Uniqueness Problem in Sound Field Reproduction
음장 재현에서의 유일성 문제

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ABSTRACT

This paper deals with a means to reproduce sound field by using Kirchhoff-Helmholtz integral equation. We control boundary value or generate sound sources on the boundary in order to control the sound field as we want. The method assumes that there is a unique relation between sound field and its boundary should. Otherwise the reproduced sound field is different from what we want generate; the original sound field. Half-infinite sound field and finite sound field are considered and whether the uniqueness is hold or not and how the reproduced field is generated are discussed in each case.

1. Introduction

Sound field reproduction based on Kirchhoff-Helmholtz integral equation is a method to generate sound field that we want by controlling boundary values, pressure or velocity. This method is divided into two types according to a point of view for each term of Kirchhoff-Helmholtz integral equation. In the first type, Green's function of free field is used and Green's function and its derivative are regarded as monopole and dipole. And boundary pressure and velocity are regarded as source input signals. So loudspeakers are located on the boundary as monopoles or dipoles. WFS(Wave Field Synthesis)[1] is an example of this type. In the other hand, in the second type, Green's function and its derivative are regarded as terms indicating propagation of boundary values and not needed to be controlled. Instead, boundary pressure and velocity are controlled by loudspeakers located outside the sound field. BSC(Boundary Surface Control)[2-3] is an example.

The Kirchhoff-Helmholtz integral equation has two terms of pressure and velocity on boundary. But some researchers[4-6] mention that two terms are not independent and ignore one of them. Or some other researchers[2-3] try to consider one term only to reduce the effort of controlling and measuring. But it can induce non-uniqueness problem.

So the objectives of this paper are to show whether sound field is reproduced uniquely by only one term and how the reproduced field is generated if uniqueness is not satisfied. It is shown mathematically and computationally.

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2. Problem Definition

2.1 The original field and the reproduced field

Left one of Fig.1 shows the original field. It is a sound field generated by original sources and the target of reproduction. The $\vec{r}$ indicates a position inside the sound field and the $\vec{r}_0$ indicates a position on the boundary. The $P(\vec{r})$ and $P(\vec{r}_0)$ indicate the complex acoustic pressure of frequency f. All description in the paper is written in frequency domain.

Right one of Fig.1 shows the reproduced field that is generated by control sources and the result of reproduction. The pressure in the reproduced field and the boundary are denoted as $\hat{P}(\vec{r})$ and $\hat{P}(\vec{r}_0)$. Source inputs that are fed into control sources are denoted as Q_1, Q_2, etc.

In most cases of sound field reproduction problem, only homogeneous field is considered. It means that there is no sound source inside the field and all sound sources are located outside. It is because sound field reproduction aims to generate a sound field for listening.

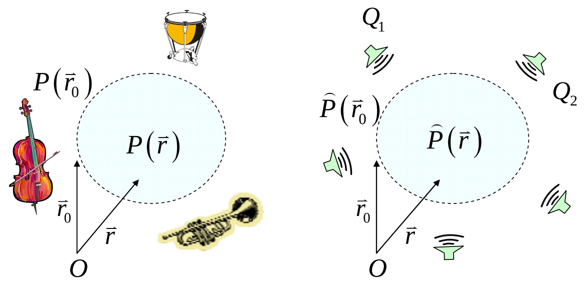

Fig. 1 Original field(left) and reproduced field(right)

2.2 Kirchhoff-Helmholtz integral equation

The Kirchhoff-Helmholtz integral equation in the threedimension is expressed as[7]

$$P(\vec{r}) = \int_{S_0} \left[G(\vec{r}|\vec{r}_0) \frac{\partial P}{\partial n}\bigg|_{(\vec{r}=\vec{r}_0)} - P(\vec{r}_0) \frac{\partial G(\vec{r}|\vec{r}_0)}{\partial n} \right] dS_0 \quad (1)$$

where $\vec{r}$ indicates a position of the field and $\vec{r}_0$ indicates a position on its boundary. All description is written in frequency domain. $G(\vec{r}|\vec{r}_0)$ is the Green's function that is defined as a function that satisfies the following inhomogeneous wave equation;

$$\nabla^2 G(\vec{r}|\vec{r}_0) + k^2 G(\vec{r}|\vec{r}_0) = -\delta(\vec{r}-\vec{r}), \quad (2)$$

where k is wave number. The particular solution of this equation that is usually called by free-field Green's function is

$$G_F(\vec{r}|\vec{r}_0) = \frac{e^{jkR}}{4\pi R}, \quad R = |\vec{r}-\vec{r}_0|. \quad (3)$$

It can be interpreted as a field by a monopole source which is located at the position $\vec{r}_0$. And its derivative can be interpreted as a field by a dipole source which is located at the position $\vec{r}_0$.

And two types of sound field are considered in this paper; half-infinite field and finite field. Left and right figure of Fig.2 show them respectively. It is well known that the uniqueness between boundary pressure and sound field is hold in half-infinite field and not hold in finite field case.

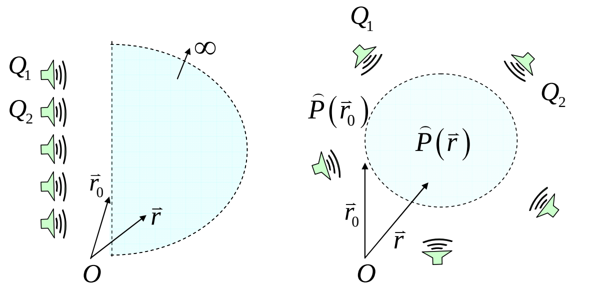

Fig. 2 Half-infinite field(left) and finite field(right)

3. Half-infinite Field Case

3.1 The Dirichlet Green's function

In the half-infinite field case, the boundary surface is composed of infinite planar boundary and half-infinite boundary. On the half-infinite boundary surface, pressure and velocity are assumed to be zero so that only planar boundary is considered.

And we can use the Green's function that satisfies Dirichlet boundary condition and remove first term in Eqn. (1) as follows;

$$P(\vec{r}) = \int_{S_0} \left[-P(\vec{r}_0) \frac{\partial G_D(\vec{r}|\vec{r}_0)}{\partial n} \right] dS_0, \quad (4)$$

where $G_D(\vec{r}|\vec{r}_0)$ is the Green's function that satisfies Dirichlet boundary condition. It is determined by

$$G_D(\vec{r}|\vec{r}_0) = G_F(\vec{r}|\vec{r}_0) - G_F(\vec{r}|\vec{r}_0') \quad (5)$$

where $\vec{r}_0'$ is the position that is symmetric to $\vec{r}_0$ as shown in Fig.3. Hence $G_D(\vec{r}|\vec{r}_0)$ has always finite value unless $\vec{r} = \vec{r}_0$.

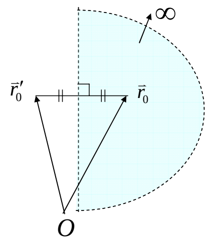

Fig. 3 Dirichlet Green's function

3.2 The first and second type of reproduction

When $\vec{r}_0$ is on the planar boundary, the Dirichlet Green's function is twice of the free-field Green's function. Therefore the derivative of the Dirichlet Green's function can indicate a dipole source. In that case, the pressure is regarded as source input signal. That is, we can reproduce the original field by using loudspeakers which are located on the planar boundary surface.

In the other hand, we can use loudspeakers as control sources which match the pressure by loudspeakers located in the outside of the sound field. It is based on the fact that the sound field is determined uniquely by the boundary pressure. We can reproduce it in this way as well.

4. Finite Field Case

4.1 The Dirichlet Green's function[10]

In the finite field case, we have to consider the entire boundary surface. And we can use the Green's function that satisfies Dirichlet boundary condition and remove first term in Eqn. (1) as in half-infinite field case but Dirichlet Green's function $G_D(\vec{r}|\vec{r}_0)$ does not have a finite value always.

$G_D(\vec{r}|\vec{r}_0)$ is obtained as the sum of free-field Green's function and a certain homogeneous solution:

$$G_D(\vec{r}|\vec{r}_0) = G_F(\vec{r}|\vec{r}_0) + G_h(\vec{r}). \quad (6)$$

On the boundary surface it is zero. So $G_h(\vec{r})$ is obtained differently according to the shape of the boundary. For example, in the case of sphere whose radius is a, $G_D(\vec{r}|\vec{r}_0)$ is obtained as

$$G_D\left(\vec{r}\middle|\vec{r}_0\right) = ik\sum_{n=0}^{\infty}\sum_{m=-n}^{n}\frac{j_n\left(kr_<\right)}{j_n\left(ka\right)}\left[h_n\left(kr_>\right)j_n\left(ka\right)-h_n\left(ka\right)j_n\left(kr_>\right)\right] \times Y_n^m\left(\theta_0,\phi_0\right)^* Y_n^m\left(\theta,\phi\right), \tag{7}$$

where j_n is spherical Bessel function, $r_<$ and $r_>$ are the smaller one and bigger one respectively among r and r_0. h_n is the first kind of spherical Hankel function and $Y_n^m\left(\theta,\phi\right)$ is spherical harmonics which is defined as

$$Y_n^m\left(\theta,\phi\right) \equiv \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}}P_n^m\left(\cos\theta\right)e^{im\phi}. \tag{8}$$

In the Eqn. (7), $j_n\left(ka\right)$ can be zero, and in that case, $G_D\left(\vec{r}\middle|\vec{r}_0\right)$ goes to infinity. It means that Eqn. (1) cannot be reduced into Eqn. (4).

The frequencies at which the Dirichlet Green's function $G_D\left(\vec{r}\middle|\vec{r}_0\right)$ goes to infinity is called by 'forbidden frequencies' because sound field can not be obtained by the boundary pressure at these frequencies[10].

4.2 The first and second type of reproduction

The Dirichlet Green's function can be expressed as a response of the sound field by a delta excitation at $\vec{r}_0$. So it does not represent a propagating wave like monopoles or dipoles. Therefore the first type of reproduction can not be applied in this form. To apply the first type of reproduction, we should use free-field Green's function and it means that both monopoles and dipoles are necessary.

In the second type of reproduction, if the Dirichlet Green's function has a finite value, we can reproduce the sound field by matching pressure on boundary because the boundary pressure determines sound field uniquely. But if the Dirichlet Green's function goes to infinity, that is, at the forbidden frequencies, we cannot reproduce in that way. It means that we need velocity information on boundary.

5. Reproduction at the forbidden Frequencies

5.1 Reproduction by matching boundary pressure on the boundary surface

Let's consider a spherical sound field whose radius is 0.5m. Loudspeakers are located around the sound field on the boundary of the sphere of radius 0.7m and assumed to be monopoles. Original field is generated by a monopole source located at (-3,0,0), it is expressed as

$$p_M\left(\vec{r}\right) = \frac{e^{jkR}}{R}, \quad R = \sqrt{\left(x+3\right)^2+y^2+z^2}. \tag{9}$$

And we discretize the entire surface for 130 points on it to have equal distance[9], and use the same number of loudspeakers.

Since the radius is 0.5m, forbidden frequencies are obtained to be frequencies where $j_n\left(0.5k\right)$ is zero. Denote the values of the argument of j_n that cause it to equal zero as ζ_{nl}. These values are known, for example, $\zeta_{01}=3.14$, $\zeta_{11}=4.49$, $\zeta_{21}=5.76$, etc[8]. Hence forbidden frequencies are 343Hz, 490Hz, 629Hz, etc.

As an example of a frequency which is not a forbidden frequency, at 500Hz, the original field and reproduced field are illustrated in Fig.4 and Fig.5, respectively. Left figure represents magnitude and right figure phase on the x-y plane where z=0. The smaller circle indicates section on the x-y plane of the interesting sound field and the bigger circle indicates section of the spherical surface where loudspeakers are located. With the pressure information on the boundary in the original field(Fig.4), we generate the reproduced field by loudspeakers(Fig.5). We can see that two sound fields are almost same inside of the small circle.

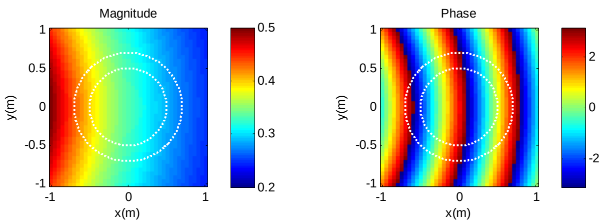

Fig. 4 Original field by monopole source (500Hz)

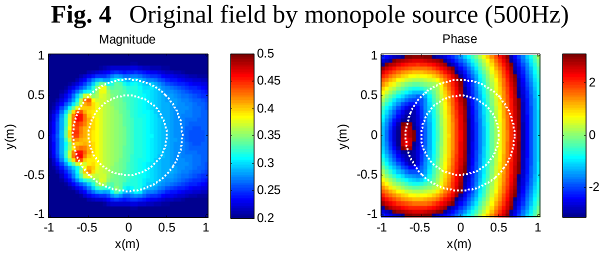

Fig. 5 Reproduced field by loudspeakers (500Hz)

But at a forbidden frequency, 343Hz, the original field and reproduced field are illustrated in Fig.6 and Fig.7, respectively. We can see that the reproduced field is different from the original one even if the boundary pressure values are identical in original field and reproduced field. As mentioned before, it is because boundary pressure doesn't determine sound field uniquely.

As frequency goes higher, the number of forbidden frequencies increases. So it can induce a fatal problem in this reproducing method.

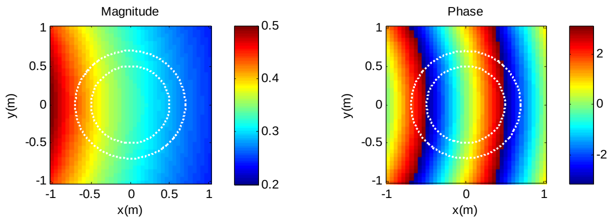

Fig. 6 Original field by monopole source (343Hz)

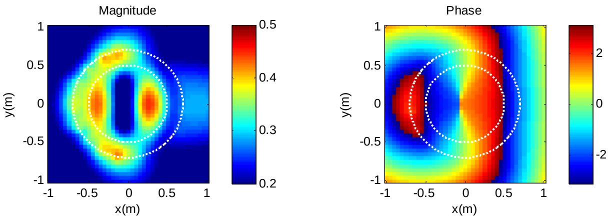

Fig. 7 Reproduced field by loudspeakers (343Hz)

5.2 Discussion the reproduction results at the forbidden frequencies

Fig.8 shows magnitude and phase of the difference between original field(Fig.6) and reproduced field(Fig.7) at 343Hz. We can see that the shape inside the small circle is (0,1) mode shape. It means that the mode is not reproduced. In other words, original field can be expressed by sum of spatial mode. But modes that have zero values on the boundary are not sampled by measuring the boundary pressure, therefore, not reproduced by loudspeakers.

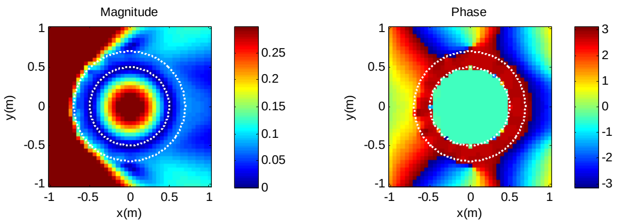

Fig. 8 Difference between original and reproduced field (343Hz)

6. Conclusion

Sound field reproduction based on Kirchhoff-Helmholtz integral equation use the relation between pressure or velocity information on the boundary and sound field.

In the half-infinite field case, we can reduce a term in K-H integral equation by using Dirichlet Green's function. The derivative of the Dirichlet Green's function has dipole form; it means that the field can be reproduced by only dipole sources. And the Dirichlet Green's function has always finite value, and pressure on the boundary determines sound field uniquely; it means that the field can be reproduced by matching only boundary pressure.

But in the finite field case, Dirichlet Green's function does not represent a propagating wave. So it cannot be considered as loudspeakers. It means that both monopole and dipole sources are needed. Also it diverges at forbidden frequencies so that boundary pressure cannot determine sound field. It means that the field cannot be reproduced by matching only boundary pressure.

The simulation results show that reproduced field is different from original field at the forbidden frequencies. And the difference is identical to mode shape, it means that the mode is not sampled or reproduced.

In conclusion, in the sound field reproduction of finite sound field, both monopoles and dipoles should be used(the first type) and boundary pressure and velocity should be sampled and matched(the second type).

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