RELIABILITY ESTIMATION AND RBDO USING KRIGING METAMODEL AND GENETIC ALGORITHM

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Abstract

In this study, effective methods for reliability estimation and reliability-based design optimization(RBDO) are proposed using kriging metamodel and genetic algorithm. In our previous study, we proposed the accurate method for reliability estimation using two-staged kriging metamodel and genetic algorithm. In this study, the possibility of applying the previously proposed method to RBDO is examined. The accuracy of that method is much improved than the first order reliability method with similar efficiency. Finally, the effective method for RBDO is proposed and applied to numerical examples. The results are compared to the existing RBDO methods and shown to be very effective and accurate.

INTRODUCTION

In order to improve qualities of products, it is important to design parts and systems maintaining adequate reliability. So far, there have been a lot of researches on estimating reliability effectively and accurately. Monte-Carlo simulation(MCS) is accurate method but heavy computation work is required. The first order reliability method(FORM) and the second order reliability method(SORM) uses linear and quadratic approximation of a limit state equation, respectively. FORM is accurate and efficient method when the limit state equation is linear, whereas SORM is used when the nonlinearity of the limit state equation is high. In addition to those method, methods using metamodel such as response surface method(RSM) and kriging[1] are studied. Metamodel is a surrogate of behavior of real complex structures and has been widely used to optimization problems[2]. Some researches have been performed on applying kriging metamodel to estimation of reliability[3-4].

In the field of reliability-based design optimization(RBDO) considering the uncertainty in the optimization procedure, there also have been many research on it. Reliability index approach(RIA) and performance measure approach(PMA) based on the reliability index are mainly studied. But these methods are not effective because they have the double loop structure. Single loop single vector(SLSV)[5], sequential optimization and reliability assessment(SORA)[6], and direct decoupling approach(DDA)[7] are proposed to convert the double loop structure to single loop or serial loop in order to improve the efficiency. Lee and Jung[8] proposed krigng based RBDO with constraint boundary sampling. But, more effort is required to improve the efficiency of RBDO method in order to apply RBDO to large structural problems.

We already proposed the reliability estimation method using two-staged kriging methamodel and genetic algorithm in our previous study[3]. We applied the proposed method to numerical examples containing normally distributed random variables and the accuracy of the proposed method was also investigated. In this study, the proposed method is applied to problems containing variously distributed random variables and the accuracy and the efficiency are compared to FORM, the typical method of obtaining the reliability index, in order to check out the possibility of applying the method to RBDO. Finally, the effective method for RBDO is proposed and applied to numerical examples.

RELIABILITY ESTIMATION USING KRIGING METAMODEL

The flowchart of the previously proposed reliability estimation method is shown in Figure 1[3]. Mathematical examples containing variously distributed random variables are selected and shown in Equations (1)-(6). The distributional characteristics of the random variables are shown in Table 1.

$$Ex1)\ g(x) = \exp(0.4x_1 + 7) - \exp(0.3x_2 + 5) - 200 \tag{1}$$

$$Ex2)\ g(x) = x_1^3 + x_1^2 x_2 + x_2^3 - 18 \tag{2}$$

$$Ex3)\ g(x) = \frac{6x_1x_3x_5(x_2 - x_3)}{x_2x_6x_5^3 - x_2(x_6 - x_7)(x_5 - 2x_8)^3} - x_4 \tag{3}$$

$$Ex4)\ g(x) = x_1 + 2x_2 + 2x_3 + x_4 - 5x_5 - 5x_6 \tag{4}$$

$$Ex5)\ g(x) = x_1x_2 - 78.125x_3 \tag{5}$$

$$Ex6)\ g(x) = \cos^{-1}\left(\frac{x_1 + 0.5(x_2 + x_3)}{x_4 - 0.5(x_2 + x_3)}\right) - \frac{6}{180}\pi \tag{6}$$

The results obtained by the proposed method and FORM[9] are shown in Table 2. From the results, the probability of failure by the proposed method is more accurate than FORM. In a few examples, the efficiency of FORM is better than the proposed method, but there is substantial error in the probability of failure.

In addition to the mathematical examples, truss problems such as the three-bar truss and the 23-bar truss shown in Figure 3 and 4, respectively, are tested. The vertical displacement at node 4 of the three-bar truss should be less than allowable displacement v_{max}. For the 23-bar truss, failure occurs when the vertical displacement at center node exceeds 15mm. The mean value and coefficient of variance(COV) are shown in Table 3. COV is obtained by dividing the standard deviation by the mean value. The results are shown in Table 4. In case of the three-bar truss, the probability of failure by proposed method is almost equal to MCS. For the 23-bar truss, the number of function call by the proposed method is a little larger than FORM. But the proposed method is more accurate than FORM. Through the mathematical examples and the truss examples, it is shown that the proposed reliability estimation method is more accurate than FORM with similar efficiency.

RBDO USING KRIGING METAMODEL

Proposed method for RBDO

In RBDO, the design points are continuously changed as the optimization proceeds. Since kriging metamodel can accumulate the previously used sample points, the additional sample points should be reduced in the optimization procedure. The proposed method for RBDO using kriging metamodel is shown in Figure 4. First, the initial sample points are selected to construct kriging metamodel for each probabilistic constraint. 0, ±1 σ , and ±3 σ away axial points from the mean value are used as the initial sample points. So, the number of the initial sample points is $4n+1$. σ indicates the standard deviation of each design variables and n stands for the number of design variables. And then, one additional sample point is added continuously until the most probable point(MPP) is converged by kriging metamodel. MPP is obtained by inverse reliability analysis(IRA) and the additional sample point is selected from the location where the mean square error of kriging metamodel is maximum. The additional sampling point is obtained by genetic algorithm[3]. When the new design point is obtained, the accumulated sample points are selected as the initial sample points for the next iteration and the additional sample points are added to the kriging metamodel until the MPP is converged. This procedure is iterated when the convergence criteria are satisfied.

Numerical examples and discussion

The proposed method for RBDO is applied to numerical examples and the accuracy and the efficiency are compared to the existing RBDO methods such as RIA, PMA, SLSV[5], SORA[6], and DDA[7].

Example 1

The first numerical example[10] is shown in Equation (7). It has two random variables and probabilistic constraints.

$$
\begin{aligned}
&\text{Minimize } f = \left(\pi d_1^2 + d_2\right) \\
&\text{subject to } \Pr\left[g_1 = 1 - \frac{x_1^3 x_2}{95.5} \geq 0 \right] \leq \Phi\left(-\beta_1^{\text{target}}\right),\ \Pr\left[g_2 = 1 - \frac{x_1^2 x_2}{70.7} \geq 0 \right] \leq \Phi\left(-\beta_2^{\text{target}}\right), \\
&\qquad 1.0 \leq d_1 \leq 2.0,\ 20.0 \leq d_1 \leq 50.0,\ x_1 \sim N\left(d_1, 0.1\right), x_2 \sim N\left(d_2, 0.3\right),\ \beta_1^{\text{target}} = \beta_2^{\text{target}} = 3.1
\end{aligned}
\tag{7}
$$

The initial design point is (1.5, 35) and all random variables are statistically independent and have normal distribution. The optimization results are summarized in Table 5. In Table 5, β^i_{MCS} stands for the reliability of the *i-th* probabilistic constraint at the optimum and is evaluated by MCS with a ten-million sample size in order to confirm whether the target reliability of probabilistic constraints is satisfied. All methods result in almost same optimum and the evaluated reliability satisfies the target reliability. The number of function call is a summation of the number of analyses for the objective function and the probabilistic constraints. It is shown that RIA and PMA are not effective methods because of the double loop structure. SLSV, SORA, and DDA have relatively small computation work. The efficiency of the proposed method is the best among the compared RBDO methods.

Example 2

The second example[11] is shown in Equation (8). It has two random variables and three probabilistic constraints.

$$
\begin{aligned}
&\text{Minimize } f=\left(d_1+d_2\right)\\
&\text{subject to } \Pr\left[g_1=1-\frac{x_1^2 x_2}{20}\geq 0\right]\leq\Phi\left(-\beta_1^{\text{target}}\right),\ \Pr\left[g_2=1-\frac{\left(x_1+x_2-5\right)^2}{30}-\frac{\left(x_1-x_2-12\right)^2}{120}-1\geq 0\right]\leq\Phi_2\left(-\beta^{\text{target}}\right),\\
&\qquad\Pr\left[g_3=1-\frac{80}{\left(x_1^2+8x_2+5\right)}\geq 0\right]\leq\Phi\left(-\beta_3^{\text{target}}\right),\ 0.0\leq d_1\leq 10.0\ \text{ for } i=1,2,\\
&\qquad x_i\sim N\left(d_i,0.3\right)\ \text{ for } i=1,2,\ \beta_1^{\text{target}}=\beta_2^{\text{target}}=\beta_3^{\text{target}}=3.0
\end{aligned}
\tag{8}
$$

The initial design point is (5.0, 5.0) and all random variables are statistically independent and have normal distribution. The optimization results are summarized in Table 6. All methods result in almost same optimum and the evaluated reliability almost satisfies the target reliability. Since g_3 is inactive at the optimum, the corresponding reliability becomes infinite. Among the various methods, the proposed method for RBDO is the most effective. Through the numerical examples, it is verified that the efficiency of the proposed method is much improved than the existing methods.

CONCLUSIONS

In this study, the effective and accurate methods for reliability estimation and RBDO are proposed using kriging metamodel and genetic algorithm. The proposed reliability estimation method is applied to problems containing variously distributed random variables and it is shown that the proposed method is more accurate than FORM with similar efficiency. The effective method for RBDO is also proposed and applied to numerical examples. The results are compared to the existing RBDO methods and shown to be very effective and accurate.

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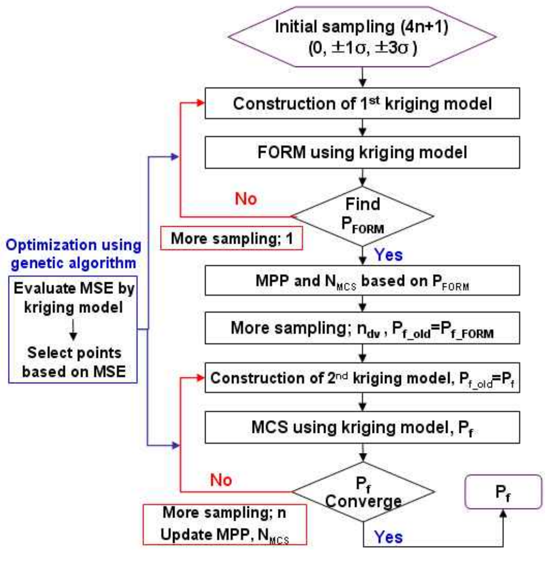

Figure 1 – Flow chart of the proposed method for reliability estimation

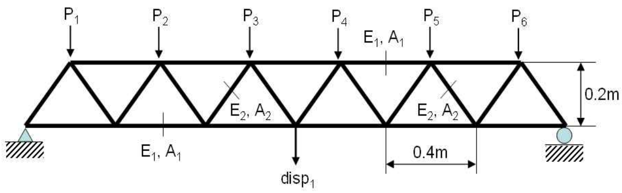

Figure 3 – 23-bar truss problem

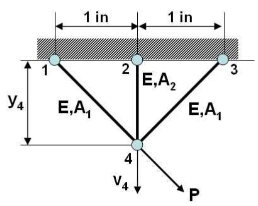

Figure 2 – 3-bar truss problem

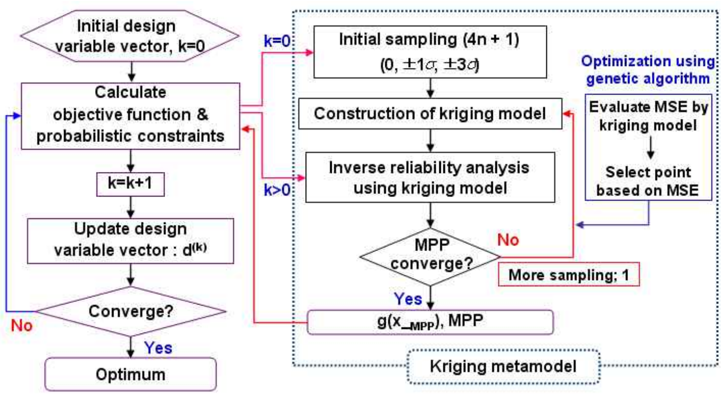

Figure 4 – Flow chart of the proposed method for RBDO

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Table 1 Statistical parameters of mathematical examples

Problem	Variables and statistical parameters
Ex1	x_1~N(0, 1), x_2~N(0, 1)
Ex2	x_1~N(10, 5), x_2~N(9.9, 5)
Ex3	x_1~N(6070, 200), x_2~N(120, 6), x_3~N(72, 6), x_4~N(170000, 4760), x_5~N(2.3, 1/24), x_6~N(2.3, 1/24), x_7~N(0.16, 1/48), x_8~N(0.26, 1/48)
Ex4	x_1~LN(120, 12), x_2~LN(120, 12), x_3~LN(120, 12), x_4~LN(120, 12), x_5~LN(50, 15), x_6~LN(40, 12)
Ex5	x_1~N(2×10^7,5×10^6), x_2~N(1×10^{-4},2×10^{-3}), x_3~Gumbel(4, 1)
Ex6	x_1~Beta(5, 5), $55.0269\le x_1\le 55.5531$, x_2~N(22.86,0.0043), x_3~N(22.86,0.0043), x_4~Rayleigh(0.1211), $x_4\ge 101.45$

Table 2 Reliability results of mathematical examples

Problem	P_f (number of function call)		
	FORM (HL-RF)	Proposed method	MCS
Ex1	3.365×10^{-3} (15)	3.604×10^{-3} (11)	3.627×10^{-3} (1×10^8)
Ex2	6.204×10^{-2} (69)	5.650×10^{-3} (13)	5.811×10^{-3} (1×10^8)
Ex3	8.558×10^{-1} (36)	8.745×10^{-1} (49)	8.709×10^{-1} (1×10^6)
Ex4	9.432×10^{-3} (28)	1.219×10^{-2} (43)	1.216×10^{-2} (1×10^8)
Ex5	4.468×10^{-4} (28)	6.707×10^{-4} (22)	6.690×10^{-4} (1×10^9)
Ex6	8.499×10^{-2} (20)	7.319×10^{-2} (21)	7.120×10^{-2} (1×10^8)

Table 3 Statistical parameters of truss problems

Problem	Variables	Distribution	Mean	COV
3-bar truss	E	Normal	1×10^7 lb/in^2	0.03
	A_1~A_2	Normal	1.0 in^2	0.02
	Y_4	Normal	1.0 in	0.05
	P	Normal	2×10^4 lb	0.10
	v_{max}	Normal	1×10^{-3} in	0.05
23-bar truss	E_1~E_2	Lognormal	2.1×10^6 GPa	0.10
	A_1	Lognormal	0.002 m^2	0.10
	A_2	Lognormal	0.001 m^2	0.10
	P_1~P_6	Gumbel	50 kN	0.15

Table 4 Reliability results of truss problems

Problem	P_f (number of function call)		
	FORM (HL-RF)	Proposed method	MCS
3-bar truss	5.249×10^{-2} (28)	5.178×10^{-2} (31)	5.171×10^{-2} (1×10^5)
23-bar truss	5.018×10^{-3} (44)	9.700×10^{-3} (61)	8.812×10^{-3} (5×10^5)

Table 5 Comparison of optimization results for example 1

Method	$f(d^*)$	d_1^*, d_2^*	No. of fun. call	β^1_{MCS}	β^2_{MCS}
RIA	37.368	2.000, 24.802	345	4.311	3.096
PMA	37.370	2.000, 24.804	209	4.283	3.104
SLSV	37.368	2.000, 24.802	105	4.311	3.096
SORA	37.370	2.000, 24.803	150	4.291	3.102
DDA	37.368	2.000, 24.802	148	4.311	3.096
Proposed method	37.373	2.000, 24.806	67	4.311	3.109

Table 6 Comparison of optimization results for example 2

Method	$f(d^*)$	d_1^*, d_2^*	No. of fun. call	β^1_{MCS}	β^2_{MCS}	β^3_{MCS}
RIA	6.726	3.439, 3.287	630	2.972	3.048	Infinite
PMA	6.731	3.441, 3.290	540	2.982	3.059	Infinite
SLSV	6.729	3.441, 3.287	269	2.982	3.056	Infinite
SORA	6.726	3.439, 3.287	348	2.972	3.048	Infinite
DDA	6.726	3.439, 3.287	240	2.972	3.048	Infinite
Proposed method	6.726	3.439, 3.287	139	2.972	3.048	Infinite