# A Consideration of Analytical Thermodynamic Modeling of Bipropellant Propulsion System

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## Abstract

This paper is to consider analytical thermodynamic modeling of bipropellant propulsion system. The objective of thermodynamic modeling is to predict thermodynamic conditions such as pressures, temperatures and densities in the pressurant tank and the propellant tank in which heat and mass transfer occur. In this paper also it shows analytic equations that calculate the evolution of ullage volume and interface areas. Since the ullage interface areas are time-varying, (the liquid propellant volume decreases as the rocket engine is firing; the change of ullage volume correspond to the change of liquid propellant volume) for a numerical convenience non-dimensionalized correlations are commonly used in most literatures with limitations; a few percentages of inherent error. The analytic equations are derived from analytic geometry, subsequently without inherent error. Those equations are important to calculate the heat transfer areas in the heat transfer equations. It presents the comparison result of both analytic equations and correlation method.

## Introduction

An analytical thermodynamic modeling of bipropellant propulsion system is to predict thermodynamic conditions such as pressures, temperatures and densities in the pressurant tank and the propellant tank in which heat and mass transfer occur, so it makes sure that the loading pressurant and propellant are sufficient during mission life and the temperatures of equipment are within allowable limits. The thermodynamic modeling as a design and analysis tool can be applicable to many bipropellant propulsion systems, for example, spacecraft: geosynchronous satellite, interplanetary probe or launcher upper stage: Ariane 5 Upper Stage EPS, Space Shuttle. Those bipropellant propulsion systems use MMH and NTO, as fuel and oxidizer, respectively.

A few important modelings in literatures have been reviewed since early 1960's. There are the same principles adopted all in literatures, those are the first law of thermodynamics and the mass conservation principle for control volumes, e.g. ullage volume, pressurant tank control volume and tank wall itself. The different points are how to deal the vaporization or condensation of vapor propellant, heat transfer relationships between interfaces and whether it operates in a blowdown mode or a pressure regulated mode.

In this paper it is focused on the convection heat transfer. The convectional method [1] correlate ullage interface areas with the change of ullage volume and it has inherent error at extreme points as Eqn (1).

$$\frac{A_u}{V_t^{\frac{2}{3}}} = 4.0675\left(\frac{V_u}{V_t}\right)^{0.6237} \quad (1)$$

But the analytic equations are derived from analytic geometry, subsequently without inherent error. The rate of heat transfer by convection is determined from Newton's law of cooling, expressed as

$$\dot{Q} = hA(T_2 - T_1) \quad (2)$$

If the heat transfer area above equation is time varying as the liquid propellant is consumed accordingly then it is one of important things to calculate the heat transfer areas according to the propellant consumption to know exact heat transfer rate. Fig 1 represents geometry to simulate an upper spherical propellant tank and Fig 2 a lower part (as shown up side down). Eqns 3 and 4 are volume equations, respectively.

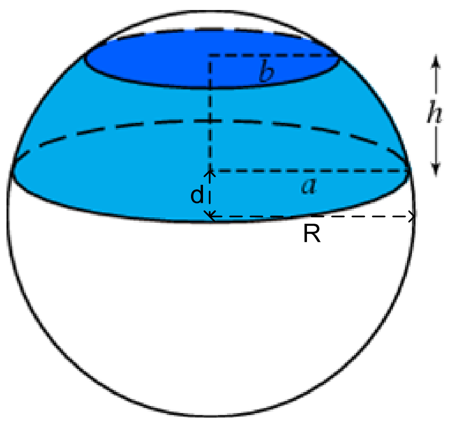


Fig. 1 A spherical segment [2]

$$V_{spherical\ segment} = \pi h\left(R^2 - d^2 - hd - \frac{1}{3}h^2\right) \quad (3)$$

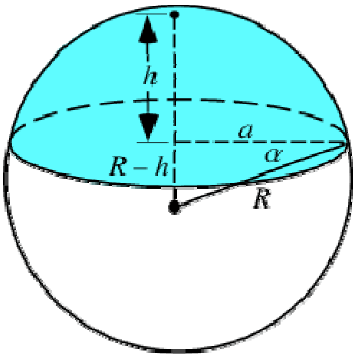


Fig. 2 A spherical cap [2]

$$V_{cap} = \frac{1}{3}\pi h^2 (3R - h) \tag{4}$$

In Fig. 3 it compares an analytic solution to the correlation solution with a sphere its diameter 1m. It presents differences in the mid range and biggest difference at right extreme point.

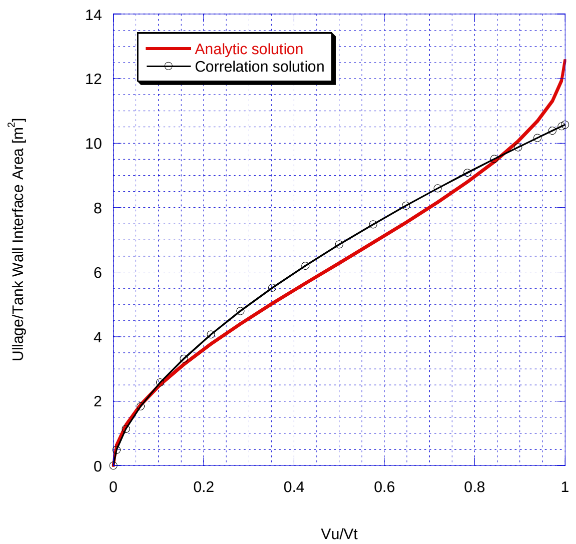


Fig. 3 Comparison of Analytic to Correlation solution

In this paper the tank model extends to a cylindrical tank with spherical ends, which is commonly used as shown in Fig 4.

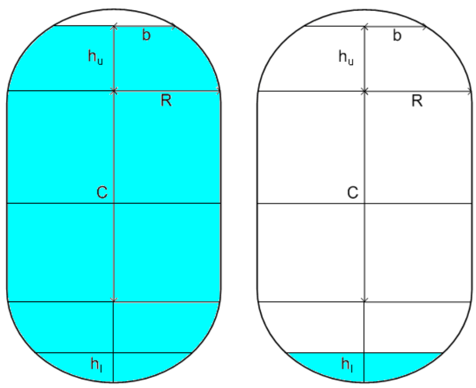


Fig. 4 Volume and surface area evolutions of a cylinder tank with spherical ends

In Fig. 5 it depicts analytical thermodynamic modeling [3]. Ricciardi and Pieragostini derived whole governing equations for entire propulsion system composed of a pressurant tank, two propellant tanks and pipeline networks but in this paper a pressurant tank and pipeline networks are deleted for comparison. Although it depicts a cylindrical tank, the actual model uses the same spherical tank as Estey et al [1]. And all other parameters are the same, too.

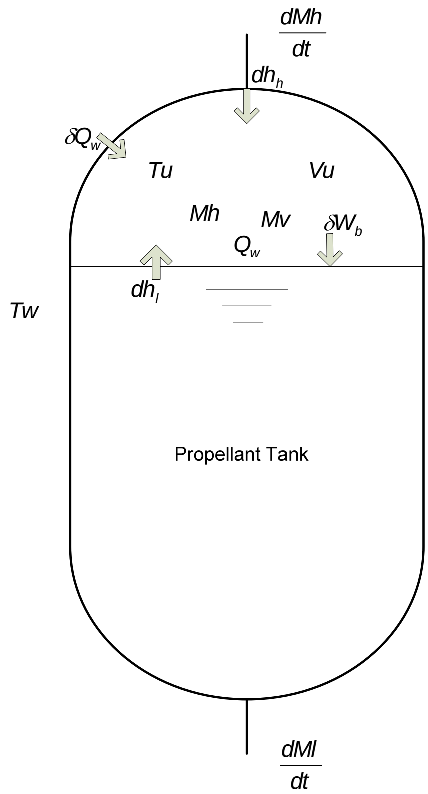


Fig. 5 Analytical thermodynamic modeling schematic

The energy balance, applicable to control volumes: ullage volume and tank wall, is expressed in the rate form as

$$\begin{aligned}&\dot{Q}_w + \dot{Q}_l - P_t\dot{V}_u + \dot{m}_g h_g + \dot{m}_v R_v T_v = \\ &\dot{m}_g C_{vg} T_g + m_g C_{vg}\dot{T}_g + m_v C_{vv}\dot{T}_v\end{aligned} \tag{5}$$

$$-hA(T_w - T_g) = m_w c_v \dot{T}_w \left(\frac{A_u}{A_t}\right) \tag{6}$$

Differentiating gas and vapour perfect gas equations, one gets

$$\frac{\dot{m}_v}{m_v} = \frac{\dot{V}_u}{V_u} - \frac{\dot{T}_u}{T_u} \tag{7}$$

$$\frac{\dot{V}_u}{V_u} = \frac{\dot{m}_l}{(\rho V_u - m_v)} - \frac{m_v}{(\rho V_u - m_v)}\frac{\dot{T}_u}{T_u} \tag{8}$$

The amount of heat exchanged with the tank wall is given as

$$\dot{Q}_w = h_c A (T_w - T_u) \tag{9}$$

The amount of heat exchanged with the liquid propellant is given as

$$\dot{Q}_l = h_c A (T_l - T_u) \tag{10}$$

The resulting set of ordinary, coupled, nonlinear differential equations for the thermodynamic variables is integrated as an initial value problem using the subroutine RKF45 in FORTRAN [4].

## Numerical Simulation Results

In Fig. 6 the comparison results between Estey *et al.* [1], Analytic model and Correlation model are shown. Differentiating the perfect gas equation, one gets

$$\frac{dP}{P} = \frac{dm}{m} + \frac{dT}{T} - \frac{dV}{V} \tag{11}$$

According to Eqn (11) and the following results of temperature and mass variations (Figs 8 and 10), the prominent factor is ullage volume variation and other variables have minor changes. Since the propellant consumption is the same to Estey *et al.*, that is ullage volume change is the same, the pressure variation follows the results of Estey *et al.* Even though Estey *et al.* set vapor pressure a variable, but no equation is present in their paper and their results say little change in the vapor pressure. However, the vapor pressure depends on the liquid temperature, so that in present work the vapor pressure is set constant because it is assumed the liquid temperature does not change. In Fig. 7 the ullage and tank wall interface area is shown as Fig. 3.

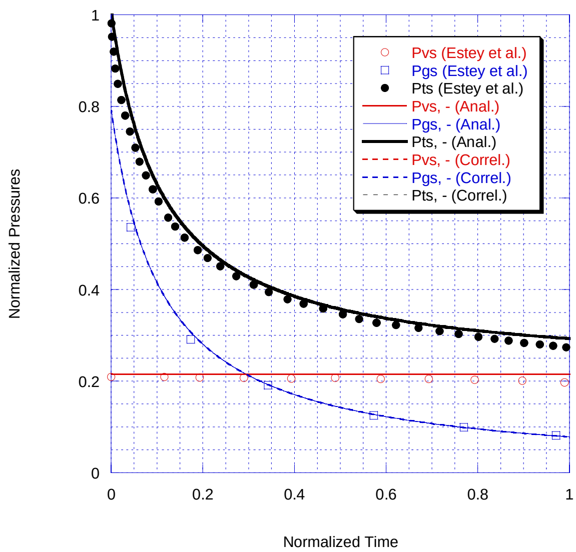


Fig. 6 Normalized Pressure Histories

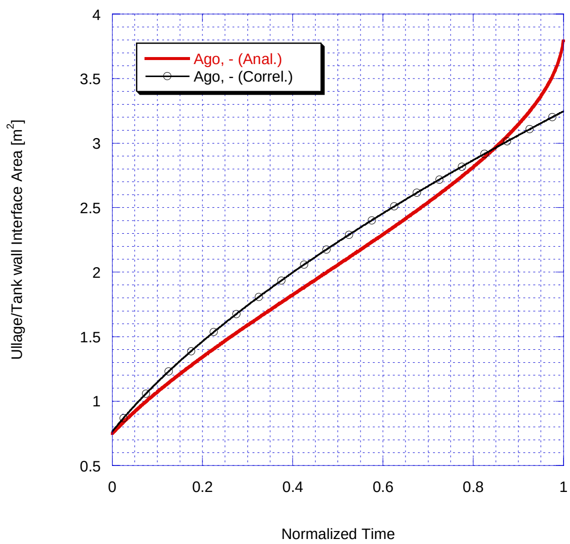


Fig. 7 Ullage/Tank wall Interface Area Evolution

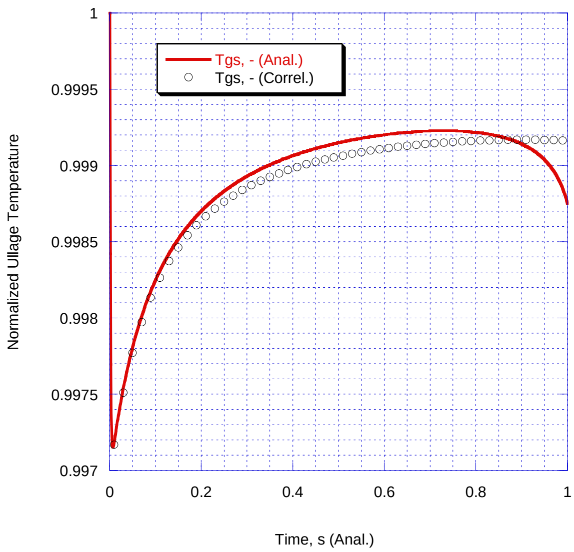


Fig. 8 Normalized Ullage Temperature Histories

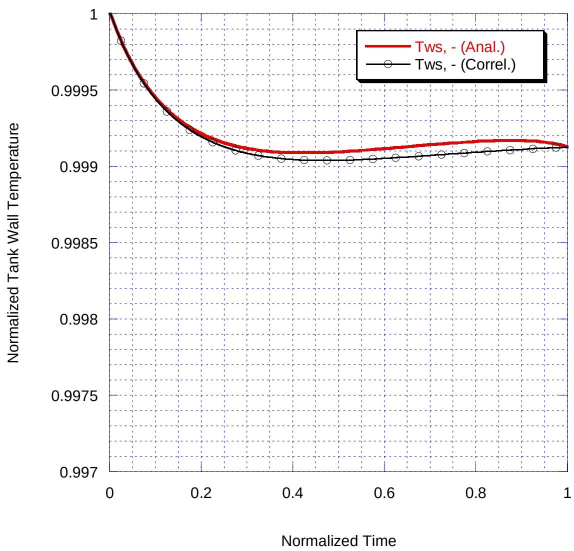


Fig. 9 Normalized Ullage Temperature Histories

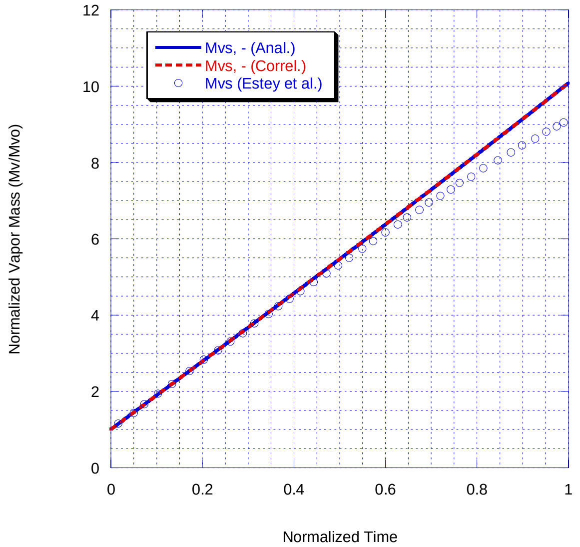


Fig. 10 Normalized Vapor Mass Histories

In Figs 8 and 9 the ullage and tank wall temperature variations are shown, in the beginning it shows adiabatic expansion process then the ullage temperature increases as the vapor mass, which is from constant liquid temperature, enters in the ullage volume. As it is expected in Fig 7, at the end of process the more cooling has occurred in the Analytic model than the Correlation model due to more area.

## Conclusion

In this work it adopts the analytic geometry equation to calculate more accurate the heat transfer area in the convection heat transfer equation planted into energy equation so that the author tried to get more accurate results and it shows a little different results but it does not make any difference with respect to whole results.

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