Development of Educational Linear Program Software Using VBA: LP-Tableau

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Abstract

A spreadsheet-based linear program (LP) software, so called, the LP-Tableau is developed to aid users to learn the simplex algorithm systematically and to understand the various conditions that make the problem unusual and, in some instances, impossible to solve. The LP-Tableau has several good features over other spreadsheet-based LP software programs available. First, the data input in the LP-Tableau is very convenient especially when small-sized problems are analyzed. Further, the LP-Tableau allows for two solution methods (i.e., Big-M and Two-Phase) and displays the whole iterations in tableau format. Thus users can easily understand the principle of the algorithm step by step. Finally, the LP-Tableau visually displays various conditions of the optimal solutions that might occur in solving the LP problems.

1. Introduction

The decision problems formulated by using deterministic decision models are frequently resolved by using linear programs (LPs). We easily encounter the LP software packages which were developed for such purpose but are different to each other, depending on the types of decision-making problems they intend to solve. They include, to name a few, MPS, Lindo Solver Suites, MINOS, Excel Solver, K-LP (Kim, et. al., 1994), LPAKO (Park, et. al., 1998) and so on (refer to the following web site for detailed descriptions, http://www-fp.mcs.anl.gov/OTC/Guide/SoftwareGuide/Categories/linearprog.html). Among others, we notice the LP software packages that operate in the Microsoft Excel spreadsheet since it is not only readily available in both the classroom and the workplace, but it is relatively easy to learn and use. The LP software packages equipped with the powerful Excel spreadsheet can provide the users with user-friendly interface for data-input and results-output by virtue of the intrinsic nature of the Excel spreadsheet. The popular LP software packages that fall into this class include Excel Solver add-ins by the Frontline company and What's best add-ins by the Lindo company. Thus many professors and lecturers who majored in management science deliver their lectures, using one of these two LP software packages (the more popular one in the classroom seems to be the Excel Solver add-ins because it is built-in in the Excel spreadsheet). Most of popular management science textbooks (for instance, Stevenson and Ozgur, 2007; Hillier and Hillier, 2007; Anderson, et. al., 2007; Winston and Albright, 2007) give a lot of space to how to use the LP software in their main contents. Despite of wide-spread use of these two popular spreadsheet-based LP software packages, however, they all lack in the detailed descriptions about the process of determining the optimal solutions although the LP algorithms generally iterate several times depending on the structure of the formulations. This finding motivates us to develop new LP software that shows the whole iterations of determining the optimal solutions. The LP-Tableau has several good features over other spreadsheet-based LP software programs currently available.

- The data input in the LP-Tableau is very convenient especially when small-sized problems are analyzed.
- The LP-Tableau allows for two solution methods: Big-M and Two-Phase. Thus it shows the results according to the user's selection.
- The LP-Tableau displays the whole iterations in tableau format. Thus users can easily understand the principle of the simplex algorithm step by step.
- The LP-Tableau visually displays various conditions of the optimal solutions that might occur in solving the LP problems.
- The LP-Tableau allows for many LP formulations on a single spreadsheet whereas previous spreadsheet-based LP software programs attached one formulation to the one spreadsheet.

This paper is organized as follows: the system characteristics are described in Section 2. The system is illustrated in detail by solving small-sized LP problems in Section 3. Concluding remarks follow in Section 4.

2. System Characteristics

The main screen of the LP-Tableau consists of four groups of items each of which is separated by a group box, as shown in Figure 1.

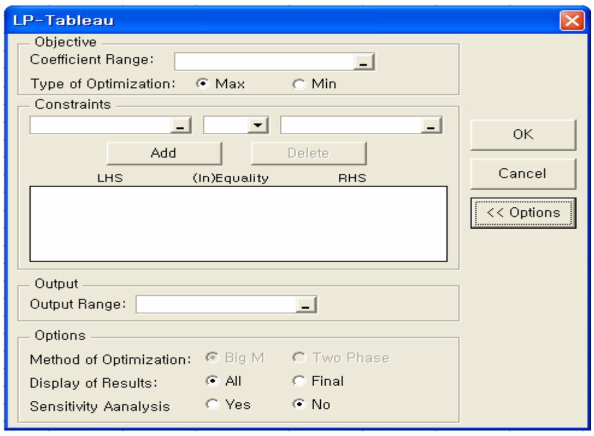

Figure 1. The main screen of the LP-Tableau

The items performing related functions are grouped in a separate group.

- Objective: Enter necessary information regarding the objective function in the LP formulation.

Coefficient range: Designate the coefficient range of the objective function that is already entered in the spreadsheet.

Type of optimization: Determine whether to maximize or minimize the objective function value.

- Constraints: Enter necessary information regarding the constraints in the LP formulation. User can enter the constraints one by one or group of constraints at once with the same equalities or inequalities. It is unnecessary to write down nonnegative constraints about the variables on the spreadsheet since the LP-Tableau assumes that all variables are nonnegative.
- Output: Designate the place where the tableaus appear on the spreadsheet.
- Options: Designate user's preference about the method of optimization, display of results, and sensitivity analysis. An initial main screen hides the "Options" portion when you run the system. Once you click the "Option >>" button, then "Options" portion is unfolded and the button changes to "Option <<".

Method of optimization: Enter your selection about which method between the Big-M and the Two-Phase you want to optimize the LP problem with. Initially, the option buttons on the Big-M and Two-Phase are disabled. Once you enter some constraints in "Constraints" and the constraints contain at least one "greater than" (i.e., >=) or "equal to" sign (i.e., =), they are turned on, which implies that some artificial variables activating one of the two methods are introduced in the formulation.

Display of results: Select the option button "All" if you want to view the whole tableaus resulting as the algorithm proceeds or select the "Final" if you want to view the final tableau.

Sensitivity analysis: Enter "Yes" if you want to view how sensitive the optimal solutions are when the objective function coefficients or resources available change. Default setting is "No."

The procedures of the LP-Tableau are coded by using the visual basic for application (VBA), which is built-in program language in the Excel spreadsheet (Bullen, et. al., 2003). The Excel spreadsheet, coupled with the VBA, have unlimited capabilities in dealing with subjects on management science (Albright, 2007). Now we shall illustrate the LP-Tableau by using some LP examples in next section.

3. Illustrations of the LP-Tableau

3.1 The LP problem resulting in the optimal solution

Let us assume that an LP problem (adopted from Bazaraa (1977)) is given as shown below.

Minimize $x_1 - 2x_2$

Subject to
$$x_1 + x_2 \geq 2$$
$$-x_1 + x_2 \geq 1$$
$$x_2 \leq 3$$
$$x_1,\quad x_2 \geq 0$$

Prior to using the LP-Tableau, you have to enter the raw data into the spreadsheet as shown in Figure 2. It is worthwhile to mention that you do not have to use any Excel mathematical function, for example SUMPRODUCT, which has to be used in the popular spreadsheet-based LP software program. This can possibly lessen the burden of inputting the data in relatively small-sized problems.

	A	B	C	D	E
1					
2	Minimize	x1	x2		
3		1	-2		
4					
5	Subject to	1	1	>=	2
6		-1	1	>=	1
7		0	1	<=	3

Figure 2. The raw data on the spreadsheet

We entered the three inequalities in the range "D5:D7" for denoting the relations of the constraints and this is not required in using the LP-Tableau. After finishing entering the parameters, you invoke the main screen of the LP-Tableau and then designate appropriate portions on the spreadsheet to the corresponding controls in the dialog box of the LP-Tableau.

Instead of capturing the whole tableaus, we will describe the final optimal tableaus resulted by adopting the Big-M and the Two-Phase methods in Figure 3 and Figure 4 respectively, in which the variables starting "S" denote slack or surplus ones and the number indicate the constraint number when we sequentially number the constraints up to down. Similarly, the variables starting "A" denote artificial ones that should be eliminated from the basis if the original problem is feasible and the number also indicates the constraint number. As usual in the Big-M method, the "M" denotes a big number.

	A	B	C	D	E	F	G	H	I	J	K	L
43												
44						FifthTableau						
45				1	-2	0	0	1M	1M	0		
46		Basis		X1	X2	S1	S2	A1	A2	S3	RHS	
47		S2	0	1	0	0	1	0	-1	1	2	
48		X2	-2	0	1	0	0	0	0	1	3	
49		S1	0	-1	0	1	0	-1	0	1	1	
50		Z		0	-2	0	0	0	0	-2		
51		C-Z		1	0	0	0	1M	1M	2	-6	
52												

Figure 3. The optimal solution resulted by the Big-M method

	A	B	C	D	E	F	G	H	I	J
52										
53					SixthTableau					
54				1	-2	0	0	0		
55		Basis		X1	X2	S1	S2	S3	RHS	
56		S2	0	1	0	0	1	1	2	
57		X2	-2	0	1	0	0	1	3	
58		S1	0	-1	0	1	0	1	1	
59		Z		0	-2	0	0	-2		
60		C-Z		1	0	0	0	2	-6	
61										

Figure 4. The optimal solution resulted by the Two-Phase method

In the Two-Phase method, the artificial variables are entirely eliminated from the starting tableau of the second phase if the LP problem is feasible since they have no chance of entering into the basis. Thus once all artificial variables are removed from the basis at the end of the first phase, the original objective function coefficients have to be introduced and pivoted for preparing the second phase.

In general, we frequently encounter the situations where we can not obtain a single optimal solution. Rather, other conditions including, for example, multiple optimal, unbounded, or infeasible occurs and those cases should be appropriately dealt with in the LP-Tableau.

3.2 The LP problems resulting in conditions other than the optimal solution

3.2.1 Multiple optimal solutions

In a sizeable LP problem, there frequently occur multiple optimal solutions. That is, the same optimal value of the objective function might be possible with a number of different combinations of values of the decision variables. The LP-Tableau detects the multiple optimal solutions by investigating *C-Z* vector for the non-basic variables and informs it by producing an additional tableau just after the first optimal solution. To illustrate the multiple optimal solutions, let us consider the following LP formulation:

Maximize $60x_1 + 30x_2$

Subject to $4x_1 + 10x_2 \leq 100$

$2x_1 + x_2 \leq 22$

$3x_1 + 3x_2 \leq 39$

$x_1, x_2 \geq 0$

Solving this LP problem, we obtain the following results shown in Figure 5.

	A	B	C	D	E	F	G	H	I	J
17					SecondTableau					
18				60	30	0	0	0		
19		Basis		X1	X2	S1	S2	S3	RHS	
20		S1	0	0	8	1	-2	0	56	
21		X1	60	1	1/2	0	1/2	0	11	
22		S3	0	0	1 1/2	0	-1 1/2	1	6	
23		Z		60	30	0	30	0		
24		C-Z		0	0	0	-30	0	660	
25										
26					ThirdTableau					
27				60	30	0	0	0		
28		Basis		X1	X2	S1	S2	S3	RHS	
29		S1	0	0	0	1	6	-5 1/3	24	
30		X1	60	1	0	0	1	- 1/3	9	
31		X2	30	0	1	0	-1	2/3	4	
32		Z		60	30	0	30	30		
33		C-Z		0	0	0	-30	0	660	
34										

Figure 5. The optimal solution resulted by the Two-Phase method

By reading the final two tableaus, we find that the LP problem results in the multiple optimal solutions and we can construct a line segment connecting these two optimal solutions such as $(11, 0) + \lambda(9, 4), \lambda \in [0, 1]$.

3.2.2 Infeasible problem

Infeasibility occurs when there are no feasible points that satisfy the given LP problem. Users who are not familiar with the LP formulations frequently produce infeasible programs. Thus it is very important to let them know in what constraint possibly causes it. As is well-known, the canonical forms of the LP problems never produce infeasible LP programs. Thus by investigating, if any, the non-zero artificial variables in the basic variables, we can identify whether the LP program is infeasible or not, which is what the LP-Tableau does.

Minimize $-x_1 - 3x_2 + x_3$

Subject to $x_1 + x_2 + 2x_3 \leq 4$

$-x_1 + x_3 \geq 4$

$x_3 \geq 3$

$x_1, x_2, x_3 \geq 0$

Solving this LP problem, we obtain the following results shown in Figure 6.

SecondTableau

		-1	-3	1	0	0	0	1M	1M	
Basis		X1	X2	X3	S2	S3	S1	A2	A3	RHS
X3	1	1/2	1/2	1	0	0	1/2	0	0	2
A2	1M	-1 1/2	-1/2	0	-1	0	-1/2	1	0	2
A3	1M	-1/2	-1/2	0	0	-1	-1/2	0	1	1
Z		-2M+0.5	-1M+0.5	1	-1M	-1M	-1M+0.5	1M	1M	
C-Z		2M-1.5	1M-3.5	0	1M	1M	1M-0.5	0	0	3M+2

Figure 6. The infeasible problem

Two artificial variables, "A2" and "A3" were used in the second and third constraints to form the initial basis (i.e., identity matrix) and they remain in the basis with non-zero values in the final optimal tableau. The LP-Tableau informs us those artificial variables by coloring them in red.

If at least one of the basic variables in the optimal tableau equals to zero, then the solution is called degenerate optimal solution. The LP-Tableau identifies it by checking the revised RHS and informs it by coloring the zero value in red. The LP-Tableau detects an unbounded solution by investigating the column vector of entering non-basic variable and informs it, if it exists, by coloring the non-positive column vector in red.

3.3. Sensitivity analysis

As most of educational or commercial LP software programs equips with the sensitivity analysis function, the LP-Tableau also has the sensitivity analysis function and display the results of analysis on the final tableau for user's convenience. Let us reconsider the LP problem used in Section 3.1 for the sensitivity analysis.

Final Tableau

		1	-2	0	0	1M	1M	0			
Basis		X1	X2	S1	S2	A1	A2	S3	RHS	Range of	Feasibility
S2	0	1	0	0	1	0	-1	1	2	RHS1	(-Inf, 3]
X2	-2	0	1	0	0	0	0	1	3	RHS2	(-Inf, 3]
S1	0	-1	0	1	0	-1	0	1	1	RHS3	[2, Inf.)
Z		0	-2	0	0	0	0	-2			
C-Z		1	0	0	0	1M	1M	2	-6		
Range of		C1	C2								
Optimality		[0,+inf.)	(-Inf., 0]								

Figure 7. The result of sensitivity analysis

The results of the sensitivity analysis for the objective function coefficients are written down in the bottom row as named "Range of Optimality" and the results of the sensitivity analysis for the RHS are written beside the revised RHS as named "Range of Feasibility," where "Inf." denotes the infinite number. The values denote not incremental but the minimum and maximum within which the parameters considered are allowed to change.

4. Concluding Remarks

In viewpoint of education, students can easily conduct various managerial experiments with help of the Excel spreadsheet and VBA. In line with this trend, we developed new spreadsheet-based LP software, the LP-Tableau. By virtue of using the VBA as a programming language, we can utilize multitude of useful functions inherent in the Excel spreadsheet. The main feature of the LP-Tableau over other spreadsheet-based LP software programs is to display the whole process of determining the optimal solution in complete tableau format.

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