Conceptual Design Trade Offs between Solid and Liquid Propulsion for Optimal Stage Configuration of Satellite Launch Vehicle

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Abstract

The foremost criterion in the design of a Satellite Launch Vehicle (SLV) is its performance capability to boost the designated payload to the desired mission orbit; it starts from focusing on the SLV configuration to achieve the velocity requirements (ΔV) for the mission. In this paper we review an analytical approach which is suitable enough for preliminary conceptual design and is used previously to optimize stage configurations for Two Stage to Orbit SLV for Low Earth Orbit (LEO) Missions; we have extended this approach to Three Stage to Orbit SLV and compared different propellant options for the mission. The objective is to minimize the Gross Lift off Weight (GLOW). The primary performance figures of merit were the total inert weight of the SLV and the payload weight that the SLV could lift into LEO, given candidate propulsion systems. The optimization is achieved by configuring the ΔV between stages. A comparison of configurations of single-stage and multi-stage SLVs is made for different propellants. Based upon the optimized stage configurations a comparative performance analysis is made between Liquid and Solid fueled SLV. A 3 degree of freedom trajectory-analysis program is modeled in SIMULINK and used to conduct the performance analysis. Furthermore, a cost analysis is performed on our stage optimized SLVs. The cost estimation relationships (CER) used give us a comparison of development and fabrication costs for the Liquid vs. Solid fueled SLV in man years. The pros and cons of the production, operation ability, performance, responsiveness, logistics, price, shelf life, storage etc of both Solid and Liquid fueled SLVs are discussed. The statistics and data are used from existing or historical (real) SLV stages.

Introduction

It is well known that the aerospace industry is more sensitive to vehicle weight as a primary figure of merit for vehicle designs than any other industry. This is because weight (or mass) is a strong driver on vehicle performance and cost, and so takes a central role in the vehicle design process. In fact, vehicle weight is so important that competitive advantage is often sought largely or exclusively on the basis of having a lighter weight than the competitor. Experience has shown that the cost of a given class of SLV is roughly proportional to the GLOW. The boost phase of an orbital space mission is critical to the design process since the initial weight of the vehicle, which is generally related to its cost, is at its maximum at lift-off. The m_{pay} is a small fraction of the total mass.

Placing an object into orbit is technically demanding and expensive too. A rough rule of thumb is that a modern SLV can deliver into orbit a m_{pay} that is only a few percent of its overall mass. Since the size of the SLV scales with the m_{pay}, there is a tremendous incentive to increase the SLV performance by keeping the mass of SLV as low as possible. We will later discuss that apart from GLOW there are other figures of merit too which strongly dominate and effect the design and performance of a SLV.

Exiting SLVs can be categorized as:

- Any number of stages, inline and/or piggy-back
- Liquid, solid, hybrid, air breathing propulsion systems.
- Mono-propellant, bi-propellant, tri-propellant
- Pump-fed or pressure-fed
- Expendable or recoverable

The research work is in progress for the design of single-stage-to-orbit (SSTO), but no Earth-launched SSTO SLV has ever been constructed. Current orbital launches are either performed by multi-stage fully expendable launchers or by the Space Shuttle which is multi-stage and partially reusable. It is extremely difficult to design a structure which is strong, safe, very light, and economical to build. The problem originally seemed insuperable, and drove all early designers to multistage rockets. In this study multi-stage expendable SLVs are considered for small m_{pay} to LEO.

SLVs are unique forms of transportation since they are the only systems that accelerate continuously throughout their performance envelope. Consequently, velocity is the fundamental measure of performance for any SLV. The ability of any SLV to achieve orbital velocity comes primarily from its propulsion efficiency, with weight and drag acting against it. Available technology constrains the specific impulse (I_{sp}), therefore the trade off has to be made. For each propulsion stage, the amount of work that needs to be done is calculated. This is measured in terms of velocity changes, ΔV. We can estimate the velocity that SLV should provide.

$$\Delta V_{design} = \Delta V_{burnout} + \Delta V_{gravity} + \Delta V_{drag} \qquad (1)$$

$\Delta V_{burnout}$ is the velocity required for the desired orbit. We add the losses, $\Delta V_{gravity}$ and ΔV_{drag}, to the $\Delta V_{burnout}$ to obtain the required ΔV_{design}. These losses are sensitive to the T/W_o. A low T/W_o ratio causes gravity losses to be high because the vehicle spends more time in ascent, while high T/W_o causes drag losses to be high because of the higher velocities achieved in the atmosphere. The T/W_o is a key SLV design parameter because it dictates the vibration, acoustic and dynamic load environment for the spacecraft. If there were no atmosphere and topographical variations, an optimum SLV trajectory would be very similar to a Hohmann transfer and gravity losses would be minimized by thrusting normal to the radius vector. To accurately estimate gravity losses we need to know a precise ascent profile and time of flight (TOF). But for medium-to-large SLVs on nominal trajectories the $\Delta V_{gravity}$ losses fall between 750 and 1,500 m/s. For the current inventory of large, expendable launch vehicles, ΔV drag losses are less than 3% of the total change in velocity required, about 20 to 40 m/s. The percentage decreases as the size of the vehicle decreases. Once we know ΔV_{design} from mission requirements, we can estimate the m_{prop} required for the SLV. [1, 2]. Several definitions are useful at this point. The flight vehicle mass is the sum of the propellant mass, structure mass, including mass of the fairing, and the mass of everything above the launch vehicle interface, including mass of the spacecraft bus, payload, and any upper stages.[3] The total mass of a multistage SLV includes the masses of propellants and their tanks, engines, feeding system and m_{pay}. We use mass fractions to describe the portion of the flight vehicle devoted to certain sections. The propellant mass fraction is the mass of propellant divided by the total flight vehicle mass; the structure mass fraction or deadweight fraction is the structural mass, including the mass of the fairing, divided by total flight vehicle mass; and the payload mass fraction is the payload mass divided by total flight vehicle mass. Typical values for propellant, structure, and payload mass fractions are 0.85, 0.14, and 0.01, respectively.[4,5] The structural weight of the vehicle, on the other hand, is much more difficult to estimate, but it depends on the weight of fuel carried. Using the data for stages from operational SLVs with different propellant combinations and including the engines, the vehicle configuration is estimated. The reduction in GLOW achieved by using three stages instead of one is 25%, while the reduction in weight compared to two stages is about 8%.

Whenever we encounter missions requiring a large ΔV, we run the risk of not being able to perform the missions with certain technologies. From Table 1, we find that a typical launch ΔV ranges from about 8.8 ~ 9.3 km/s. Negative sign indicates beneficial effect of rotation. Third stage of Ariane 44L uses a continuous burn into a geosynchronous transfer orbit. For Shuttle the Injection occurs at ≈ 111 km. An additional ΔV= 144 m/s is required to circularize at apogee.

Table 1
Velocity Budgets to LEO for selected SLVs [6]

Vehicle	V_{LEO}	$\Delta V_{GRAVITY}$	$\Delta V_{STEERING}$	ΔV_{DRAG}	ΔV_{ROT}*	$\Sigma\Delta V = \Delta V_{PROP}$
Ariane A-44L	7802	1576	38	135	-413	9138
Atlas I	7946	1395	167	110	-375	9243
Delta	7842	1150	33	136	-347	8814
Saturn V	7798	1534	243	40	-348	9267
Titan IV/ Centaur	7896	1442	65	156	-352	9207
Space Shuttle	7794	1222	358	107	-395	9086

For different propellant types, historical or empirical data can relate a vehicle's inert mass to the propellant mass required. To do this, we use the concept of the *inert mass fraction*

$$f_{inert} = (m_{inert}) / (m_{prop} + m_{inert}) \tag{2}$$

The inert mass includes everything except the payload and propellant masses. The inert mass usually includes tank structure, support structure, engines, the propellant feed system, fairings, electronics, and any number of other non-reactive (inert) components. We use historical data to predict a typical inert mass fraction. Historical numbers only predict what has been done in the past. New technologies, new technology applications, or new requirements may change the number drastically. We further assume that the inert mass is only a function of propellant mass. However, such factors as the thrust level and pressures (i.e., propellant storage pressure, combustion chamber pressure, pump pressure) can significantly affect the mass of the structure and, in particular, the mass of the engine. With all of this, the f_{inert} approach is quite powerful and can give us very good preliminary design results. Using the ideal rocket equation, we get

$$m_{prop} = \frac{m_{PAY}\left[\exp\left(\frac{\Delta V}{gI_{SP}}\right)-1\right]}{1-f_{inert}\exp\left(\frac{\Delta V}{gI_{SP}}\right)}\left(1-f_{inert}\right) \tag{3}$$

In the formulation below, we find there is a "not feasible" condition that gives us a relationship between the mission ΔV, I_{sp}, and f_{inert} . The term "payload" means different things to different people. To a spacecraft designer, the payload could be a telescope or communications system attached to a spacecraft bus. To a SLV designer, the payload is the entire spacecraft (bus and instrument). Equations (4), (5) give a fundamental limit on vehicle performance. If the denominator is less than or equal to zero, it is impossible to build the vehicle as conceived. When the denominator goes to zero, the propellant required is infinite. If the denominator is negative, the propellant mass is negative. Neither of these possibilities is feasible:

$$\left(1 - f_{inert} \exp\left(\frac{\Delta V}{g I_{SP}}\right)\right) \le 0 \qquad (4)$$

Solving this (4), we get a "*non-feasible condition*" for our mission:

$$I_{SP} \le \frac{\Delta V}{\ln\left[\dfrac{1}{f_{inert}}\right] g} \qquad (5)$$

Equation (5) explains that for a given mission (ΔV) and a given technology (f_{inert}), the I_{sp} must be above a certain value for the system to work. Notice that the payload, initial, or final masses do not show up in this equation, implying that the equation limits the ratio of masses (for example, m_{pay} / m_i) and that the vehicle's absolute size has no fundamental physical limit.

We usually strive to achieve low m_{inert} (subject to the usual constraints such as cost), so our vehicle is lighter and, hopefully, less costly. The choice of mass fraction depends on how conservative we wish to make the structure, the stage number, and the expected propellant mass and density, and on how innovative we are. Reaction-control systems (attitude-control systems) are relatively small and have f_{inert} near 0.7 (only 30% of the total system mass is propellant). Other space propulsion systems (upper stages, kick motors, and maneuvering systems) average a mass fraction of 0.17.[6] To decide on an appropriate mass fraction, we need to look at our system and decide whether we want a conservative mass estimate (higher fraction) or an aggressive mass estimate (lower fraction).

Table 2 First Stages of Common Launch Vehicles[4]

Stage	Propellant Mass (kg)	Gross Mass (kg)	Sea-Level I_{sp}	f_{inert}
Atlas-E	112,900	121,000	233	0.067
Atlas-I	138,300	145,700	239.75	0.051
Atlas-II	155,900	165,700	240.75	0.059
Atlas-IIA	155,900	166,200	241.7	0.062
Atlas-IIAS	155,900	167,100	241.7	0.067
Delta	96,100	101,700	263.2	0.055
Titan-II	118,000	122,000	281	0.033
Titan-III	134,000	141,000	287	0.050
Titan-IV	155,000	163,000	287	0.049
Saturn S1-B	408,000	444,000	232	0.081
Saturn S1-C	2,080,000	2,210,000	264	0.059
Ariane-H150	155,000	170,000	409	0.088
Proton	410,200	455,600	285	0.100

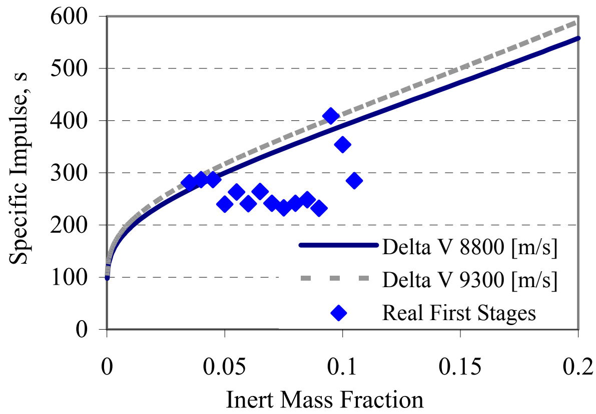

Fig. 1. Feasible Regions for Launch Systems

The two curves shown Fig.1 represent the minimum possible I_{sp}, given a certain structural technology (f_{inert}), to perform a launch mission. Data for existing or historical (real) first stages is overlaid from [4,5]. Finding technology that can lower the f_{inert} can relieve us from a requirement for high I_{sp}. But existing systems are pretty good, and it is difficult to drastically improve structural technology, we can drastically improve the "integrated f_{inert}" (the average mass fraction, integrated over a mission) by discarding inert mass as it becomes unnecessary. However, increasing the number of stages increases the cost of our system, if we choose ***n*** stages, the cost for this system can be greater than ***n*** times the cost of a single stage. So the need arises for stage optimization.

Sizing Procedure for Multistage SLV

The velocity increment for the SLV is the sum of those for the individual stages where ***n*** is the total number of stages.

$$\Delta V_{TOTAL} = \sum_{i=1}^{n} \Delta V_i \qquad (6)$$

- ΔV distribution is postulated and the resulting payload fraction calculated.
- The ΔV distribution is varied until the payload fraction is maximized.
- Once the ΔV distribution is selected, vehicle sizing is accomplished by starting with the uppermost or final stage (whose payload is the actual deliverable payload) and calculating the initial mass of this assembly.
- This assembly then forms the payload for the previous stage and the process repeats until all stages are sized.

Propellants for each stage

As propellant density increases, f_{inert} decreases. But large dispersions in Fig.2. indicates that other factors play a major role in these results. The density group-ings indicated depend on the propellant combinations used. When examining candidate propellant combinations, two of the most important considerations are I_{sp} and propellant density. I_{sp}, is the most important indicator of performance. Density of propellants is important too, as that dictates the overall size and mass of propellant tanks and propellant fluid management. Most storable propellants have disadvantage in I_{sp} as compared to cryogenic propellants. On the other hand, cryogenic propellants combinations experience a penalty in fluid density. The density of liquid hydrogen is 7% that of water, where most other propellants are 60%-140% the density of water [7)]. There is a common perception that the choice of propellants can be based on the density of the propellants. Further, this perception drives us to choose denser, and usually lower- I_{sp} propellants (such as RP-l/LOx) for lower stages and less-dense, higher-I_{sp} propellants (such as H_2/LOx) for upper stages. We reason that higher-density propellants allow us a better (lower) f_{inert}, which leads us to a lighter first stage. Although this reasoning may be correct depending on the mission and requirements, the overall vehicle mass usually increases above what is achievable with higher-performing propellants. The Saturn family of launch vehicles used RP-l/LOx on the first stage and H_2/LOx on second and third stages. This approach is now universally accepted. However, these vehicles were huge because they were intended for the very large ΔV mission of going from the Earth's surface to the Moon and back. If designers had made the first stage with H_2/LOx, it would have been too big to transport to the launch site and would have made vertical assembly and operation of the vehicle even more difficult than it was. Although the mix of propellants was appropriate for Saturn-V, it may not be appropriate for other missions.

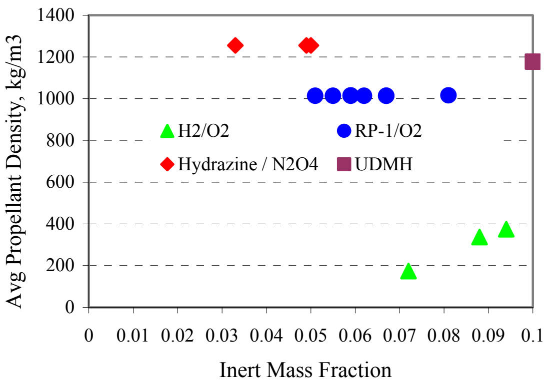

Fig. 2. Inert-mass Fraction versus Average Propellant Density for the Vehicles Listed in Table 2.

The average propellant density from the oxidizer-to-fuel ratio (O/F) for the individual systems, determines the mass of fuel and oxidizer based on the propellant mass:

$$\begin{aligned} m_{fuel} &= (m_{prop}) / (1 + O/F) \\ m_{OX} &= (m_{prop} \times O/F) / (1 + O/F) \\ V_{fuel} &= m_{fuel} / \rho_{fuel} \\ V_{OX} &= m_{OX} / \rho_{OX} \\ \rho_{average} &= m_{prop} / (V_{fuel} + V_{OX}) \end{aligned} \tag{7}$$

In Fig 2. the propellants the middle band contains RP-l/LOx systems (O/Fs about 2.25) and the left-hand band reflects values for hydrazine/N_2O_4 (O/Fs about 1.9). Clearly, f_{inert} decreases as propellant density increases, but large dispersions indicate that there are other important factors at play.

Inert-mass fraction for each stage

In order to get pragmatic results, using Ref. (4), (5) and (6) we can make a reasonable choice.

Fraction of ΔV to each stage

Let f_1 —> fn_{stage} be the fraction for each stage;
1 refers to the first stage,
n_{stage} refers to the last stage.

$$\begin{aligned} &f_1 + f_2 \ldots\ldots + f_{nstage} = 1 \\ &f_1\, \Delta V_{tot} = \Delta V_1 && (\Delta V \text{ on first stage}) \\ &f_i\, \Delta V_{tot} = \Delta V_i && (\Delta V \text{ on } i^{th} \text{ stage}) \\ &f_{nstage}\, \Delta V_{tot} = \Delta V_{nstage} && (\Delta V \text{ on last stage}) \end{aligned} \tag{8}$$

Stage Size

We start at the uppermost stage and work back to the first stage. The payload for each succeeding stage includes the previous stages and the actual payload for the mission. This process allows us to size the individual stages and the entire vehicle. Ref. (6) adopts this approach for two stage SLV stage optimization using conventional methods; we have further extended this approach to Three Stage SLV. We assumed an average ascent ΔV = 9000 m/s and m_{pay} of 1 kg. The choice of m_{pay} allowed us to normalize all of the other masses. By simply multiplying all of the *normalized masses* by the actual m_{pay} we get a preliminary design estimate of the actual SLV Take off Mass (m_{o1}). The model for Mass Estimation for a multi-stage SLV is adopted from (3). For n_{stage} SLV the take-off mass m_{o1} should be

$$m_{0i} = m_{propi} + m_{ki} + m_{0(i+1)}$$

$$m_{o1} = \frac{m_{o1}}{m_{o2}} \cdot \frac{m_{o2}}{m_{o3}} \cdot \ldots\ldots \frac{m_{oi}}{m_{oi+1}} \cdot \ldots\ldots \frac{m_{on}}{m_{pay}} \cdot m_{pay} \tag{9}$$

Minimized the vehicle mass by optimizing the ΔV fraction allotted to each stage

By varying f_1 through f_{nstage} we determined the combination that minimizes the vehicle's initial mass. The objective is to minimize the initial take off mass m_{o1} for the vehicle b optimal allocation of ΔV to each propulsion stage.

Optimization Results

We start by looking at the SSTO. No optimizing was required because all of the ΔV goes onto the only stage. Only the H_2/O_2 system and the hydrazine system turn out to be feasible for this mission, given our assumed numbers. The results are shown in Table 3. Table 4 shows the results of TSTO. Two-stage vehicle using propellants with higher I_{sp} is lighter than a three-stage vehicle using one stage with a low I_{sp}. The mass of the all-solid vehicle is still quite high compared to the one using liquids. This explains why existing vehicles, such as Scout and Pegasus, have so many stages.[4)]

Table 3
Results of the Single-Stage-to-Orbit SLV

Variables	H_2/O_2	Hydrazine / N_2O_4
Specific impulse (s)	410	290
Inert-mass fraction	0.075	0.035
Propellant mass (kg)	26.06	127.04
Inert mass (kg)	2.11	4.61
Final mass (kg)	3.11	5.61
Initial mass (kg)	**29.17**	**132.64**
Mass of payload /initial mass	3.43 %	0.75 %
Minimum feasible I_{sp}	354.2 s	273.66 s

Table 4
Results of the Two-Stage-to-Orbit SLV

Design Variables	All H_2/O_2	RP-1 / H_2	N_2H_4 /H_2	All Solids
Stage 1 - I_{sp} (s)	410	290	290	260
Stage 2 - I_{sp} (s)	435	435	435	290
Stage 1 - f_{inert}	0.095	0.070	0.050	0.100
Stage 2 - f_{inert}	0.100	0.100	0.100	0.080
Stage 1 - ΔV (m/s)	4140	2610	2880	3780
Stage 2 - ΔV (m/s)	4860	6390	6120	5220
Stage 1 - m_{prop} (kg)	9.066	12.328	12.558	63.179
Stage 2 - m_{prop} (kg)	2.668	5.648	4.956	9.708
Stage 1 - m_{inert} (kg)	0.952	0.928	0.661	7.020
Stage 2 - m_{inert} (kg)	0.296	0.628	0.551	0.844
Initial vehicle mass (kg)	**14.106**	**20.531**	**19.726**	**81.752**
Payload mass/Initial mass	7.1 %	4.9 %	5.1%	**1.2%**

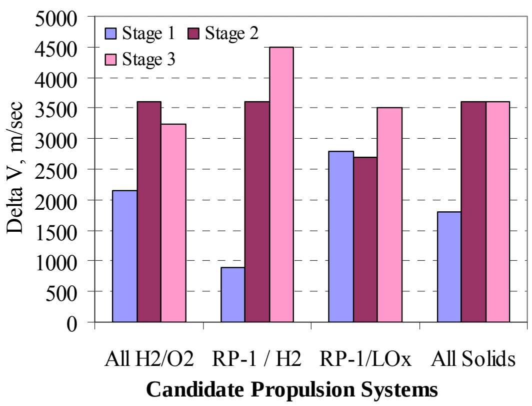

Fig. 3. Optimized ΔV Distribution for the Candidate Propulsion Systems

Table 5
Results of the Three-Stage-to-Orbit SLV

Design Variables	All H_2/O_2	RP-1 / H_2	RP-1 / LO_2	All Solids
Stage 1 - I_{sp} (s)	410	290	290	260
Stage 2 - I_{sp} (s)	435	435	290	290
Stage 3 - I_{sp} (s)	435	435	320	290
Stage 1 - f_{inert}	0.24	0.1	0.07	0.2
Stage 2 - f_{inert}	0.4	0.4	0.08	0.4
Stage 3 - f_{inert}	0.36	0.5	0.08	0.4
Stage 1 - ΔV (m/s)	2160	900	2790	1800
Stage 2 - ΔV (m/s)	3600	3600	2700	3600
Stage 3 - ΔV (m/s)	3240	4500	3510	3600
Stage 1 - m_{prop} (kg)	5.122	3.782	21.829	23.974
Stage 2 - m_{prop} (kg)	3.799	5.629	7.0218	14.873
Stage 3 - m_{prop} (kg)	1.301	2.361	2.5461	3.267
Stage 1 - m_{inert} (kg)	0.538	0.285	1.643	2.664
Stage 2 - m_{inert} (kg)	0.422	0.625	0.6523	1.293
Stage 3 - m_{inert} (kg)	0.145	0.262	0.23653	0.284
Initial vehicle mass (kg)	**12.326**	**13.945**	**34.928**	**47.356**
Payload mass/Initial mass	8.113	7.171	2.863	2.112

To check the variations caused by I_{sp} and f_{inert} RP-1/LOx is used in analysis instead of RP-1/H_2. We get an initial mass for the all-H_2/O_2 vehicle of 12.312 kg and 47.356 kg for the all-solids vehicle. The optimized results are shown in Table 5.

Performance Analysis

Conceptual designs of SLVs cannot be conducted without the cooperation of numerous disciplines, including several analysis technologies: aerodynamic analysis, propulsion analysis, structural analysis, trajectory analysis, heating analysis, controls analysis, cost analysis, operations analysis, and so on. The approach used in this study is simpler and of lower fidelity than the latest sophisticated computation methods, but needs lesser computation time so it is useful for our conceptual design estimates.

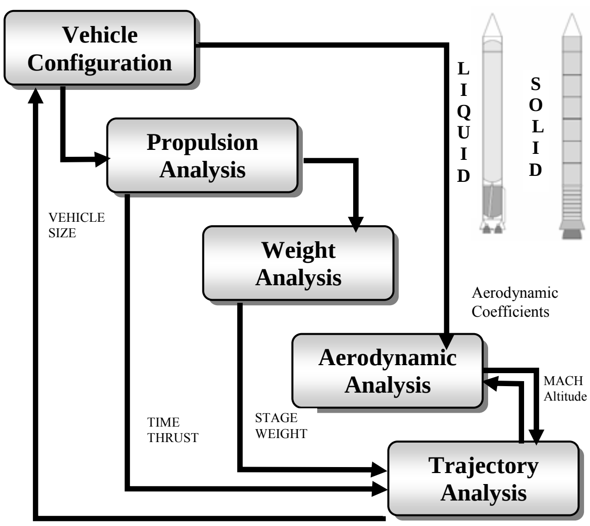

Fig. 4 SLV Performance Analysis Approach

The trajectory analysis depends on inputs from the aerodynamic analysis, weight and propulsion analysis, and these results are used to compute the flight trajectories. A three degree-of-freedom (3-DOF) trajectory model was developed in SIMULINK. The SLV is treated as a point-mass, flight in 2D over spherical non-rotating earth was assumed, which implied that the Coriolis and centrifugal pseudo forces were negligible. The 1976 standard atmosphere (no winds) was used. The trajectory was optimized during maneuvering phase, after transonic phase SLV turning was considered under gravity. Trajectory inputs includes vehicle (e.g., GLOW, liftoff T/W, propellant mass flow rate, aerodynamic coefficients and reference area). Terminal constraints on altitude, velocity, and flight-path angle as well as maximum in flight dynamic pressure, α during maneuvering phase, pitch rate, and normal force limits are enforced. The flight trajectory of SLV is composed of several phases.[3)] Each stage shuts down one after another and separates to shed m_{inert}. Because, the equations of motion change discontinuously at the shutdown points, the trajectory must be divided into intervals, the number of which corresponds to number of stages. Initial and terminal times of the burn intervals are given by the propulsion analysis. The phases are as follows:

Vertical Launch Phase: starts from the time of lift off from launch pad and lasts up to 5s. An initial condition is that the flight path angle is 90 deg. After launch, the SLV accelerates by the power of the 1st stage rocket motor and flies vertically for 5s before the next phase of pitch over.

Launch Maneuver Phase (Pitch over): This phase starts after the end of vertical launch phase up to the start of transonic phase. α is constrained to approach zero before start of transonic phase (Mach = 0.8) and remain zero till end of transonic phase (Mach = 1.3). Launch Maneuver control angle φ is one of the design variable, it represents the pitch angle.

Powered Ascent Phase: starts from end of transonic phase up to the shutdown of 3rd stage motor. We constrain the flight-path angle of SLV to decrease monotonously. α must approach to zero at the time of first stage separation, however, there is no such restriction on separation of upper stages.

Coasting Phase: starts at the end of 2nd stage, the final stage and payload separates and zooms up with no thrust in an elliptical orbit whose apogee is at 500 km.

Kick Phase: Finally, an apogee kick puts the payload into a 500 km circular LEO. The constraint conditions in these phases are the angle of attack, dynamic pressure, normal acceleration, and body axial acceleration. The set of motion equations are derived from [3)]

$$
\begin{aligned}
\dot{V} &= \frac{T\cos\alpha - D}{m} - g\sin\vartheta \\
\dot{\vartheta} &= \frac{T\sin\alpha + L}{mV} - \frac{g\cos\vartheta}{V} + \frac{V}{R_e + h}\cos\vartheta \\
\dot{h} &= V\sin\vartheta \\
\dot{l} &= \left(\frac{R_e}{R_e + h}\right)V\cos\vartheta \\
\alpha &= \eta + \varphi - \vartheta \\
\eta &= \frac{l}{R_e} \\
\varphi &= \varphi_{pro}(t) \\
L &= C_L^{\alpha}\frac{1}{2}\rho v^2 A_{ref} \\
D &= C_D\frac{1}{2}\rho v^2 A_{ref}
\end{aligned}
\qquad (10)
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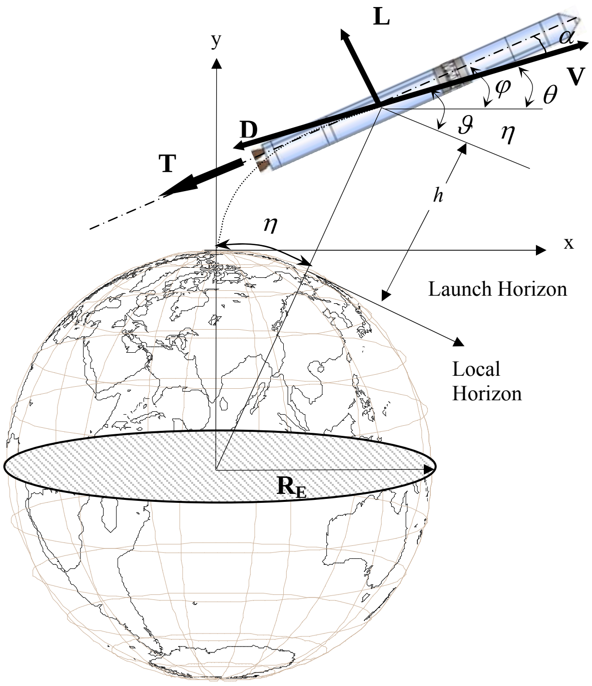


Fig. 5 Forces acting on a SLV during flight

The weight analysis computes the component masses of the SLV from the geometric and flight performance parameters on the design variables and the amount of propellant provided by the trajectory analysis.[3)] The performance of an engine can be expressed with characteristic velocity C^*, I_{sp}, and their efficiencies. The performance factors are influenced by types of propellants, mixture ratio, chamber pressure p_e, and geometric parameters such as area of the nozzle throat A_t and area expansion ratio. When rocket engines are operated, a thrust chamber is in high temperature and pressure condition and thermodynamic properties of hot combustion gas can seldom be measured. Meanwhile, chamber pressure p_c and thrust T can be easily measured, compared to other properties. So, in order to evaluate the performance of a liquid engine, C^*, I_{sp} and their efficiencies are predicted theoretically.

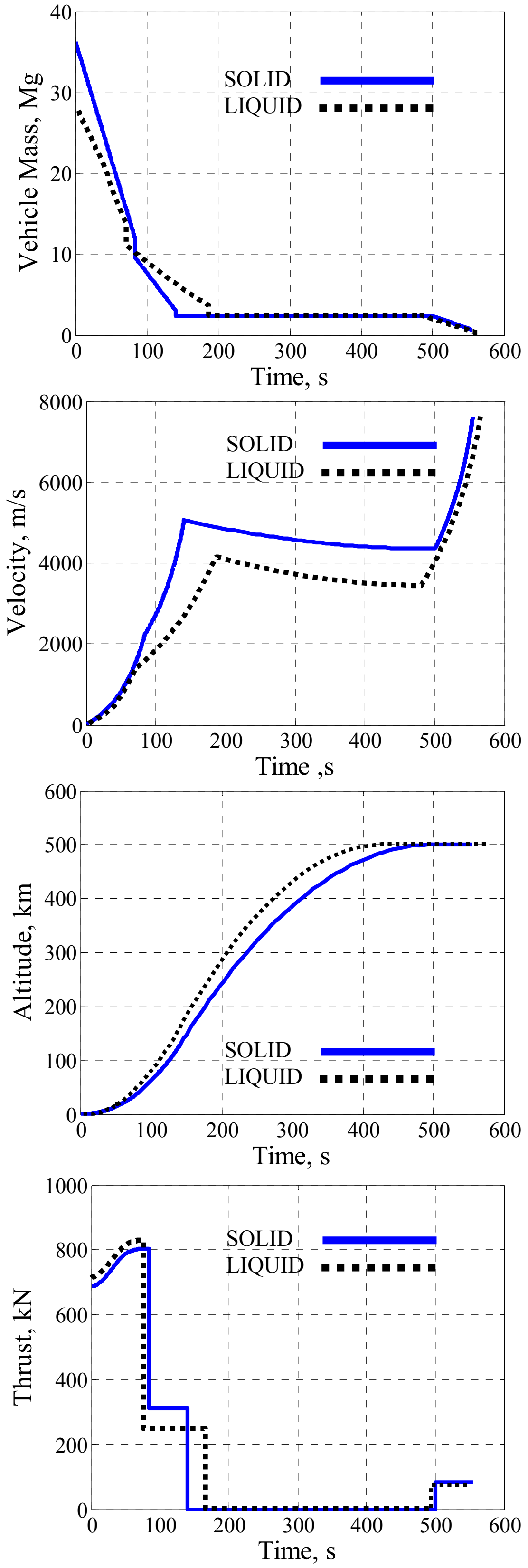


Fig. 6. Performance of LIQUID FUEL (Kerosene (RP-1) and Liquid O_2) SLV vs. SOLID FUEL SLV

LOx/Kerosene is a common propellant in the US and in Russia, and it has been used since the early age of liquid propulsion. A huge number of LOX/Kerosene engines are propelling launchers.[8)] Representative high thrust engines are listed in Table 6.

Table 6
Engines using LOx/Kerosene

Engine	Launcher	T (kN)	I_{SPvac} (s)	MR	p_c (bar)	Mass (kg)
RD-170	ENERGIA	7908	336	2,63	250	9500
FI	SATURNV	7776	304	2,27	67,7	8444
RD-180	ATLAS III	4152	338	2,72	257	5545
RS-27	Delta	1023	295	2.25	48	1027
RD-107	Soyouz	1000	313	2,47	58.5	1155
RD-108	Soyouz	912	315	2,39	51	1250
RD-120	Zenith	833	350	2,6	163	1125
NK 33	NI	1687	331	2,586	145	1242
MA-5 Boost.	Atlas	1910	265	2,25	49.5	1513
MA-5 Sust.	Atlas	270	220	2,27	51	470

Cost Analysis

There is a widespread assumption throughout much of the aerospace industry that lowest cost is attained by attaining the highest I_{sp} and lowest GLOW. This belief is fostered by cost models that equate low cost with low system weight. It is this belief that spurs research and development of high performance liquid rocket propulsion especially liquid hydrogen based booster applications. All launch vehicle cost models are based on the mass of the system being developed or produced. The normal cost model relationship is:

$$C = f \times m_{inert}^{\ x} \tag{11}$$

where

C is the cost

f and *x* are historically derived coefficients

The natural and wide spread assumption from such cost models is that the lower the mass of a system the lower the cost. Low mass is attained from either higher engine performance (I_{sp}) or from use of light weight material. In other words, the engineering profession looks to mass reduction as a key to cost reduction. In fact, mass reduction is not the only strategy that leads to lower development, fabrication and operational costs. An alternative is to examine the relationships for different types of rocket stages. The cost estimation relationships (CER) for different types of launch vehicles differ significantly.[9)]

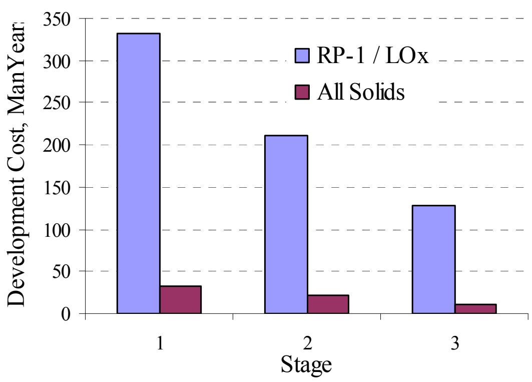
Table 7
Cost Estimation Relationships

Cost[man-years*]	SRM	Liquid
Development	$19.4 * m_{inert}^{0.507}$	$260 * m_{inert}^{0.487}$
Fabrication	$3.17 * m_{inert}^{0.35}$	$4.8 * m_{inert}^{0.485}$

*Many Year: A unit measuring the work of one person in a year, based on a standard number of man-days and Man hours. It describes the amount of work done by an individual throughout the entire year. It will be different for various industries depending on the average number of hours worked each week and the number of weeks worked per year.[10)]

SRMs are much simpler to design because they have very few moving parts and because the total parts count is orders of magnitude lower than for liquid rocket stages. Simplicity is a key to low launch vehicle stage development costs. Generally, one would expect that SRMs will cost between ten and twenty-five times less (depending mostly on scale) to develop than for a liquid rocket stage. Generally, one would expect that SRMs will cost between two and ten times less (depending mostly on scale) to fabricate than for a liquid rocket stage. It becomes clear that SRMs are significantly more cost effective than liquid rocket stages when the cost model coefficients are compared. SRMs are very simple while all current liquid rocket booster systems are relatively complex. SRMs have few parts while liquid systems have many parts. Many of the parts on liquid rockets are moving or rotating while there are few moving parts on SRMs. Many of the functions on SRMs such as nozzle cooling are much simpler than those found on liquid rocket systems. All this means that less engineering design and analysis is required during the development phase. The fabrication cost estimation relationship coefficient for SRMs is near that of liquid rocket systems. It is the exponent for solid rocket motors that is very low making the relative cost of liquid rocket systems high by comparison. The reason for the difference is again simplicity of SRMs compared to the complexity of pump-fed liquid rockets.

Based on the CER given by ref[9] we perform a cost analysis on our stage optimized SLV from Table 5.



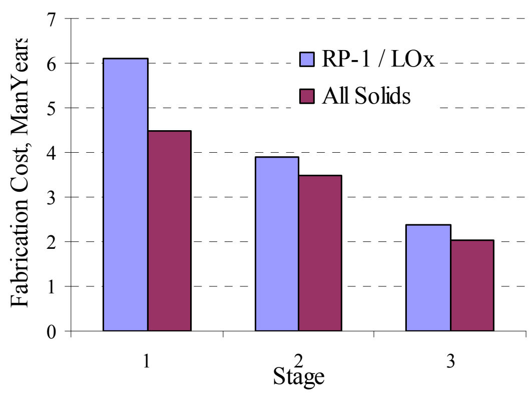


Fig. 7. Cost Comparison between Solid and Liquid SLV from Table 5, based on CERs from Ref[9]

Discussion

Due to the fact that no single propulsion system has all desirable properties, the selection of the propellant combination is a compromise of many parameters as discussed below:

Table 8
Propellant Selection Tradeoffs

Economic	• availability in large quantities • low cost • logistics of production • simplicity of production process
Performance	• specific impulse • effective exhaust velocity • low molecular mass
Hazards	• corrosive effects • explosive hazards, fire hazards toxicity
Desirable Properties	• low freezing point • high boiling point • low vapor pressure • high specific gravity • chemical stability • high specific heat • high thermal conductivity • low viscosity

SRM capability exceeds all other in the availability, logistics, price, shelf life, storage and physical hazards like corrosion, explosion, fire, toxicity, accidental spills etc. SRM are the oldest rocket system, easily sized, simple to operate. But, they are not Re-startable, non or only partially Throttle able, offer low I_{sp}, and give off toxic emissions including chloro-fluorocarbons. Every American launch vehicle family has at least one member that uses some form of solid propulsion as a major contributor to liftoff thrust. Scout, Pegasus, Taurus and Lockheed Launch Vehicle are all composed exclusively of SRMs. The Titan IV descends from a line of launch vehicles that since 1965 relied on a complete stage strapped onto the central core vehicle. [4] Since 1969, the Delta family has relied on SRMs to augment the first stage liquid rocket core. Atlas IIA was the last of the launch vehicle families to use SRM augmentation when the use of Castor strap-on SRMs created the Atlas IIAS. Space Shuttle has always used the overwhelming power of twin SRMs to provide a major share of the total system propulsive impulse.

While widely used SRMs have some very serious drawbacks. Once ignited there is generally no abort mode available to shut down the motor. SRMs are also characterized by high levels of vibration and thrust pulsation making it necessary to provide protection for delicate payloads. SRMs also have heavy cases and insulators resulting in high burn-out weight. But both SRM and LIQUID propellant rockets require the development of large pressure vessels (the tanks for the liquid propellants, and the whole rocket for solid propellants). The engineering for the LIQUID containing pressure vessel is considerably simpler. The liquid tanks are under regulated pressure, while the casing of the SRM is hammered by pressure spikes from combustion instability. The LIQUID tanks have the propellant in benign contact with the load carrying structure, while SRM require insulation to keep hot gases from burning through the walls. LIQUID tanks are in one piece, while in large sizes solids have jointed pressure vessels. Having filled a liquid propellant tank, there is nothing much to inspect. For SRM, one worries about voids and cracks in the propellant, batch to batch variation in the mechanical and burning properties of the propellant, the internal bonding of the propellant to insulation material and inhibitor coatings, the effect of the ambient temperature on propellant mechanical and burning, properties, the age and prior storage conditions of the propellant grain etc. Engineering of the tanks of large, pressure fed LIQUID propellant rockets is actually considerably simpler than that of pump fed liquid propellant rockets. With higher pressures, the walls are thicker and the structure is much stiffer due to internal pressure which sharply reduces the need for clever internal structures to provide stiffness and carry loads.

In spite of the relatively more difficult engineering in making pressurized tanks for pump fed rockets, in-flight failure of LIQUID propellant tanks on pump fed rockets is almost unheard of. On the other hand, in-flight failures of the pressure vessel of SRM on various vehicles have generated an entire gallery of pictures of fireballs in the sky. LIQUID propellant rockets have a variety of subsystems not present on SOLIDS, any one of which can represent a failure point. The complexity and number of such systems however is markedly less on pressure fed rockets than on pump fed systems. Relative reliability of pressure fed and pump fed LIQUID rockets can be judged by comparing the excellent reliability record of the pressure fed second stage of the Delta II, and the various pressure fed engines in the Apollo with the repeated failures of Russian, European, Japanese and American pump fed upper stages. [4,5)]

One main consideration is the I_{sp}, for the LIQUID propellants are higher than the SRMs. Large mass fraction is required when the velocity required is greater or for the strategic applications. I_{sp} is a function of the propellant characteristics and the designer selected pressures. SRMs with the highest flame temperature and lowest average molecular weight in the exhaust flow yields the highest I_{sp}. LIQUID propellants have the advantages of being a well known and tested technology, versatile and reliable, but the future booster designs will be based on SRMs because they offer a responsive launch vehicle; no hazardous fueling in the field or cryogenic "topping off" is required and can sit for years, already to be fired at a moment's notice, without concern. Responsive Space lift can be fully supported by all-solid propellant, standardized launchers, similar to ICBMs. Much of the current development focus has been placed on LIQUID (liquid oxygen/kerosene-based) vehicles, quite similar to the Thor and Atlas of the 1950~60's. The performance of various LIQUID boosters is discussed in [11)].There is no need to restart the evolutionary process again; attention should be paid to develop all-solids vehicles to meet future needs. There is a backlog of satellite launches due to non-availability of compatible launch system. There exist many payloads with no identified launch systems. There is requirement of reliable, lower cost launch system. Several decades' old proven technologies of SRMs can be employed for cheaper access to space even for developing nations. Solid fueled SLVs are more reliable and low-cost solution for small payloads (up to 1 ton) to Low Earth Orbit (LEO) [12,13)] SRMs can be combined to form family of launchers to match various payloads and mission requirement. Such systems have early flight readiness and require minimal launch permanent crew.

Conclusion

For multistage SLV, the overall payload fraction depends on how the ΔV requirement is partitioned among stages. Payload fractions will be reduced if the ΔV is partitioned suboptimally. The optimal distribution must be determined by trial and error. Results reveal that to maximize payload fraction for a given ΔV requirement:

- Stages with higher I_{sp} should be above stages with lower I_{sp}.
- More ΔV should be provided by the stages with the higher I_{sp}.
- Each succeeding stage should be smaller than its predecessor.

We have shown how staging is a valuable tool and helps in a satellite launch mission, but increasing the number of stages may sometimes lead to more complexity, increase in cost, and decrease in reliability of the Launch Mission. The selection of the propellant and optimal number of stages are all trade offs the Mission Designers have to analyze in very early design phase, which can save much time and cost at the later stages of mission execution. We chose LEO mission as an example but this type of analysis applies to any space mission that involves staging and requires a large velocity increment. The objective was to minimize the GLOW [N] or Initial Take off Mass [kg]. The primary performance figures of merit were the total inert weight of the SLV and the payload weight that the SLV could lift into LEO, given the candidate propulsion systems, more variables can be added and fine tuned to get more pragmatic results for a detailed analysis. It was a preliminary conceptual design approach, but our results show that our basic assumptions and estimates are reasonable enough and can be used as initial estimates for detailed trade-off analyses. The trajectory evaluation method devised accurately predicted the performance for the SLVs studied, and is not time intensive and is also tolerant of small trajectory modifications.

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Appendix A

Nomenclature

C	= *Cost* [Man Years]
D	= Drag force [Newton]
f_{inert}	= inert mass fraction
g	= acceleration due to gravity [m/sec^2]
$GLOW$	= Gross Lift Off Weight [N]
h	= Height Above Ground (Altitude) [m]
I_{sp}	= average specific impulse [s]
L	= lift force [Newton]
l	= Ground Range [m]
m_{0i}	= liftoff weight of ith stage rocket [kg]
$m_{0(i+1)}$	= payload of the ith stage rocket [kg]
m_{fuel}	= Fuel mass [kg]
m_{inert}	= mass excluding propellant + payload [kg]
m_{ki}	= mass of structure of ith stage rocket [kg]
m_{OX}	= Oxidizer Mass [kg]
m_{propi}	= mass of propellant of ith stage rocket [kg]
m_{prop}	= propellant mass [kg]
O/F	= Oxidizer to Fuel Ratio
p_{ci}	= Chamber pressure of ith stage [Pa]
p_{ei}	= Nozzle exit pressure of ith stage [Pa]
Re	= Radius of Earth [m]
ρ	= density [kg/m^3]
T/W_o	= initial thrust-to-weight ratio
V_{fuel}	= Fuel Volume [m^3]
V_{OX}	= Oxidizer Volume [m^3]
α	= angle of attack [rad]
η	= Range Angle [rad]
ϑ	= flight path angle [rad]