

## Vibration Analysis of Bladed Disk using Non-contact Blade Vibration System

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### Abstract

The blade vibration problem of bladed disk is the most critical subject to consider since it directly affects the stable performance of the engine as well as life of the engine. Especially, due to complicated vibration pattern of the bladed disk, more effort was required for vibration analysis and test. The research of measuring the vibration of the bladed disk, using NSMS(Non-intrusive stress measurement) instead of Aeromechanics testing method requiring slip ring or telemetry system with strain gauge, was successful. These testing can report the actual stresses seen on the blades; detect synchronous resonances that are the source of high cycle fatigue (HCF) in blades; measure individual blade mis-tuning and coupled resonances in bladed disks. In order to minimize the error being created due to heat expansion, the tip timing sensor is installed parallel to the blade trailing edge, yielding optimal result. Also, when working on finite element analysis, the whole bladed disk has gone through three-dimensional analysis, evaluating the family mode. The result of the analysis matched well with the test result.

### Introduction

When developing the bladed disk, the vibration problem of rotor blades is directly related to stable performance as well as HCF life of the airfoil. Therefore, while going through development examination, general process is to attach strain gauge onto blades, measuring resonance and magnitude of vibration stress acting on the blades. However, in order to receive the signals sent from the rotating rotor blades, special tools such as slip ring or telemetry system is required. Also, in order to measure the blades of bladed disk with high temperature, it is necessary to attach strain gauge for high temperature purpose. However, due to technical problems, the cost and the required period is immense. To make situation worse, because of short HCF life of strain gauge for high temperature purpose, it is very difficult to obtain continuous examination data. Also due to complexity of the measurement method, there is a huge burden on investment group about loss. In order to prevent such disadvantages and risks of using the method of measuring vibration of the rotor blade using the strain gauge, the new system that would replace the original method was implemented: NSMS (Non-intrusive Strain Measurement System). NSMS measures the vibration occurring at the tip

of the blade, using TOA(Time Of Arrival)<sup>(1)</sup>. Not only this system would reduce the required period and the cost for the process, but also the new system strengthens the weak point that can be found when measuring the vibration using strain gauge. Unlike blade with no resonance, when there is a resonance occurring on the blade, the change in phase shift or TOA can be noticed. NSMS is a technology, which attaches the sensor onto the case, measuring the change in phase shift or TOA.<sup>(2,3)</sup> Recently, this technology was implemented by several advanced and superior companies, due to brilliant development of sensor or DSP(Digital Signal Processing). Especially, the most significant advantage of NSMS would be that it is possible to measure the vibration of the every single blade. When using the strain gauge, due to limitation of number of usable signals from signal transmission device, strain gauges are attached to certain number of blades only. Therefore, it is impossible to measure the vibration of the every single blades of the assembly. By measuring the vibration of whole blades, it is possible to confirm the coupling effect of disc and blades, as well as phenomenon resulted from mistuning between each blade. To add another disadvantage of using strain gauge method is that, the attached strain gauge on the blade would affect aerodynamics as well as providing additional weight or diminution effect, making it tough to measure the actual vibration. NSMS method, which get rids of the disadvantages caused when using strain gauge, is complimented and commonly used by many turbo-machinery companies due to brilliant advance in sensor technology and DSP development. NSMS measurement method attaches the measurement sensor onto the case, measuring the deflection of the blade.<sup>(4,5)</sup> Once deflection of the blade was measured, unique oscillation frequency and phase shift are analyzed, reconstructing the vibration mode. Afterwards, the process can be finished as maximum vibration stress is predicted.

### Blade Resonance Analysis Method

The method of the single degree of freedom (SDOF) curve fit is based upon the assumption that each blade is a single degree of freedom resonator. The assumption is a good one if the bladed disk is a weakly coupled system. In the weakly coupled bladed disk, system modes are so localized that one blade dominates each system mode. In such a system, each blade is acting like a simple resonator. NSMS measures tip deflection of a blade only once per

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revolution. For an SDOF resonance, this measurement will result in typical signal as shown in Fig.1. The value in the Fig.1 is a sensor phase angle relative to the vibration node. Depending on the sensor location, the signal for the same SDOF system response will vary. The method of the SDOF curve fit uses these signals to identify the parameters of resonance frequency, 0-peak amplitude and damping in the resonance. The simplest SDOF curve fit uses the data from only one sensor. With multiple sensors, the resonance can be identified multiple ways. Identification of the same resonance with multiple SDOF curve fits with these different sensors should give the same parameters. Theoretically, a signal from one sensor is enough to identify all resonance parameters. For more complicated coupled modes, each blade participates in several system modes and exhibits multiple closely-spaced resonances. It is then not correct to model the response of each blade as that of a SDOF resonator. Alternative modal analysis techniques are then applicable.

To perform Circumferential Fourier fit, three or more sensors at the same chord-wise position are required. At least three sensors are required to obtain three unknowns amplitude, phase and DC offset of the arrival delay. In addition, since blades vibrate with harmonics of excitation engine order, the excitation engine order should be known before Circumferential Fourier fit is performed. If there are more than three sensors, a least-mean-squares fit can be used; and error in the fit is then also available. One example of Circumferential Fourier fit is shown in Fig 2. Four data points from four sensors are obtained.

The equation to fit is where is measured arrival delay, is unknown DC offset, is unknown vibratory amplitude, is the engine order of the response and is unknown phase. If the equation is rearranged with knowns one side and unknowns the other, it becomes the (1). The equation (1) can be solved using least mean square arithmetic.(6)

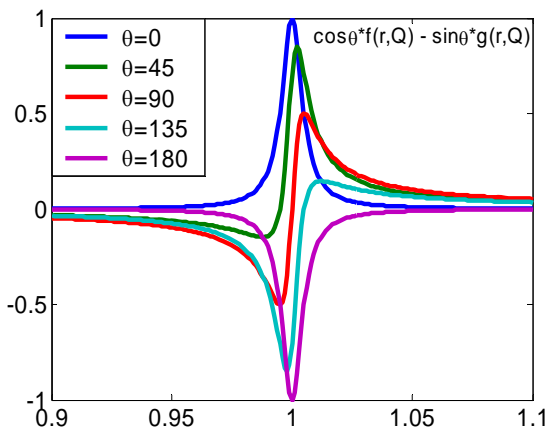


Fig. 1 Magnitude as a function of RPM. Observed patterns of blade delay for a SDOF resonator. Depending on sensor location, the signal for the same resonator is different.

$$\begin{pmatrix} \delta_1 \\ \delta_2 \\ \delta_3 \\ \delta_4 \end{pmatrix} = \begin{bmatrix} 1 & \sin E\theta_1 & \cos E\theta_1 \\ 1 & \sin E\theta_2 & \cos E\theta_2 \\ 1 & \sin E\theta_3 & \cos E\theta_3 \\ 1 & \sin E\theta_4 & \cos E\theta_4 \end{bmatrix} \begin{pmatrix} dc \\ A \cos E\varphi \\ A \sin E\varphi \end{pmatrix} \quad (1)$$

or

$$\bar{\delta} = M \bar{p} \quad (2)$$

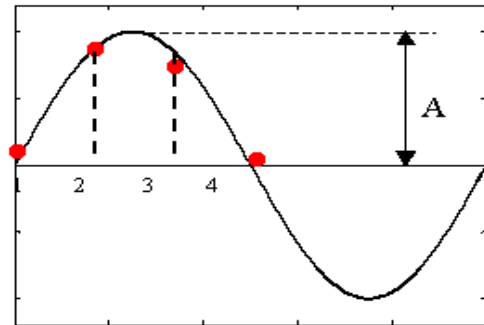


Fig. 2 Magnitude as a function of sensor position. Red dots indicate measurement at four sensors along the same chord-wise position. Black line is vibratory trajectory of a blade.

### Test Setup

The main sensor used in this test was optical Dual Light Probes (DLPs) which are small, un-cooled optical probe used for engine test. This sensor, shown in Fig. 3, consists of two optic fiber bundles with a skew angle of approximately 15 degree between their axes. Each optic fiber bundle consists of a center fiber, which carries laser light and six surrounding receive fibers. The spread angle of each light beam exiting the center fiber is about 15 degree (total included angle). As a blade passes in front of the probe, it creates a reflected optical pulse on each channel of the sensor.

Using NSMS, the distance of flight from one light beam to the next can be measured. As the passing blade moves closer to the probe, the distance of flight decreases. These probes can be used in single-channel mode, measuring only tip timing, or in dual-channel mode, measuring both tip timing and tip clearance. Best axial positioning of the sensors depends on the shape of the mode or modes of interest. Ideally a measurement should be taken at the location of the largest deflection to maximize the sensitivity to the mode. When more than one mode of interest is involved, the axial position of the sensor should be the one that has appreciable sensitivity to all modes. The shape of the bladed disk is shown in Fig. 4.

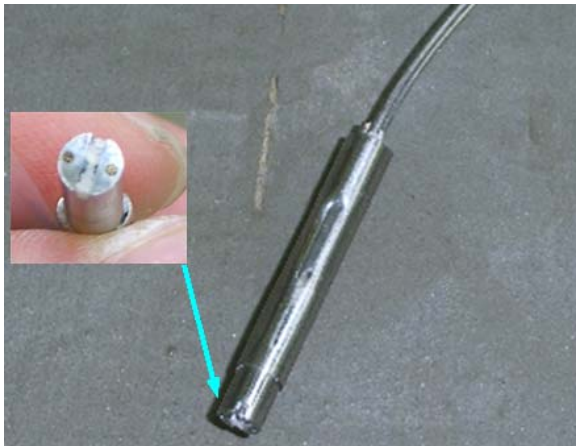


Fig. 3 Small, un-cooled optical probe used for engine test, measuring both tip clearance and tip deflection. Probe uses two fiber bundles skewed at about 15 degree.

The mode shape has large modal displacements at trailing edge in the vibration analysis. Therefore, when a optical point sensor is located near the trailing edge to measure circumferential tip deflection. A small difference in axial positioning of the sensor could result in large difference in reported tip deflection. In order to minimize the error being created due to heat expansion, the tip timing sensor is installed parallel to the blade trailing edge,

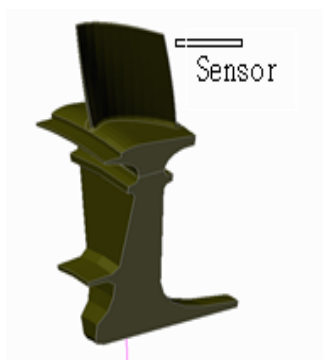


Fig. 4 Shape of bladed disk. a optical point sensor is located near the trailing edge to measure circumferential tip deflection.

The Circumferential Fourier analysis technique benefits from appropriate circumferential spacing of the sensors. A good circumferential spacing is one that gives large condition number for the matrix  $M$ . Thus, the best sensor spacing depends upon the engine order of the response. The sensor locations need not be located within a single vibration cycle, but can be spread around the circumference. However, spacing the sensors closely makes the data insensitive to whole-rotor motions due to whirl and unbalance. For the test, a uniform circumferential spacing of 11.0 degree was used considering the nozzle passing

frequency(NPF). This configuration of sensor locations gives large condition number. Fig. 5 shows the installation drawing of multiple sensors.

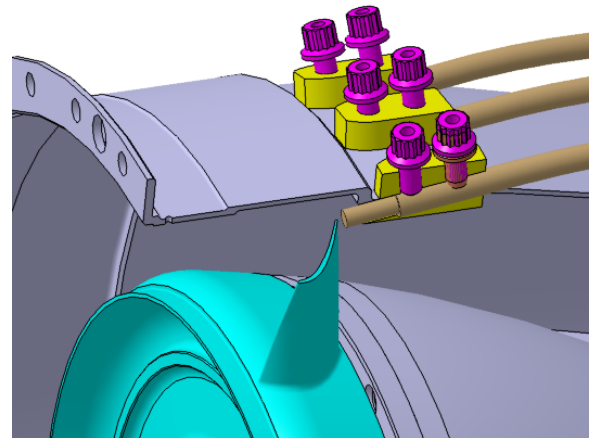


Fig. 5 Drawing for sensor holders. Four optical sensors were used. A circumferential spacing of 11 degree was used.

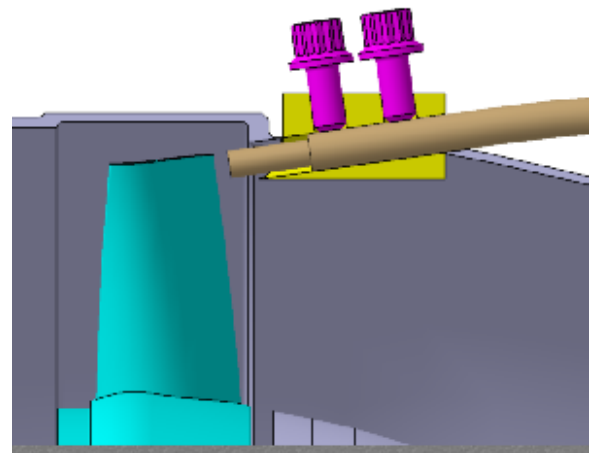


Fig. 6 Details of the installation of optical sensor in the test

Fig. 6 shows a detailed photo of a single optical sensor installation. A reference signal of once-per-revolution (1/rev) is needed to calculate tip deflection of every blade from the corresponding time-of-arrival data. This signal was generated with an eddy current sensor. Blade #1 is defined to be the first detected blade after 1/rev pulse. Blades are numbered in the order of their passage.

## Analysis Result

### Finite Element Analysis Result of the Bladed Disk

For various designs of reconstruction of blade, it should be possible to predict precise measurement of natural frequency as well as vibration mode. For the structural analysis method for single blade, there are simple beam shape, plate shape, and three dimensional Finite element

model. For these cases, it is necessary to consider the following factors: the cross section of the blade's shape (torsion and taper), flexibility of root of blade, effect of shroud, effect of rotation, and change in material property due to high temperature. However, for structural analysis method for multiple blades, it is necessary to consider the factors such as rigidity of disc and flexibility of root of blade. In finite element analysis, first process is to create the precise three-dimensional finite element model. Using the created three-dimensional model, add the geometrical shape and material property of the bladed disk, which is to be analyzed. When on the process, it is necessary to expand the range of analysis from model considering single blade to model considering multiple blades at the same time. The process can be verified by comparing the result of natural frequency and vibration mode obtained from this process, to vibration test. The fig 7 shows the shape of the bladed disk, and the fig 8 shows the result of three dimensional vibration analysis of the bladed disk. After the analysis, due to flexibility of disc, appearance of a lot of bladed disk's family mode was confirmed. Such family mode was constantly appearing upto approximately 30kHz.

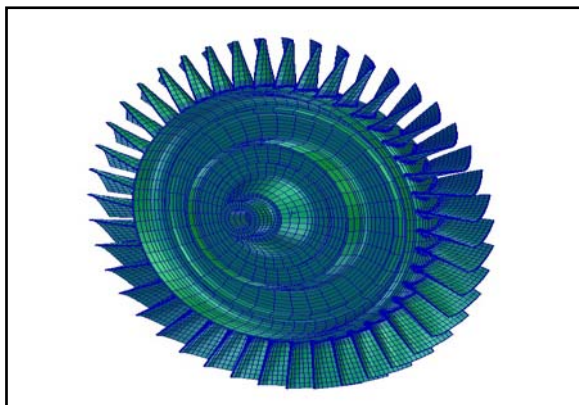


Fig. 7 Shape of the full model of the bladed disk

Also, the fig 9 shows the Campbell diagram, which is used to verify the existence of resonance on bladed disk within operating field. Through the analysis, it was possible to verify the resonance on the bladed disk. Also, the vibration mode phenomenon of the bladed disk at that point, was in shape of family modes, and disc was in shape of 2 ND(Nodal Diameter). However, it was hard to verify the shape of the blades due to superposition and coupling of the modes. The resonance of the family mode is caused by resonance excitation due to unsteady aerodynamic force of the frontal vanes. Also it was analyzed that the mode of bladed disk was moving as in shape of traveling wave.

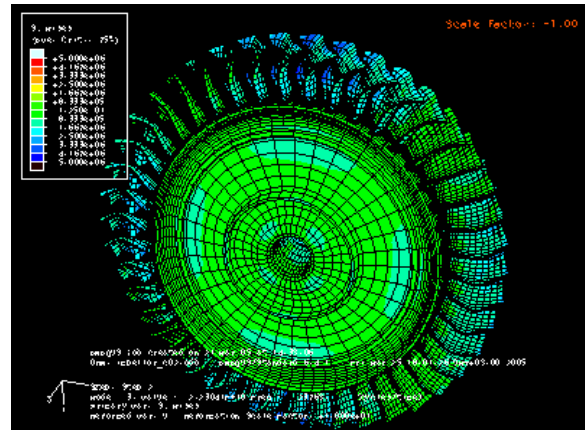


Fig. 8 The result of three dimensional vibration analysis of the bladed disk. Vibration mode phenomenon of the bladed disk was in shape of family modes, and disc mode was in shape of 2 ND.

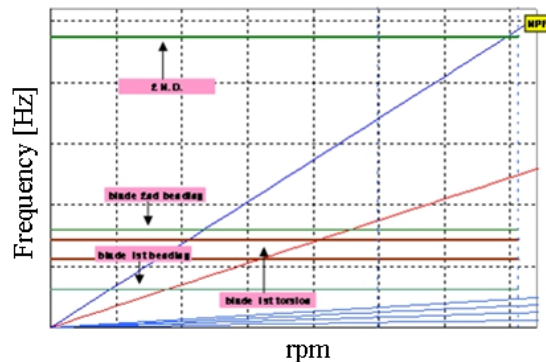


Fig. 9 Campbell diagram, which is used to verify the existence of resonance on bladed disk within operating field. Through the analysis, it was possible to verify the resonance on the bladed disk.

### Test Result

Two approaches were used to extract vibration data: SDOF curve fit and Circumferential Fourier fit. Among several modes in the RPM sweep range, the modes excited by NPF were analyzed. The processed data of tip vibration from NSMS tip timing data is shown in Fig 10. In the RPM sweep range, there are several regions of resonance. Among the resonance regions, the region inside the green box has the largest resonance response. This resonance is excited by NPF and its mode shape is the first shown in Fig. 8.

The SDOF curve fit was applied to the data of the resonance region indicated with the green box. Theoretically, data from only a single sensor is sufficient to identify all the resonance parameters. Like a simple resonator, NSMS measurement of a blade by a tip-timing sensor has very specific signal that can be identified.

Fig. 10 shows clear NSMS signal of a blade's response measured at two different sensor locations (The following figure, which shows the signal

measured from past experiment, is used as an example in order to help understanding NSMS). They are like signal of a SDOF resonator shown in Fig.1. The SDOF curve fit is performed on each sensor's signal to identify the response parameters. The solid line is the SDOF curve fit.<sup>(7)</sup>

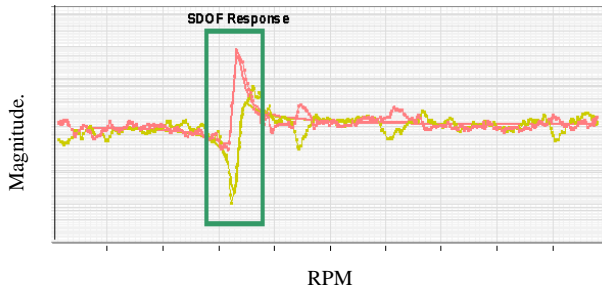


Fig. 10 Magnitude as a function of rpm . The portion of a blade response inside the green box resembles a SDOF resonance. The SDOF curve fit is shown as a solid line. Each color represents data from a different sensor.

The result of the measurement is shown in the fig. 11. The figure shows that the result of operational field is very similar to the analyzed result from finite element analysis. Fig. 12 shows all of the curve fit results for all acquired runs. From the data, the reported amplitude varies by a factor 3~5 from blade to blade and 25 % from run to run. Also, the resonance RPM varies by 6% from blade to blade and 2% from run to run.

### High Cycle Fatigue Evaluation of the Bladed Disk

To evaluate the HCF life of the blade of the bladed disk, the goodman diagram.<sup>(8)</sup> The following fig. 13 below depicts the goodman diagram in a graphical form. If the vibratory stress measured in the NSMS exists within the stable area, it implies that blade will have infinite HCF life, and if the vibration stress exists in unstable area, it implies that there is a possibility, where the blade will break apart. A lot of researchers have tried to verify the analysis method of the HCF life using goodman diagram. They have concluded that the method has tendency to underestimate the actual HCF life. However, the reason why Goodman diagram is commonly used for the evaluation of the engine components, is due to simple application and conservative results that method yields. When using the goodman diagram, it is necessary to consider average stress effect.

Also, it is said that normally, goodman diagram is calculated with consideration of lowered life compared to ratio of ultimate tensile strength. Endurance limit and ultimate tensile strength that are being used in high frequency fatigue damage evaluation in goodman diagram process, the reliability of life evaluation of those two factors are increased by using statistically calculated minimum value. While

measuring the vibration of bladed disk using NSMS, the vibration is measured to be some values, which is considered to be within the stable area in goodman diagram, having infinite HCF life. Also, the value of the fatigue margin was measured to be greater than critical level, concluded to be structurally safe.

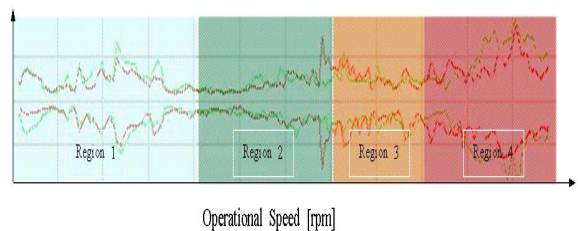


Fig. 11 Magnitude as a function of rpm. Each color represents data from a different sensor.

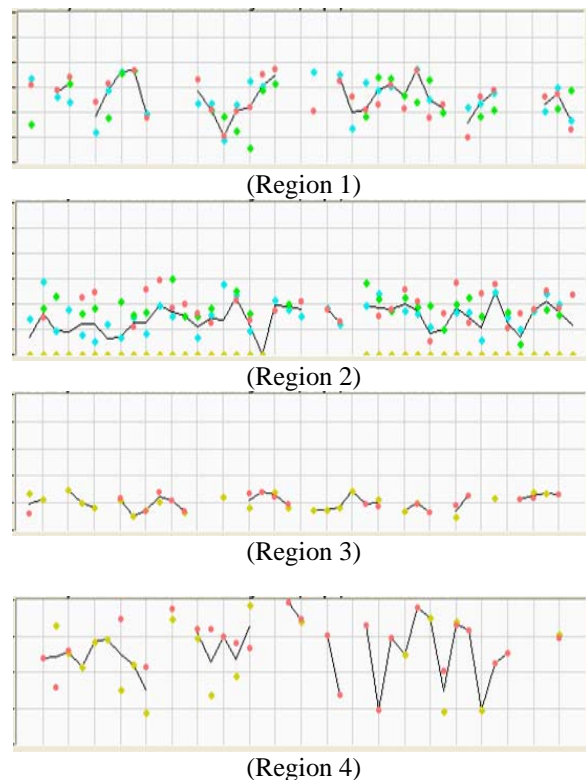


Fig. 12 Magnitude as the function of blade number. Results from the curve fit. Each figure represents data from the result of operational field.

mode	Blade Displacement	Analysis Result	Test Result
6	2.0	17358	18000
7	1.0	18701	19333
8	1.5	20248	21 000
10	2.0	22504	22700

Table 1. The figure shows that the result of operational field is very similar to the analyzed result from Finite Element analysis

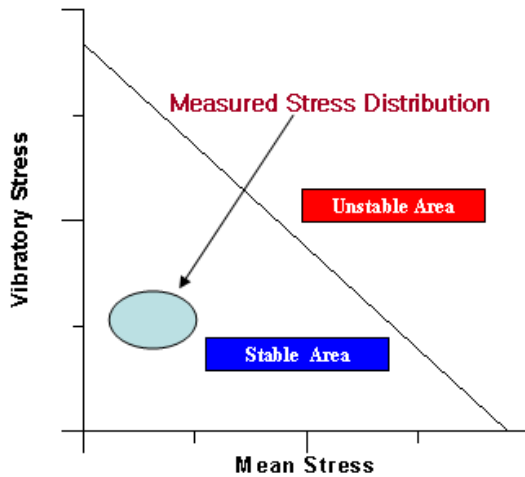


Fig. 13 Goodman diagram. If the vibratory stress measured in the NSMS exists within the stable area, it implies that blade will have infinite HCF life

### Conclusion

When dealing with the process of measuring the blade top using tip timing sensor, the position of installation of the sensor was changed to be parallel to the blade in order to minimize the tip timing error being created due to heat expansion difference between rotor and casing. This change in position of installation of sensor leads the experiment to be completed successfully.

The research of measuring vibration of bladed disk, using NSMS method, was successful. In Finite Element Analysis, the bladed disk is constructed as three-dimensional model. The result of the analysis shows that the resonance of the family mode is created due to unsteady aerodynamic force of frontal vanes. The family mode of bladed disk having form of traveling wave is confirmed, and the result of the examination and the analysis are highly synchronized. Also, lifespan analysis of the blade is done by using method of Goodman. Finally, the research has confirmed bladed disk is structurally safe.

### Nomenclature

#### Symbols

$\theta$	Sensor Location
$r$	Normalized Shaft Rate
$Q$	Resonance Factor
$f(r, Q)$	Function of normalized RPM
$g(r, Q)$	Function of normalized RPM
$E$	Assumed Wave Number
$\delta_k$	Measurement Vector
$A$	Measured Amplitude of Vibration
$\varphi$	Measured Phase of Vibration

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