Regional Control of Vibration

진동의 영역 제어

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ABSTRACT

Generally, a linear vibration theory regards a vibratory system as the superposition of many degrees of vibratory system. Modal analysis stems, in fact, considers the vibration system as what has input, output, and transfer function that relates the input and output. When we want to control, however, the vibratory system, we define, first, the object function that can be vibration energy of certain vibratory system. Then, we try to find the transfer function that can minimize the object function. We can readily extend this approach to control the distributed vibration system. For example, the vibrations of a vehicle, including ships and trains. In this case, we may want to minimize the vibration of the area we select. For example, minimize the vibration of the passengers' seat, but allowing the vibration of other area; for example engines and wheels. This paper introduces a general theory that can control the vibration of the selected area, which can be called as "regional control of vibration." In fact, this is the extended theory of well known sound control of "bright zone"(Choi and Kim, 2002).]. Several illustrative examples demonstrate the applicability and properties that are not available if we use modal analysis method.

1. INTRODUCTION AND OBJECTIVES

The vibration control has been mainly focused on suppressing or, sometimes, amplifying the magnitude of displacement, velocity, or acceleration of the vibration of one or several vibratory system. For example, we often want to minimize the vibration of a seat or seats of passenger car, by controlling the sources of vibration or their related transmission paths. This means that the relations between the target vibration or vibrations and the vibration sources are to be considered to have linear association between them: amplitude and phase relations. It is also common to see the relations in a matrix form, which generally gives us modal matrix. It essentially expresses the amplitude and phase relation between each excitation to the out put responses in frequency domain.

One may extend these linear relationships to more general contents. For example, we may want to suppress or magnify the vibration of several vibratory systems provided that we have given excitation or vibratory systems. Then our objective is to make some vibratory system vibrates with the "wanted" amplitude, velocity, or acceleration, but others do with regard to the values we specify. Then the vibration problem is to be a integral, or summation form rather than what we have seen in modal domain approach. One may also wants to make the vibration level different between the elements; one group makes big vibration and the other group does small one. This leads us, then, to have a maximization or minimization problem between the quantities that we specify. For example, maximizing the ratio of the vibration energy between the elements that we select is the vibration energy contrast problem.

We, first, introduce the theoretical formulation that finds the control parameters; their magnitude and phase relationships that can minimize the vibrations of selected vibratory elements by keeping the power of the control sources, which can be also other vibratory elements. Next addresses the way to maximize the vibration energy ratio between the groups of vibratory systems. That can be regarded as maximization problem between two vibratory systems. In fact, this theoretical formulation essentially stems from the theory proposed by Choi and Kim, which handles acoustically bright and dark zone control.

2. PROBLEM DEFINITION

A vibratory system is composed of several vibratory elements, which are affected by one another. To control the vibration of the elements, control sources are introduced. Let us assume a vibratory system as shown in Fig. 1. It has M vibratory elements, v_1, v_2, …, v_M, which have each level, $X_1(\omega)$, $X_2(\omega)$, …, $X_M(\omega)$ and N control sources, s_1, s_2, …, s_N, with the levels, $Q_1(\omega)$, $Q_2(\omega)$, …, $Q_N(\omega)$. Hence the level of m-th element $X_m(\omega)$ can be expressed as

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$$X_m(\omega) = \sum_{i=1}^{N} G_{mi}(\omega) Q_i(\omega) + \sum_{j=1}^{M} T_{mj}(\omega) X_j(\omega) + C_m \quad (1)$$

where $G_{mi}(\omega)$ indicates the relation between m-th element level and i-th source level and $T_{mi}(\omega)$ shows interactions by the other vibratory elements. The vibratory element vibrates with the constant level $C_m(\omega)$ by driving excitations.

By using vector expressions and omitting ω, Eq. (1) can be rewritten as

$$X_m = \mathbf{G}_{m\bullet}\mathbf{Q} + \mathbf{T}_{m\bullet}\mathbf{X} + C_m \quad (2)$$

where

$$\begin{aligned} \mathbf{G}_{m\bullet} &= \begin{bmatrix} G_{m1} & \cdots & G_{mN} \end{bmatrix} \\ \mathbf{Q} &= \begin{bmatrix} Q_1 & \cdots & Q_N \end{bmatrix}^T \\ \mathbf{T}_{m\bullet} &= \begin{bmatrix} T_{m1} & \cdots & T_{mM} \end{bmatrix} \\ \mathbf{X} &= \begin{bmatrix} X_1 & \cdots & X_M \end{bmatrix}^T \end{aligned} .$$

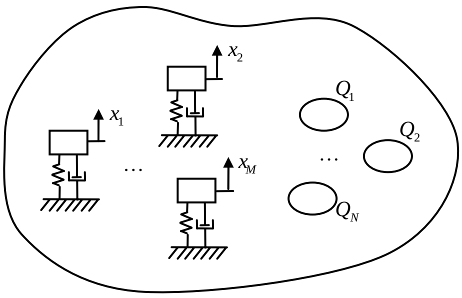

Fig.1. A vibratory system

When we treat all elements of system, the matrix form of Eq. (2) is

$$\mathbf{X} = \mathbf{GQ} + \mathbf{TX} + \mathbf{C} \quad (3)$$

Moving the second term of RHS to LHS and introducing $\tilde{\mathbf{G}}$ and $\tilde{\mathbf{C}}$, we have

$$\begin{aligned} \mathbf{X} &= (\mathbf{I} - \mathbf{T})^{-1}(\mathbf{GQ} + \mathbf{C}) \\ &= \tilde{\mathbf{G}}\mathbf{Q} + \tilde{\mathbf{C}} \end{aligned} . \quad (4)$$

If we want to suppress the vibrations of some areas or magnify those of others, then we can achieve it by minimizing the vibration energy of elements in the region requiring small vibration or by maximizing the levels of elements in the region requiring big vibration. So we can call the regions as 'dark zone' and 'bright zone' as in the work of Choi & Kim(2002). Then let us denote the subscripts of the elements in 'dark zone' as $m = D_1, D_2, \cdots$, and those in 'bright zone' as $m = B_1, B_2, \cdots$. Then we can express the former $\mathbf{X}_D$ and the latter $\mathbf{X}_B$ as follows.

$$\begin{aligned} \mathbf{X}_D &= \begin{bmatrix} X_{D_1} & X_{D_2} & \cdots \end{bmatrix}^T \\ \mathbf{X}_B &= \begin{bmatrix} X_{B_1} & X_{B_2} & \cdots \end{bmatrix}^T , \end{aligned} \quad (5)$$

where D indicates 'dark' and B does 'bright'. And $\mathbf{X}_D$ is expressed as

$$\mathbf{X}_D = \tilde{\mathbf{G}}_D\mathbf{Q} + \tilde{\mathbf{C}}_D . \quad (6)$$

The vibration energy of 'the dark elements' is proportional to $\mathbf{X}_D{}^H\mathbf{X}_D$, so that we define e_D as,

$$\begin{aligned} e_D &= \mathbf{X}_D{}^H\mathbf{X}_D \\ &= \left(\tilde{\mathbf{G}}_D\mathbf{Q} + \tilde{\mathbf{C}}_D\right)^H \left(\tilde{\mathbf{G}}_D\mathbf{Q} + \tilde{\mathbf{C}}_D\right) \\ &= \mathbf{Q}^H\tilde{\mathbf{G}}_D{}^H\tilde{\mathbf{G}}_D\mathbf{Q} + \mathbf{Q}^H\tilde{\mathbf{G}}_D{}^H\tilde{\mathbf{C}}_D + \tilde{\mathbf{C}}_D{}^H\tilde{\mathbf{G}}_D\mathbf{Q} + \tilde{\mathbf{C}}_D{}^H\tilde{\mathbf{C}}_D . \end{aligned} \quad (7)$$

From the same procedure, the vibration energy of the bright elements e_B is obtained as follows:

$$e_B = \mathbf{Q}^H\tilde{\mathbf{G}}_B{}^H\tilde{\mathbf{G}}_B\mathbf{Q} + \mathbf{Q}^H\tilde{\mathbf{G}}_B{}^H\tilde{\mathbf{C}}_B + \tilde{\mathbf{C}}_B{}^H\tilde{\mathbf{G}}_B\mathbf{Q} + \tilde{\mathbf{C}}_B{}^H\tilde{\mathbf{C}}_B . \quad (8)$$

We can think three problems here; One is to minimize the energy of dark elements e_D, another is to maximize the energy of bright elements e_B, the other is to maximize the ratio of e_B and e_D. But generally in vibration problems, brightness control is not in use.

3. SOLUTION METHOD

3.1 Darkness Control

The problem to minimize e_D is mathematically equal to that of active noise control by Nelson & Elliott(1995). The energy of dark zone e_D is real value. If $\tilde{\mathbf{G}}_D{}^H\tilde{\mathbf{G}}_D$ is positive definite, e_D has a unique global minimum. In case that $D = N$, the solution is obtained as

$$\begin{aligned} \mathbf{Q} &= -\left(\tilde{\mathbf{G}}_D{}^H\tilde{\mathbf{G}}_D\right)^{-1}\tilde{\mathbf{G}}_D{}^H\tilde{\mathbf{C}}_D \\ &= -\tilde{\mathbf{G}}_D{}^{-1}\tilde{\mathbf{C}}_D \end{aligned} \quad (9)$$

In case that $D > N$ or $N < D$, the solution is obtained as follows:

$$\mathbf{Q} = -\left(\tilde{\mathbf{G}}_D{}^H\tilde{\mathbf{G}}_D\right)^{-1}\tilde{\mathbf{G}}_D{}^H\tilde{\mathbf{C}}_D \quad (10)$$

3.2 Contrast Control

There are some regions where high level of vibration is acceptable, and we don't need to control vibration there. But also we can choose the solution that makes more vibration in the regions as long as it is effective for suppressing vibration in other regions. In this case, we can select 'contrast control'. The contrast β is defined as

$$\beta = \frac{e_B}{e_D} = \frac{\mathbf{X}_B{}^H \mathbf{X}_B}{\mathbf{X}_D{}^H \mathbf{X}_D} \tag{12}$$

The cost function J of this problem is defined as

$$J = e_b + \beta\left(J_0 - e_D\right) \tag{13}$$

and $\mathbf{q}_C$ that maximize the cost function J will be obtained as the solution of contrast control. (Details are not dealt with in this paper.)

4. SIMPLE CASES

To evaluate the formulations, let us consider simple cases of darkness control that we already know the solution. But it is noteworthy that the formulations can be applied not only to these simple cases but also to complicated systems and distributed parameter systems.

4.1 One damped simple oscillator and one control source

Fig.2. illustrate the vibratory system. External force $f\left(t\right)$ are applied to damped simple oscillator, and control source $q(t)$ suppress vibratory system. To minimize the energy of vibration, we have to express $X\left(\omega\right)$ in the form of Eq. (4). $C\left(\omega\right)$ is the level of vibration in the frequency domain by only the force $f\left(t\right)$, which is obtained form the following equation:

$$m\ddot{c}\left(t\right) + b\dot{c}\left(t\right) + kc\left(t\right) = f\left(t\right). \tag{14}$$

Letting $c(t) = C(\omega)e^{j\omega t}$, we get

$$C\left(\omega\right) = \frac{F\left(\omega\right)}{k - m\omega^2 + j\omega b}. \tag{15}$$

And $G\left(\omega\right)$ is the response to the level by the force of unity:

$$G\left(\omega\right) = \frac{1}{k - m\omega^2 + j\omega b}. \tag{16}$$

Finally the solution $Q\left(\omega\right)$ that minimizes the vibration energy is obtained as

$$Q\left(\omega\right) = -F\left(\omega\right). \tag{17}$$

It means that $Q\left(\omega\right)$ and $F\left(\omega\right)$ have same magnitude and out of phase so that the resultant force is zero. It is a natural result.

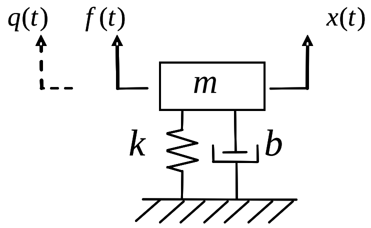

Fig.2. case 1

4.2 Two damped simple oscillators(coupled) and one control source

Let us consider another system illustrated in Fig. 3. In this case, we want to suppress vibration of m_2 by applying control source to it, when the external forces initially excite each element. From the following equations of motion

$$(k_1 - m_1\omega^2 + j\omega b_1)X_1 = (k_1 + j\omega b_1)X_2 + F_1$$
$$\left\{(k_1 + k_2 - m_2\omega^2) + j\omega(b_1 + b_2)\right\}X_2 = Q_2 + (k_1 + j\omega b_1)X_1 + F_2, \tag{18}$$

we know each term of Eq. (3). According to the objective of this problem, Eq. (6) becomes

$$X_2 = \tilde{G}_{22}Q_2 + \tilde{C}_2, \tag{19}$$

where

$$\tilde{G}_{22} = \left(k_1 - \omega^2 m_1 + j\omega b_1\right)[(k_1 - \omega^2 m_1 + j\omega b_1)\{k_1 + k_2 - \omega^2 m_2 + j\omega(b_1 + b_2)\} - (k_1 + j\omega b_1)^2]^{-1} \tag{20}$$

$$\tilde{C}_2 = \left\{(k_1 + j\omega b_1)F_1 + (k_1 - \omega^2 m_1 + j\omega b_1)F_2\right\}[(k_1 - \omega^2 m_1 + j\omega b_1)\{k_1 + k_2 - \omega^2 m_2 + j\omega(b_1 + b_2)\} - (k_1 + j\omega b_1)^2]^{-1} \tag{21}$$

Finally the solution of darkness control is

$$Q_2 = -\frac{k_1 + j\omega b_1}{k_1 - \omega^2 m_1 + j\omega b_1} F_1 - F_2 . \qquad (22)$$

Therefore, the response of the vibratory system to above control source is

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} \dfrac{F_1}{k_1 - \omega^2 m_1 + j\omega b_1} \\ 0 \end{bmatrix} . \qquad (23)$$

Comparing Eq.(23) with Eq.(21), we can know that vibratory element m_2 is suppressed by using 'darkness control'.

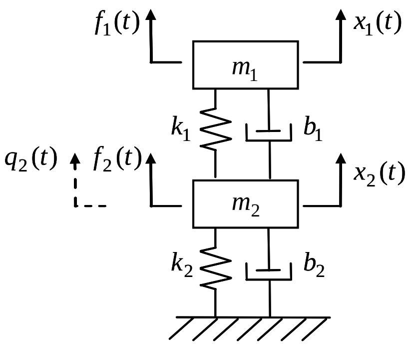

Fig.3. case 2

5. CONCLUSION

We have introduced a general theory that can control the vibration of the selected area, and applied it to simple cases: 1, and 2-DOF vibratory system. In each case, we used equations of motion in matrix form. To minimize vibration energy, we obtained the level of control source from 'darkness control'. According to results, we knew that solutions were natural.

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