordering.

A) PAM-oriented ordering

Let $z = (z_1 z_2 \dots z_N)$ be N integers in Z. Illustrated in Fig.1 is the approach to specify the optimal testing order for $x_k \in Z$ at layer k, simply by comparing x_k with appropriate b-boundaries and c-boundaries [9]. b-boundaries are constructed as the line-boundary between the two adjacent corresponding constellation points, while c-boundaries are constructed as the line-boundary passing the corresponding constellation point. For example, it is easy to see from Fig.1 that, z_3 is the first values to be tested. In addition, $x_n \leq R_{k,k}z_3$, the optimal testing order for all for values in z are (z_3, z_2, z_4, z_1). In case of $x_n \geq R_{k,k}z_3$, the detection order will be (z_3, z_4, z_2, z_1).

It is easy to see that for real-valued decoupled system, PAM-oriented ordering can also be directly applied.

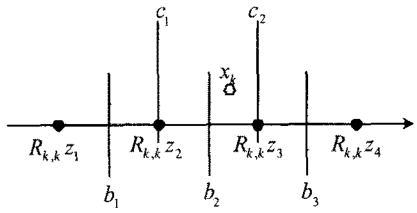

Figure 1: Illustration of PAM-oriented ordering technique

B) QAM-oriented ordering

The illustration of QAM-oriented ordering is given for 16-QAM in Fig.2 [10]. In the first step, through the first quadrant detection of x_n, the region where x_n is located is roughly selected by comparing real and imaginary parts (so-called I and Q components) with the real and imaginary axes. In the next step, we shift the coordinator so that the new origin coincides with the center of the selected region. Then, through the quadrant detection, a smaller region where x_n is located is detected. By repeating this process N times (N=3 for 16-QAM), we can divide and construct the detection order for all constellation.

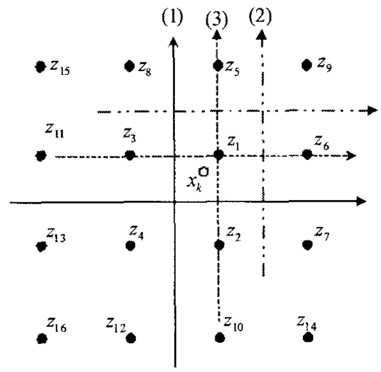

Figure 2: Illustration of QAM-oriented ordering technique

C) PSK-oriented ordering

Due to the special constant-modulus structure of M-PSK the above two ordering techniques can not be applied. A very simple method of specifying the optimal testing order for all signal points at level k just by comparing the slopes, which is simply resulted from real division operator, of straight lines passing the origin is depicted in Fig.3 [9]. For simplicity, we consider a 8-PSK constellation, and let $z = (z_1, z_2, \dots, z_8)$ be 8 signal points from that constellation. First, the slope of the straight line passing the origin and the received point can be computed as:

$a_k = Im\{x_n\}/Re\{x_n\}$ (13)

Where Im{} and Re{} denote the real and imaginary parts, respectively. Then, by comparing a_k in (13) with the slopes b1, b2, b3 and b4 of the four solid lines, we are able to locate region when the received point x_n belongs to. As a result, the first symbol to be tested, namely, z_l, and the slope c_l of the dashed line passing the origin and the point $R_{k,k}z_l$ can easily be obtained. For each given M-PSK, it has its own pre-determined b-boundaries and c-boundaries. Finally, by comparing a_k with c_l, the detection orders of the remaining symbols can be achieved without difficulty. For example, one can observe from Fig.4 that $b_2 \le a_k \le b_3$, therefore, z_2 will be the first symbol to be tested. In addition, since $a_k \le c_2$, the optimal testing order for 8 signal points is $(z_2, z_1, z_3, z_8, z_4, z_7, z_5, z_6)$. In case $a_k \ge c_2$, the optimal order becomes $(z_2, z_3, z_1, z_4, z_8, z_5, z_7 z_6)$.

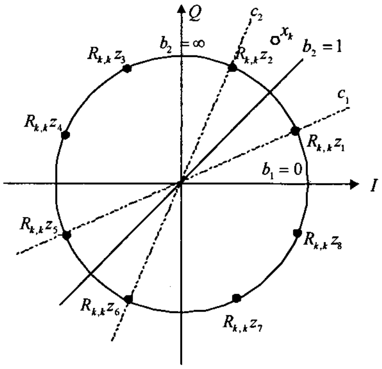

Figure 3: Illustration of PSK-oriented ordering technique

V. Performance and complexity

In this section, the computer simulations are implemented to evaluate and compare the bit-error-rate (BER) performance and complexity of the ordering Sphere Decoders and their counterparts.

Fig.4 illustrates the BER performances of brute-force maximum-likelihood, the PAM-oriented ordering SD, the QAM-oriented ordering SD and PSK-oriented ordering SD. In our simulation, we assume that the signals are transmitted on a burst-by-burst basic. In addition, the channel is assumed to be quasi-static within each burst and randomly change from on burst to the next. The 6 transmit - 6 receive antenna system employs 8-PSK, 16-QAM scheme. The complex system is real-valued decoupled to apply PAM-oriented ordering SD. As we can be seen, the BER performances are almost identical. This means that the PAM-oriented ordering SD, QAM-oriented ordering and PSK-oriented ordering SD are optimal decoders.

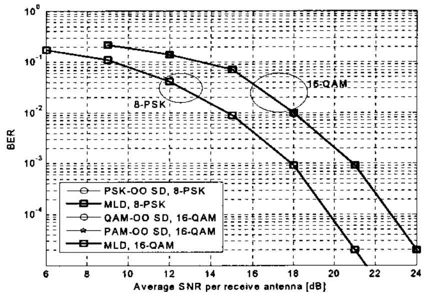

Figure 4: BER performance comparisons

The average complexities of the PAM-oriented ordering SD, QAM-oriented ordering, the PSK-oriented ordering and the linear MMSE-SQRD-based decoder [4] are compared in Fig. 5. The set-up for the simulation is similar to that for Fig.4. As can be seen from Fig.5, the average complexity of the ordering SD classes can be comparable with that of linear decoder while they allow to obtain ML-like BER performance.

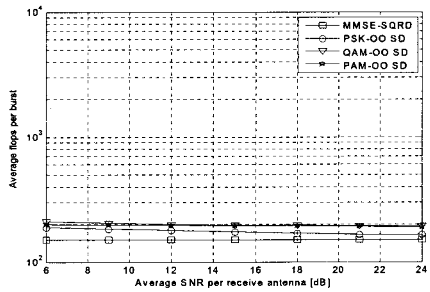

Figure 5: Average Complexity Comparison

V. Conclusion

This study has presented a review on three simply geometrical approaches for optimally ordering the tested set of the sphere decoders. Each of them is proposed for certain signal constellation characteristics, namely PAM, QAM and PSK. By applying these simply geometrical approaches, the decoders not only are accelerated in reaching the optimal solution, but also is significantly reduced the number of computations required for preparing the set of tested candidates. It is also worth mentioning that these approaches are not limited in the context of SDs, they can be applied to any decoders exploiting tree-search, such as QRD-M decoder.

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