Reliability Based Real-time Slope Stability Assessment

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SYNOPSIS: A reliability based slope stability assessment method is proposed and examined considering the variation of matric suction which is measured by a real time slope monitoring system. Mean value first order reliability method and advanced first order reliability method are used to calculate reliability indices of a slope. The applicability of methods is compared by applying them to the range of matric suctions measured by the real-time monitoring system. Sensitivity analysis is also performed to examine the contribution of random variables to the reliability index of slope. Finally, the proposed method is applied to a model slope. The results show that the reliability index of slope can be used for efficient slope management by quantifying the risk of slope in real time.

Key words: Slope, Matric suction, Reliability analysis, Real-time monitoring

Introduction

Rainfall-induced slope failures are one of the most frequent natural hazards in heavy rainy season in many country. Generally these kinds of failures are triggered by reduced soil strength which is induced by rainfall infiltration. Thus measuring the hydraulic soil properties in a soil slope is primarily important for estimating the slope stability. Also many researches have been performed to monitor the slope behavior automatically (Ng et al, 2003; Li et al, 2005; Lee & Kim, 2007). However the preceding researches have been focused on soil property measurement, although effective and reasonable interpretation of measured data for decision making is equally important for effective slope maintenance.

Generally soil is non-homogenous and anisotropic. Therefore it implies inherent variability. Besides, measurement errors are naturally involved due to its complex structure. As a result, there are many uncertainties in real time measured soil properties which are related with rainfall infiltration to soil. Accordingly, consideration of soil uncertainties should be required for reasonable real time slope stability assessment.

The objective of this paper is to present a reliability based slope stability assessment method which takes into account automatically measured matric suctions. MVFORM(Mean Value First Order Reliability Method) and AFORM(Advanced First Order Reliability Method) are adopted to calculate reliability indices of slope. The results are compared in the range of measurable matric suction by a real-time monitoring system. The contribution of random variables to the reliability index of slope is also examined by sensitivity analysis. Finally, the proposed method is applied to a model slope and the results are interpreted using 2D imaging illustration.

Factor of Safety of Slope Considering Matric Suction

The variation of the pore water pressure distribution with time as well as the initial matric suction must be considered to include the effect of the rainfall conditions in the slope stability analysis. The slope stability analysis can be conducted using the Limit Equilibrium Method with the modified Mohr-Coulomb shear strength criterion, which contains the matric suction term. The slope stability analysis should consider the possibility of surface failures as well as deep circular failure condition. The slope failure type can be affected mainly by soil characteristics and rainfall conditions. The equation (1) is applicable to an unsaturated soil slope considering the matric suction and stress state(Cho & Lee, 2001).

$$FOS = \frac{c' + (u_a - u_w)\tan\phi^b + \gamma z_w \cos^2\alpha \tan\phi'}{\gamma z \sin\alpha \cos\alpha} \qquad \textbf{(1)}$$

where FOS=factor of safety; c'=cohesion; γ=unit weight; $(u_a - u_w)$=matric suction; ϕ'=friction angle, ϕ^b=an angle defining the increase in shear strength by an increase in matric suction; z_w=vertical depth of slope; and α=slope angle.

The value ϕ^b approaches the effective friction angle ϕ' at low matric suction values (Fredlund et al., 1987). Therefore, the same values for ϕ^b and ϕ' were adopted in this study for simplicity. Then, equation (1) can be rewritten as follows.

$$FOS = \frac{c' + \{\gamma z_w \cos^2\alpha + (u_a - u_w)\}\tan\phi'}{\gamma z \sin\alpha \cos\alpha} \qquad \textbf{(2)}$$

Reliability Analysis of Slope Considering Matric Suction

Conventional deterministic stability assessment based on the factor of safety for a slope could not consider the inherent soil variability. Instead reliability analysis which can consider soil uncertainties gives more quantitative and reasonable evidence to assess the stability of rainfall-induced slope. The probability of failure can be defined as

$$p_f = P[G(X) < 0] = \iint_{G(X)<0} f(X)dx \tag{3}$$

where $G(X)$=limit-state function; and $f(X)$= joint probability density function of random variables.

Limit state surface $G(X) = 0$ separates safe and non-safe regions in random variable X space. The probability of failure can be calculated by integrating $f(X)$ over failure domain, $G(X) < 0$. Limit state function can be defines as $G(X) = FOS(X) - 1$ for general stability problems. For rainfall-induced slope stability problem with random variables such as cohesion c', friction angle ϕ', and matric suction $(u_a - u_w)$, the limit state surface can be expressed as

$$G(c', \phi', (u_a - u_w)) = \frac{c' + \{\gamma z_w \cos^2\alpha + (u_a - u_w)\}\tan\phi'}{\gamma z \sin\alpha \cos\alpha} - 1 \tag{4}$$

In practical reliability analysis, calculation of equation (3) is very difficult, because identification of joint probability density function, $f(X)$, is hard and multiple integration over failure domain is sometimes impossible. Therefore reliability index-based approximated approach using first order moment of limit state function is adopted in this study.

As defined in equation (5a), reliability index-based approach calculates the reliability index β using mean value and standard deviation of random variables. Then a probability of failure P_f can be obtained by equation (5b).

$$\beta = \frac{\mu_G}{\sigma_G} \tag{5a}$$

$$P_f = 1 - \Phi(\beta) \tag{5a}$$

where μ_G=mean value of limit state function; σ_G= standard deviation of limit state function; and Φ=cumulative normal density function.

In order to the obtain probability of failure using equation (5), mean value and standard deviation of limit state function are needed. MVFORM (Mean Value First Order Reliability Method) is simplest and easiest method to approximate the mean value and standard deviation of limit state function. This method uses the first order moment of limit state function at mean value point of random variables to obtain the mean value and standard deviation of limit state function (Cornell, 1971). From equation (4), mean value and standard deviation of limit state function for rainfall-induced slope stability problem using MVFORM can be written as follows.

$$\mu_G = \frac{\mu_{c'} + \{\gamma z_w \cos^2\alpha + \mu_{(u_a-u_w)}\}\tan\mu_{\phi'}}{\gamma z \sin\alpha\cos\alpha} - 1 \quad \textbf{(6)}$$

$$\sigma_G^2 = \frac{\sigma_{c'}^2 + \left(\gamma z_w \cos^2\alpha + \mu_{(u_a-u_w)}\right)^2 \sec^4\sigma_{\phi'} + \sigma_{(u_a-u_w)}^2 \tan^2\mu_{\phi'}}{\gamma^2 z^2 \sin^2\alpha\cos^2\alpha} \quad \textbf{(7)}$$

where $\mu_{c'}$, $\mu_{\phi'}$, $\mu_{(u_a-u_w)}$ and $\sigma_{c'},\sigma_{\phi'},\sigma_{(u_a-u_w)}$ are mean values and standard deviations of random variables c',ϕ' and (u_a-u_w) respectively. Equation (5a) can be used to determine the reliability index β using equations (6) and (7).

$$\beta = \frac{\mu_{c'} + \{\gamma z_w \cos^2\alpha + \mu_{(u_a-u_w)}\}\tan\mu_{\phi'} - \gamma z \sin\alpha\cos\alpha}{\sqrt{\sigma_{c'}^2 + \left(\gamma z_w \cos^2\alpha + \mu_{(u_a-u_w)}\right)^2 \sec^4\sigma_{\phi'} + \sigma_{(u_a-u_w)}^2 \tan^2\mu_{\phi'}}} \quad \textbf{(8)}$$

In spite of simple calculation in MVFORM, it has some analytical limitations to lead to a lack of invariance in calculating the probability of failure P_f (Hasofer & Lind, 1974). However, in the slope reliability analysis, MVFOSM can be used because the limit state function is quite linear in the space of cohesion c' and friction angle ϕ' (Christian et al., 1994).

AFORM(Advanced First Order Reliability Method) is a complementary method to overcome a lack of invariance in MVFORM. The reliability index can be calculated as minimum distance from the mean value points to the limit state surface (Hasofer & Lind, 1974). The reliability index based on the AFORM approach can be expressed as follows.

$$\beta_{HL} = \min_{Z\in F}\sqrt{\mathbf{Z}^T\mathbf{R}^{-1}\mathbf{Z}} \quad \textbf{(9a)}$$

$$\mathbf{Z} = \frac{\mathbf{x}-\boldsymbol{\mu}}{\boldsymbol{\sigma}} \quad \textbf{(9b)}$$

where $\mathbf{Z}$=vector of standard normalized random variables, $\mathbf{x}$=vector of random variables, $\boldsymbol{\mu}$=vector of mean value, $\boldsymbol{\sigma}$=standard deviation of random variables, $\mathbf{R}$= correlation matrix, and F = failure region. The design point of limit state function $\mathbf{Z}^*$ can be defines as

$$\mathbf{Z}^* = \beta\boldsymbol{\alpha} \quad \textbf{(10)}$$

From equation (10), though $\boldsymbol{\alpha}$ is a mathematical direction cosine vector, it can be used as influence/sensitivity factor in the reliability analysis and it satisfies following condition.

$$\sum_{i=1}^{n} \alpha_i^2 = 1 \quad \textbf{(11)}$$

where α_i^2 can be interpreted as relative contribution of each random variables to the reliability index in the sensitivity analysis. The higher the value is, the more significant random variable to affect the reliability of slope is.

Applicability of Reliability Based Slope Stability Assessment with Variation of Matric Suction

The proposed reliability index-based analysis was performed in the range of measurable matric suctions to validate for real time slope stability assessment. Cohesion c', friction angle ϕ', matric suction $(u_a - u_w)$ were considered as random variables and assumed that the values have normal distribution. The COVs of cohesion and friction angle used in this study were determined referring to Fredlund & Dahlman (1971) and Phoon & Kulhawy (1999). Statistical properties used in this study are summarized in Table 1. Especially, the COV of matric suction was determined by statistical processing using measured data of weathered granite soil which is classified as loamy sand. Fig. 1 plots the matric suction data used in this study and the COV corresponding to the matric suction is shown in Fig. 2.

Table 1. Input values of parameters used in analysis

Input parameter	Mean Value	COV (%)	References
Cohesion (c')	12.0 kPa	40	Fredlund & Dahlman (1971)
Frictional angle (ϕ')	20 °	10	Phoon & Kulhawy (1999)
Matric suction $(u_a - u_w)$	0~100kPa	20~50	
Unit weight (γ)	18.816 kN/㎥		
Angle of slope (α)	45 °	-	
Depth (z)	0.45 m	-	

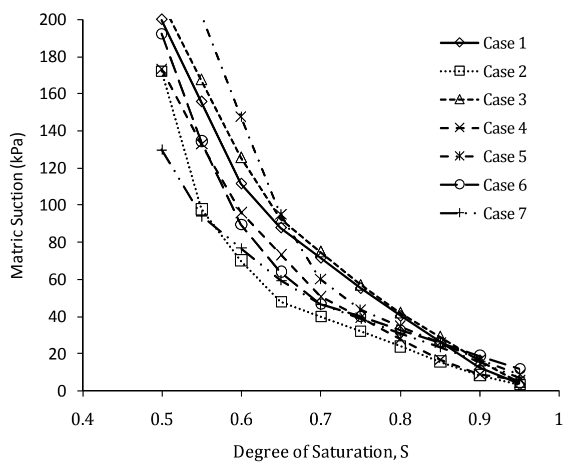

Fig. 1. SWCC used in this study

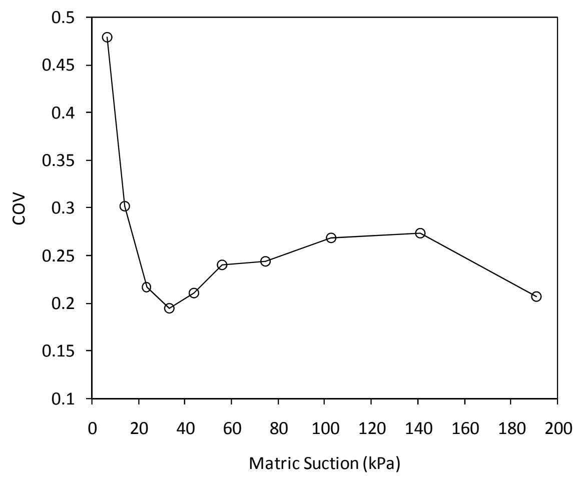

Fig. 2. COV of Matric suction variation

The reliability index calculated by the proposed method using MVFROM and AFORM is shown in Fig. 3. In low range of matric suction, MVFORM and AFROM analyze result in similar reliability index but at high suction range, MVFORM give lower reliability index than that of AFORM. These results were compared with MCSM (Monte Calro Simulation Method) to verify the accuracy of the proposed method. To conduct the MCSM analysis, random variables and statistical properties were considered as those of MVFORM and AFORM. 1,000,000 trials with latin hypercube sampling were performed for accurate analysis. The results show that AFORM give comparatively exact results in most range. However, MCSM failed to calculate the reliability index between 30 to 50 kPa. When we consider the efficiency for real-time assessment, this result indicates that MCSM is not suitable for real time slope stability assessment.

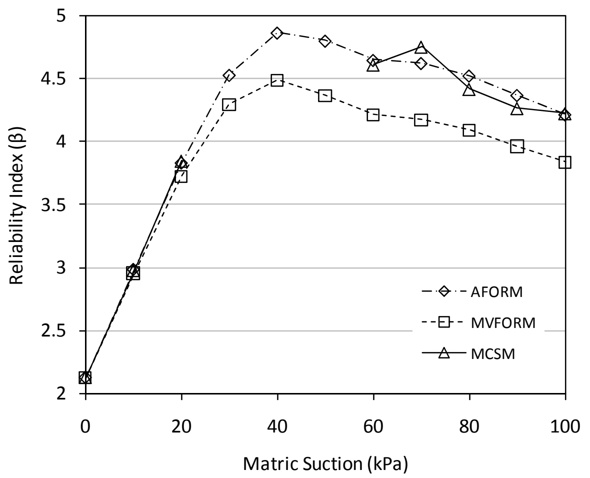

Fig. 3. Comparison of AFORM and MVFORM results with MCSM results

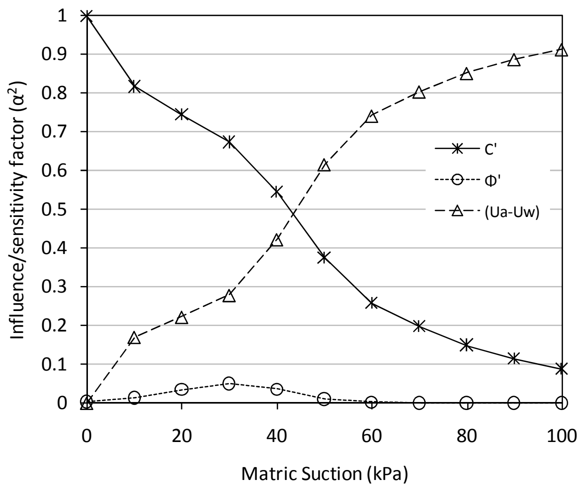

Fig. 4. Results of sensitivity analysis according to matric suction variation

To identify the contribution of each random variable to reliability index, sensitivity analysis using α_i^2 in equation (10) was performed. The results are represented in Fig. 4. In the lower matric suction range, cohesion c' is the most significant influence factor. However, the influence of matric suction $(u_a - u_w)$ becomes more important in high matric suction range. This result explains why the MVFORM results are lower at high suction range. It is because the influence of nonlinearity in matric suction term included in equation (4) becomes larger at high suction. It means that AFORM is more suitable than MVFORM in considering the matric suction variation.

Field Application

A model slope was built in a test bed with 2m height and 45^o slope angle to apply the suggested method. The rainfall amount was measured by tipping bucket rain gage produced by CAMPBELL SCIENTIFIC. They recorded rainfall events from 18th of March to 21st of April in 2008 at interval of every 10minutes. Measured rainfall amount is shown in figure 5. During the monitoring, total rainfall amount was 45.47mm and the biggest rainfall happened on March 22nd, 23.88mm of rainfall for 28hours.

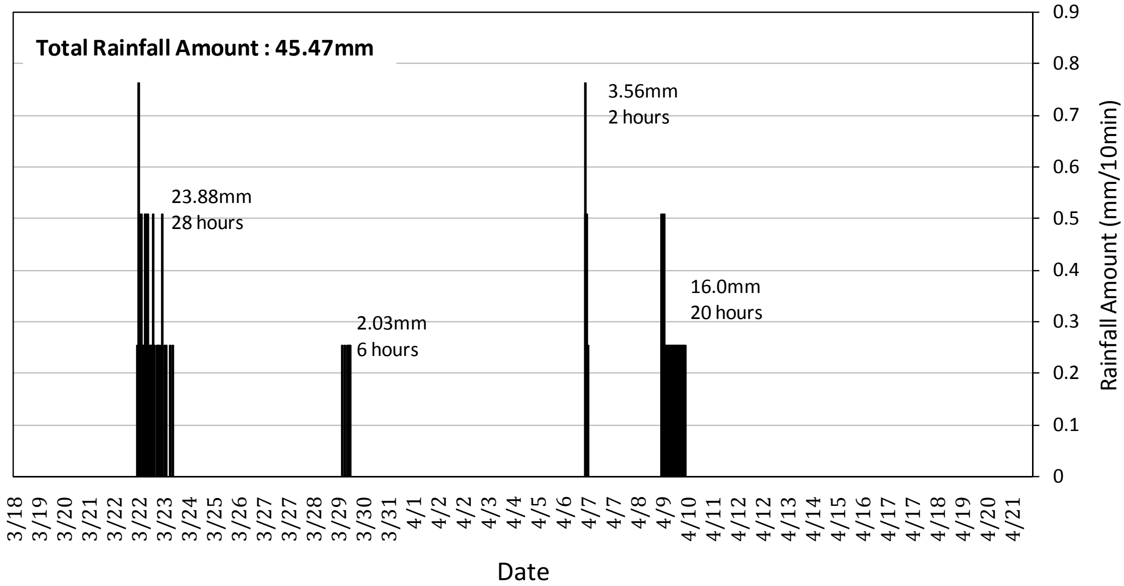

Fig. 5. Rainfall amount during monitoring

The matric suction variation was measured by tensiometers which are installed at 0.3, 0.45 and 0.6m depths in the model slope. The data were statically measured every 10 minute about for 1 month. The measured matric suction values were interpreted by 2D image plot using interpolation and extrapolation techniques as shown in fig. 6. The matric suction dropped suddenly after the rainfall began and at the surface of slope, positive pore water pressure was developed by the rainfall infiltration. After the rainfall stopped, the soil water was desiccated at the surface but some of it infiltrated into the deeper area. It means the deeper soil can have lower shear strength of soil even after the rainfall stops. The variation of matric suction at relatively deeper depth were not sensitive than that of at the surface.

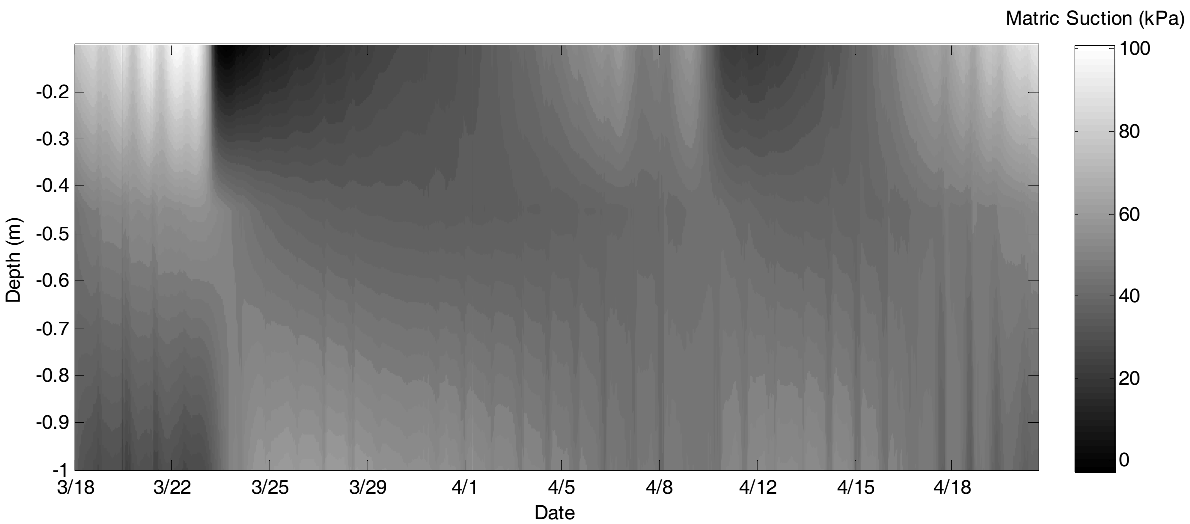

Fig. 6. Measured matric suction distribution during monitoring

The reliability index of slope considering the measured matric suction was evaluated by the proposed AFORM approach. Cohesion c', internal friction angle ϕ', and matric suction $(u_a - u_w)$ were considered as random variables as summarized in Table 1. The measured matric suction was also used to calculate the COV which is shown in fig. 2. Fig. 7 plots the calculated reliability index during monitoring. The deeper the depth is, the lower the reliability index is before the rainfall begins. However, the reliability index at the surface decreased dramatically to 2.06 during the rainfall. It is corresponding to 2% of probability for failure. It also recovers suddenly at the surface, but at relatively deep depth, it recovers in a slower rate than that at the surface. This can be explained by that the matric suction variation is the most significant factor to affect the reliability index as previously explained in the sensitivity analysis results (fig. 4) and the change of matric suction at surface is larger than that at deeper depth. According to this result, control of rainfall infiltration into slope would be the most effective way of increasing the stability. The above analysis results indicate that the probability of failure at every depth with time can be effectively quantified by the suggested method.

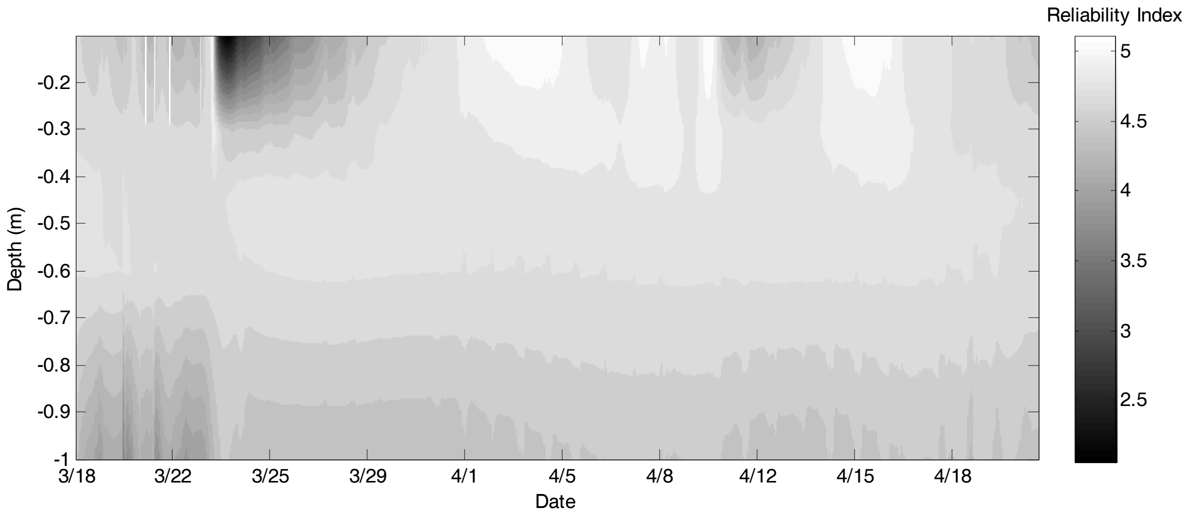

Fig. 7. Reliability index distribution during monitoring

Summary and Conclusions

A reliability based slope assessment method considering the matric suction variation was proposed in this study for real-time slope risk identification. MVFORM and AFORM based slope reliability analysis methods were adopted. The result shows that MVFORM gives lower reliability index value than AFORM. This is caused by the nonlinearity of matric suction term in the limit state function. The result of sensitivity analysis also indicates that cohesion c' is a significant variable at low matric suction value but matric suction $(u_a - u_w)$ is the most influencing factor at high suction range. Importance of friction angle ϕ' is estimated relatively low. The proposed method was applied to a model slope and the matric suction values were measured every 10minutes about for 1 month. The analysis result shows that the reliability index at surface changes a lot and it becomes small during the rainfall. The application results validate that the reliability-based slope assessment method considering the matric suction variation was capable of quantifying the slope risk and hence it can be an effective technique for slope monitoring and maintenance.

In this study, the applicability of reliability based slope stability assessment method considering the matric suction variation was presented. However, the reliability base slope stability criteria and statistical properties of soil slope should be further studied for practical reliability based slope maintenance system.

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