AJCPP 2008
March 6-8, 2008, Gyeongju, Korea

An Optimization Method Based on Hybrid Genetic Algorithm for Scramjet Forebody/Inlet Design

ZHOU Jianxing[1], PIAO Ying, CAO Zhisong[2], QI Xingming[1], ZHU Jianhong[1]*
[1]School of Aerospace, Tsinghua University, Beijing 100084, China
E-mail: zhoujx03@mails.tsinghua.edu.cn
[2]China Academy of Space Technology, Beijing, China

Keywords: AJCPP, Propulsion, Scramjet, Inlet Design, Optimization, Hybrid Genetic Algorithm

Abstract

The design of a scramjet inlet is a process to search global optimization results among those factors influencing the geometry of scramjet in their ranges for some requirements. An optimization algorithm of hybrid genetic algorithm based on genetic algorithm and simplex algorithm was established for this purpose. With the sample provided by a uniform method, the compressive angles which also are wedge angles of the inlet were chosen as the inlet design variables, and the drag coefficient, total pressure recovery coefficient, pressure rising ratio and the combination of these three variables are designed specifically as different optimization objects. The contrasts of these four optimization results show that the hybrid genetic algorithm developed in this paper can capably implement the optimization process effectively for the inlet design and demonstrate some good adaptability.

Introduction

With flight speed of aircraft increasing above Ma 3, the matching and coupling relationship between aircraft and propulsion system becomes stronger and stronger, and the forebody of the aircraft turns into one part of the propulsion system. As a result, an integrated design method considering the aircraft's forebody and the propulsion system's inlet as a whole must be adopted at design stage. For example, some of the demonstration vehicle, vision aircraft, and most realistically the propulsion testing systems are designed through this method[1-3]. Fig 1 is X-43C demonstration vehicle model from NASA, which was designed with considering the integrated effect of aircraft and propulsion system.

Fig 1 X-43C demonstration vehicle[1)]

Design of scramjet considering the forebody of the aircraft and the inlet of scramjet imposes an important role on such integrated design work, with usually taking some design factors such as the flow angles along the inlet wedge, and so on, into account[4-6]. Some of the design purposes are to minimize the total pressure losses, to minimize the resistance along the inlet passage, to maximize the pressure rising ratio, and so on. Always it is an iterative process for such design work, which needs to adjust these design factors iteratively to meet performance requirements for the scramjet system[6)]. In fact, these jobs can be induced into one class of optimization problem, which means to search globally best results for the design variables to meet some specific performance requirements among all these design variables in their solution spaces.

Compared with some different global optimization algorithms such as optimization algorithm based on differential method, enumerative algorithm, random search algorithm, and so on, genetic algorithm as a typical heuristic search algorithm succeeds with obvious advantage for solving some complex global optimization problems in many fields[7-10)]. Besides robustness to get satisfying results searched over full ranges of variables, genetic algorithms are also capable of strong local search ability. And if the gradient at local area is small, genetic algorithm also can conduct neighboring field exchange with the current optimization bit sequence as search center point. However, traditional local optimization algorithms have much more advantages over genetic algorithms in local search abilities. As a result, various hybrid genetic algorithms with better efficiency in local search ability were created and provided by many scholars, with making some amendments in their local search algorithms. The most popular method to construct such hybrid genetic algorithms is to provide an additional local optimization operator in the typical regroup cycle of genetic algorithm. Optimized with such hybrid genetic algorithm, the offspring newly born should have been locally optimized before they join into the population of next generation and thus the hybrid genetic algorithms can quickly reach the local optimization point[7-10)].

As a result, considering advantages of genetic algorithm and traditional simplex algorithm, a hybrid genetic algorithm was specifically established for the design work of forebody/inlet of scramjet. The genetic algorithm developed not only fully exerts the global search ability and global convergence ability of genetic algorithms, but also can accelerate the convergence speed because of inducing the traditional simplex algorithm as a genetic operator into the cycle. Besides, in the genetic algorithm developed in this paper, the traditional simplex algorithm was also used for a second time optimization based on the results first optimized by genetic algorithm. And the genetic algorithm can effectively compensate the disadvantages of dealing with the problem met during design work of scramjet with only single one of these two algorithms, which means slow convergence speed, lower precision of the genetic algorithm, and sensitive to the initial points of simplex algorithm, and so on. Taken these techniques for the hybrid genetic algorithm, a global optimization result can approach very quickly with the genetic algorithm developed.

Basic Ideas for Optimization Design of Scramjet Inlet

Among those design variables of forebody/inlet geometry for aircraft/scramjet integrated system, the fluid turning angles δ_i (i = 1, …, 5), which means the wedge angles for the forebody and inlet of the integrated system in geometry as factors influencing the shock waves to compress the incoming air flow[11, 12], were chosen as controlled design parameters. And the flow resistance along the passage, the total pressure recovery coefficient, the pressure rising ration and some combination of these three parameters are chosen as optimization objects for different purpose specifically. The solver for the possible results is programmed with C++ according to one dimensional aero-dynamics equations as the forebody/inlet's mathematical model. The designed hybrid genetic method will use this solver to conduct global optimization.

The Mathematical Model for Forebody/Inlet

According to the physics on oblique shock wave and the relationship of shock wave already known in aero-dynamics or gas dynamics theories[11, 12], wedges of forebody /inlet of scramjet system can induce shock waves at each wedge to compress the incoming air to some higher pressure, transfering the air's kinetic energy into potential pressure energy. For better compressing effects, the inner part of the inlet must also be used to compress the air, which were also wedges to induce some weak shock waves and called inner compressing. In the designing condition, the shock waves outside and inside the passage of scramjet inlet compress the incoming air, and should be intersected at the lip of the scramjet inlet shown in fig 2, which can reduce the drag stirred by overflow of flows when sucked into the inlet[4-7, 10, 11].

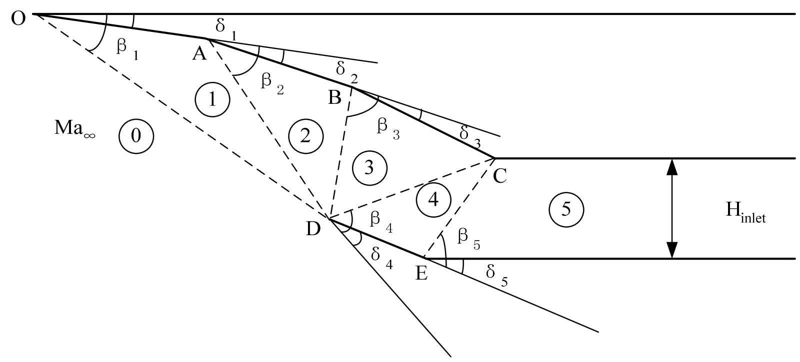

Fig2 Forebody/inlet sketch of scramjet

According to one dimensional oblique shock wave equations of aero-dynamics law, those aero-dynamics parameters of compressible incoming flow in front of and behind the i-th shock (parameters with subscript i-1 are those in front of the i-th shock, and for the parameters with subscript i are parameters behind the i-th shock), have some obvious relationship and can be shown with following relationships[10, 11]. For initial design stage, such relationship can be used to make some assessment about the designed system's performance characteristics. As for further research, it must considering the effects of real gas, such compressibility, viscosity, et al. Fig2 displays the shock and the zones divided by the shocks, and δ_i , β_i designate the flow turning angle and the corresponding shock angle at the i-th wedge.

Ratio of static pressure:

$$\frac{p_i}{p_{i-1}} = \frac{2\gamma}{\gamma+1} Ma_{i-1}^2 \sin^2 \beta_i - \frac{\gamma-1}{\gamma+1} \tag{1}$$

Ratio of density:

$$\frac{\rho_i}{\rho_{i-1}} = \frac{(\gamma+1) Ma_{i-1}^2 \sin^2 \beta_i}{2 + (\gamma-1) Ma_{i-1}^2 \sin^2 \beta_i} \tag{2}$$

Ratio of static temperature:

$$\frac{T_i}{T_{i-1}} = \frac{(1+\frac{\gamma-1}{2}Ma_{i-1}^2\sin^2\beta_i)(\frac{2\gamma}{\gamma-1}Ma_{i-1}^2\sin^2\beta_i - 1)}{\frac{(\gamma+1)^2}{2(\gamma-1)}Ma_{i-1}^2\sin^2\beta_i} \tag{3}$$

Ratio of total pressure

$$\frac{p_i^*}{p_{i-1}^*} = (\frac{\rho_i}{\rho_{i-1}})^{\frac{\gamma}{\gamma-1}}(\frac{p_{i-1}}{p_i})^{\frac{1}{\gamma-1}} \tag{4}$$

Mach number:

$$Ma_i^2 = \frac{Ma_{i-1}^2 + \frac{2}{\gamma-1}}{\frac{2\gamma}{\gamma-1}Ma_{i-1}^2\sin^2\beta_i - 1} + \frac{Ma_{i-1}^2\cos^2\beta_i}{\frac{\gamma-1}{2}Ma_{i-1}^2\sin^2\beta_i + 1} \tag{5}$$

Flow turning angle:

$$\tan\delta_i = \frac{Ma_{i-1}^2\sin^2\beta_i - 1}{[Ma_{i-1}^2(\frac{\gamma+1}{2} - \sin^2\beta_i) + 1]\tan\beta_i} \tag{6}$$

Writing equation (6) into another form with tanβ_i as an independent variable:

$$\tan^3\beta_i + A\tan^2\beta_i + B\tan\beta_i + C = 0 \tag{7}$$

Where,

$$A = \frac{1 - Ma_{i-1}^2}{\tan\delta_i(1+\frac{k-1}{2}Ma_{i-1}^2)}$$

$$B = \frac{1+\frac{k+1}{2}Ma_{i-1}^2}{1+\frac{k-1}{2}Ma_{i-1}^2}$$

$$C = \frac{1}{\tan\delta_i(1+\frac{k-1}{2}Ma_{i-1}^2)}$$

If δi and Ma_{i-1} are known, β_i can be reached through solving equation (7) with numerical method such as Newton iteration. Since the equation is a function 3-order of tanβ_i, three roots can be given from algebraic theory, each of which has a physical significance, corresponding to strong shock wave angle, weak shock angle, and meaningless root of expansion wave discontinuity. Considering the physical significance of these roots, weak shock angel root can be chosen as the result for the equation according to aero-dynamics and gas dynamics laws. And the other two roots of the equation are thrown away. This process can be accomplished through one simple program.

When knowing the shock angle β_i, one can get the other aero dynamical parameters corresponding to the zone immediately after the *i*-th shock wave with solving the ratio equations (1) through (6). And then one can get the total performance parameters through the forebody/inlet of scramjet, such as drag coefficient C_D, total pressure recovery coefficient σ, ratio of inlet rising pressure p_r, and so on. Here just give some of the definitions of these parameters:

Drag coefficient:

$$C_D = \frac{\int_O^C p\sin\delta dl}{q_0 H_0} \tag{8}$$

Total pressure recovery coefficient:

$$\sigma = \prod_{i=1}^{5} p_i^* / p_{i-1}^* \tag{9}$$

Ratio of inlet rising pressure:

$$p_r = \prod_{i=1}^{5} p_i / p_{i-1} \qquad (10)$$

Design of the hybrid genetic algorithm

The main features of genetic algorithms are group search strategy and simply genetic operators, which are different from those traditional heuristic search algorithms. Group search strategy can make genetic algorithms break the restriction of neighbor search, gathering information with distribution method in the solution space and globally searching the optimized results. However, the improvement speed of optimization result among each generation group will greatly slow down because the diversity of group is reduced at later evolving generation period of genetic algorithm. Since the global search ability of the genetic algorithms exceeds their local search ability at some degree, some special treatment should be taken to compensate the disadvantages of local search ability. In this paper, a traditional simplex algorithm was used as a genetic operator to make improvements for better local search ability of the genetic algorithm. Specifically speaking, those techniques taken in the genetic algorithm developed in the paper are as follows: real number coding method, discerning the characteristics of individual with adaptability function, design of simplex algorithm as genetic operator, with simplex algorithm to conduct a second time optimization process on the basis of the results optimized by genetic algorithm, and so on.

Real number coding method

Real number coding method was used for some higher dimensional and high precision problems described with continuing functions. Every gene of one individual in one generation is designated by a float point number, and the coding length equals to the number of strategy variables. Advantages of real number coding method are as follows: capable of displaying number with large range, improving the calculation precision of genetic algorithm, conducting genetic search through large space, reducing the calculation complex of genetic algorithm, improving the calculation efficiency, and so on.

Adaptability degree calculation

When the genetic algorithm runs, numerical value from the adaptability degree function can be used to decide which individual can be chosen to evolve into next generation. Because the adaptability degree function directly affects the performance of genetic algorithm, random competition operator was chosen as selective operator for the design in the hybrid genetic algorithm developed in this paper, which can be accomplished by comparing the value of object function for each individual. And the adaptability degree function was chosen as the optimization function for the inlet design process.

Genetic operators

The genetic operators' design includes design of cross operator, variation operator and selective operator, and so on.

(1) Cross operator

Arithmetic cross operator is one operator widely used among the real number coding of genetic algorithm. The cross operator can be described as follows:

Suppose that the cross process happens between two individuals X_1 and X_2, and both of these two new individuals can be determined after the cross operation process:

$$\begin{cases} X_1^* = aX_1 + (1-a)X_2 \\ X_2^* = (1-a)X_1 + aX_2 \end{cases} \qquad (11)$$

Where a is a number in the range of [0, 1], which can be a constant and a variable decided by the evolvement. In the algorithm in this paper, a is a random number chosen from range [0, 1].

(2) Variation operator

Traditional real number coding genetic algorithm has such disadvantages as slow speed of convergence and easy to appear premature phenomena. The possible reasons lie in that the variant operation has little disturbance to the local extreme point during the evolvement. As a result, the construction of an adequate variation operator plays a key role on wether the real number coding genetic algorithm is effective. A new random direction variation operator was designed in the hybrid genetic algorithm developed in this paper, named as determinate random direction variation operator, and described as follows:

Supposed that the chromosome of the individual chosen for variation is X_k and a disturbance direction p_k was randomly created, the whole variation operation process is to find an optimization point along the direction p_k as new chromosome, with X_k as start point. That is, to conduct a one-dimensional search operation:

$$\min \phi(\lambda) = F(X_k + \lambda p_k) \qquad (12)$$

In the algorithm developed in this paper, golden mean method was used to get the optimization step k_0, and a new chromosome X'_k was created for the individual after variation process:

$$X_k^{'} = X_k + \lambda_0 p_k \qquad (13)$$

(3) Selective operator design

Traditional standard selective operator requires that the adaptability function must be greater than zero, which brings some difficulty for the choosing of appropriate adaptability function. In another way, the selective operator based on adaptability value sequence also incurs problems such as premature phenomena and slow convergence speed. To avoid these disadvantages, a selective operator of matches' competition with higher determination and some degree of random was designed in the algorithm in this paper, whose size is 3. Because of many random factors influencing the algorithm, the optimization individual among the present group will be destroyed, which will affect the operating efficiency and convergence. Sequentially, optimization reserving strategy was taken. The individual optimized doesn't take part in the cross operation and variation operation and instead it will be used to substitute the poorest individual at current generation after the operation of cross and variation.

Hybrid genetic algorithm

Usually, two ways are used to implement the hybrid process of genetic algorithm and classical optimization algorithm. One is to take the classical optimization process as a genetic operator of the genetic algorithm to accelerate the convergence speed. The other way is to handle the optimization result of genetic algorithm with classical optimization algorithm to conduct second optimization as the final optimization result. The first way was taken in the hybrid genetic algorithm designed in this paper and the classical optimization algorithm was simplex algorithm.

The basic idea for hybrid operator is as follows. After the present group completed the operation of cross and variation, the selection process with some probability are conducted with classical algorithm to get local optimization result, which will be selected as new chromosome of new individual and then be evaluated and selected among the evolved group. In one word, the classical algorithm as a strong local search operator takes part in the whole evolution process. A self-adaptability operator p_n, which will increase with the evolving process and approach a constant p_0, was purposely designed for the hybrid operator.

$$p_n(t) = p_0 e^{-a(1-t/T)} \tag{14}$$

Where T is the biggest number of generation set in the genetic algorithm, t is the present number of evolution, and p_0 is constant and taken some number in the range of (0, 1]. p_0 is like the cross probability and variation probability, which shows the most effective degree of local search operator to each individual. Bigger is the value of p_0, more fully is that the hybrid operator exploit the selective space and with larger calculation cost as a price. And the value of p_0 is set to 0.10 in the algorithm in this paper. a is the parameter of the probability variance for control operator and was set to 1 in this paper. The purposes of designing an increasing function p_n about the variable t are as follow: at the initial stage of evolution, the probability to conduct local optimization process is too small to keep the diversity of individuals; at later stage of evolution, there was a reason to believe that the individual approaches the global optimization result, when conducting local optimization with higher probability will help accelerate and convergent to the global optimization result. Of course, conducting simplex optimization for the chromosome of selected individual at initial point will enhance the search speed locally.

The process of algorithm

The complete optimization framework designed in this paper is shown in fig3 and the main stages are as follows:

Stage I: description of problems.

Stage II: genetic operations to create initial group and conduct related genetic operation.

(1) Determine the upper and lower limits of design parameters to randomly create a parent group and set the principal parameters of the algorithm;

(2) Evaluation of the group: calculate the basic attributes for each individual, such as to calculate the object function, to determine the best and poorest individual, and so on;

(3) Conduct cross operation for the group based on the cross probability p_c.

Stage III: Local optimization operations, to quickly find the local optimization result based on some of the chromosome evolved through stage II.

(4) Conduct variance operation along the direction determined according to the variance probability p_m, and conduct local search with simplex algorithm among the group based on the probability p_n calculated by self-adapt hybrid operator;

(5) Select new group with game competition operator according to the optimization reserving strategy.

Stage IV: make decision based on some related conditions and finish.

(6) Conduct group adaptability degree calculation. If the error of the adaptability degree is smaller than the setting limit or the iteration time exceeds the maximum iteration number allowed, finish evolution. If not, go to step (3) to conduct a new evolution process.

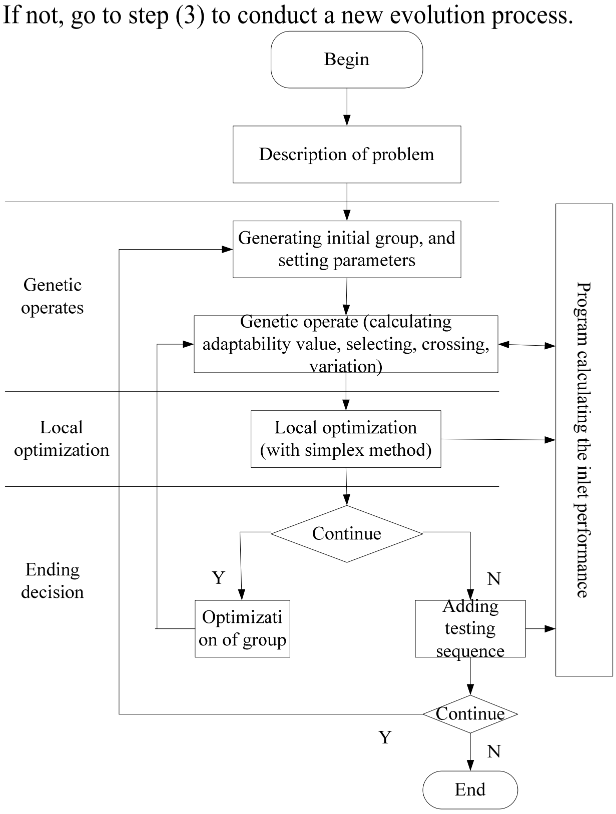

Fig3 Optimization framework of forebody/inlet with hybrid genetic algorithm

Optimization of forebody/inlet and results

Choose the performance number such as the drag of inlet C_D, the total pressure recovery coefficient σ, the pressure rising ratio p_r of inlet and a combination of these three numbers as optimization objects. Given the height of the passage outside of the inlet, and with the design parameters of wedges angles δ_1, δ_2, δ_3, and δ_4 as variable optimized with the hybrid algorithm, one can determine the forebody/inlet geometry of scramjet. As a result, the design problem can be described as follows:

Maximize: $\{-C_D, \sigma, p_r\}$

Subject to:

$$\begin{aligned} &\delta_{i\min} \le \delta_i \le \delta_{i\max}; \\ &\delta_1 + \delta_2 + \delta_3 = \delta_4 + \delta_5; \\ &L_{OC} \le 3.0 \\ &\dot{m}_c \ge 5.0kg/s \end{aligned} \tag{15}$$

To make the scramjet to work properly, some restrictions are imposed onto the optimization problem, such as the length of the inlet, the flow flux of the inlet, and so on.

The geometry of the scramjet was optimized with the hybrid algorithm developed in this paper, which means find the global optimization through the solution space. According to the optimization object described in equations (15), the optimization function can be transformed into four optimization problems: minimize C_D, maximize σ, and maximize p_r, and maximize a combination of these three numbers $f = 1/C_D + \sigma + p_r$.

The parameters set for the optimization process of the hybrid algorithm for the four optimization problems are: size of the group is set to 30, cross probability in one generation is 0.9, variance probability in evolvement is 0.05, and number of evolve generation is 100. And the designed altitude that the scramjet will work is 30 km, Mach number 6.0, angle of attack 0 degree. After optimization process of the four problems, the designed parameters and corresponding performance parameters are shown in table 1.

Table1 Optimization results for the four objects

Objects	Designed parameters（°）					Performance results		
	δ_1	δ_2	δ_3	δ_4	δ_5	C_D	σ	p_r
$\min C_D$	1.409	1.912	6.136	2.277	7.179	0.109	0.906	9.929
$\max \sigma$	0.514	1.635	6.044	2.756	5.436	0.162	0.926	7.666
$\max p_r$	1.482	1.933	6.161	2.435	7.142	0.110	0.908	10.173
$\max f$	1.463	1.874	6.128	2.466	6.998	0.112	0.909	9.950

Conclusion

Through the hybrid genetic algorithm designed purposely for the forebody/inlet design and some of the optimization results, some conclusions can be drawn as follow:
(1) The hybrid genetic algorithm takes the advantages from global convergence of genetic algorithm and the local search ability of simplex algorithm, which makes it exceed the normal genetic algorithm in the speed of convergence speed and avoids the disadvantage of traditional simplex to trap into local optimization result;
(2) The optimization results conducted with the hybrid algorithm show that it can be used to implement the optimization problems met in the design of scramjet and related engineering works.

References

1) McClinton, C. R. and Hueter, U.: NASA's Advanced Space Transportation Hypersonic Program, AIAA-2002-5175.
2) Hunt, J. L., Pegg, R. J. and Petley, D. H.: Airbreathing Hypersonic Vision-Operational-Vehicles Design Matrix, AIAA-1999-01-5515.
3) Cockrell C. E., Auslender A. H., Wayne Guy R., et al: Technology Roadmap for Dual-Mode Scramjet Propulsion to Support Space-Access Vision Vehicle Development, AIAA-2002-5188.
4) Safarik P., Polak A.: Optimal Shock Wave Parameters for Supersonic Inlets, Journal of Propulsion and Power, 12(1), 1996, pp. 202-205.
5) Xu X., Cai G. B.: Design and Optimization Method for Two Dimensional Scramjet Inlet, 22(6), 2001, pp. 468-472.
6) Smart M. K.: Optimization of Two-dimensional Scramjet Inlets, Journal of Aircraft, 36(2)1999, pp. 430-433.
7) Li, M. Q., Kou J. S., Lin, D., et al: *Genetic algorithms: principles and applications*, Science Press, Beijing, 2002, pp. 59-65.
8) Rudnic E. M., Patel J. H., Greenstein G. S., et al: A genetic algorithm framework for test generation, IEEE Trans. on Computer-Aided Design, 16(9), 1997, pp. 1034-1044.

9) Goldberg D. E.: *Genetic algorithms in search, optimization, and machine leaning*, Addison-Wesley Publishing Company, Massachusetts, 1989.
10) Wang X. P., Cao L. M.: *Genetic algorithms: theory, application and program*, Xi'an Jiaotong University Press, Xi'an, 2002.
11) Xu H. F.: *Principles of aerodynamics*, Beijing Institute Press, Beijing, 1987.
12) Zucrow, M. J. and Hoffman, J. D.: *Gas dynamics*, John Wiley & Sons Inc., New York, 1976.