질량 변화에 따른 Lumped Mass Beam Model 의 이론적 동특성 규명

Theoretical Approach; Identification of Dynamic Characteristics for Lumped Mass Beam Model due to Changes of Mass

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ABSTRACT

This paper predicts the changes of natural frequencies due to the changes of mass at different point mass stations by using iterative calculation Transfer Matrices Method for different boundary conditions of a single beam structure (fixed-free and fixed-fixed beam). Firstly, the first three natural frequencies of an original beam are obtained using Transfer Matrices Method to verify the accuracy of the obtained results. The results are then compared with the exact solutions before purposely changing the parameter of mass. Both beams are modeled as discrete continuous systems with six-lumped-mass system. A single beam is broken down into a point mass and a massless beam which represent a single station and expressed in matrix form. The assembled matrices are used to determine the value of natural frequencies using numerical interpolation method corresponding to their mode number by manipulating some elements in the assembled matrix.

1. Introduction

The mass, stiffness, and damping properties of a structure determine its dynamic characteristics in term of natural frequencies and their corresponding modes. Structural modification is the process whereby desired dynamic behavior is obtained by changing the parameter of mass, stiffness or damping properties. In this paper, the dynamic characteristics of a single beam are changed by simply adding more, and removing the existing mass element at different point mass station by considering the two different boundary conditions of a single beam structure (fixed-free and fixed-fixed beam)

Jung-Youn Lee [(1)] investigated the changes of dynamic characteristics of a gamma-shaped-beam-element-structure by using Transfer Matrices Method and compared the obtained result through experiment. In the following years, Jung-Youn Lee [(2)] proposed a numerical calculation method by considering the amount of change of generalized mass based on variation of lumped masses to predict the new dynamic characteristics of a single fixed-free beam due to mass modification. These researches explained that, by considering the changes of parameter of the original structure, the new dynamic characteristics in term of natural frequencies can be predicted. By detailed mathematical understanding of the shifting natural frequencies of a structure due to structure modification, we are able to design a structure based on certain dynamic characteristic requirements. For example, Dmitri [(3)] had proposed a numerical calculation method to design a mechanical structure based on certain dynamic characteristics by changing the mass and stiffness of the original structure which led to achieve the desired natural frequencies.

Examples given above describe the application of changing some parameter of a structure and the methods used to predict the new dynamic characteristics due to its structure modification. However, none of them considered the most optimal location where the mass of the structure can be modified. In real application, some structural modification requires the changes of mass of original structure but at less modified point. In this paper, as a preliminary study, two boundary conditions of beams are considered and a single beam is divided into six stations which consist of point mass and massless beam. Each mass of point masses is purposely increased and reduced in order to investigate how this mass parameter modification at different point mass influences the dynamic characteristics of the beam.

2. Theory

As mentioned earlier, a single beam can be expressed as lumped mass model which consists of a number of stations. Each station is a combination of a point mass and massless beam. Figure 1 shows a single beam which is modeled as 6 lumped mass model.

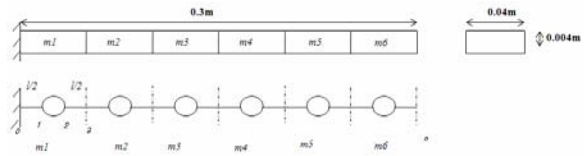

Fig. 1.Six Lumped Mass Beam Model

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The total mass of the beam is equally distributed in the lumped mass beam model. Considering the free body diagram, slope and deflection of both elements, 4-by-4 matrices can be formed as presented in eq. (1) ,(2),and(3).

Massless Beam Matrix(0-1)

$$\begin{pmatrix} Y_1 \\ \theta_1 \\ M_1 \\ V_1 \end{pmatrix} = \begin{bmatrix} 1 & \frac{l}{2} & \frac{l^2}{8EI} & \frac{l^3}{48EI} \\ 0 & 1 & \frac{l}{2EI} & \frac{l^2}{8EI} \\ 0 & 0 & 1 & \frac{l}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} Y_0 \\ \theta_0 \\ M_0 \\ V_0 \end{pmatrix} \quad (1)$$

Point Mass Matrix(1-2)

$$\begin{pmatrix} Y_2 \\ \theta_2 \\ M_2 \\ V_2 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ m\omega^2 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} Y_1 \\ \theta_1 \\ M_1 \\ V_1 \end{pmatrix} \quad (2)$$

Massless Beam Matrix(2-3)

$$\begin{pmatrix} Y_3 \\ \theta_3 \\ M_3 \\ V_3 \end{pmatrix} = \begin{bmatrix} 1 & \frac{l}{2} & \frac{l^2}{8EI} & \frac{l^3}{48EI} \\ 0 & 1 & \frac{l}{2EI} & \frac{l^2}{8EI} \\ 0 & 0 & 1 & \frac{l}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} Y_2 \\ \theta_2 \\ M_2 \\ V_2 \end{pmatrix} \quad (3)$$

Thus, the total matrix of a single station of point mass and two half massless beam can be represented by substituting eq.(1)and (2) into eq.(3) which yield a new matrix as below,

$$\begin{pmatrix} Y_3 \\ \theta_3 \\ M_3 \\ V_3 \end{pmatrix} = \begin{bmatrix} 1+\frac{l^3m\omega^2}{48EI} & \frac{1}{2}\left(1+\frac{l^3m\omega^2}{48EI}\right)l+\frac{l}{2} & \frac{\left(1+\frac{l^3m\omega^2}{48EI}\right)l^2}{8EI}+\frac{3l^2}{8EI} & \frac{\left(1+\frac{l^3m\omega^2}{48EI}\right)l^3}{48EI}+\frac{7l^3}{48EI} \\ \frac{l^2m\omega^2}{8EI} & \frac{l^3m\omega^2}{16EI}+1 & \frac{l^4m\omega^2}{64E^2I^2}+\frac{l}{EI} & \frac{l^5m\omega^2}{384E^2I^2}+\frac{l^2}{2EI} \\ \frac{lm\omega^2}{2} & \frac{l^2m\omega^2}{4} & \frac{l^3m\omega^2}{16EI}+1 & \frac{l^4m\omega^2}{96EI}+l \\ m\omega^2 & \frac{lm\omega^2}{2} & \frac{l^2m\omega^2}{8EI} & 1+\frac{l^3m\omega^2}{48EI} \end{bmatrix} \begin{pmatrix} Y_0 \\ \theta_0 \\ M_0 \\ V_0 \end{pmatrix} \quad (4)$$

The above matrix represents a single lumped mass model and the number of above matrix can be considered as the number of degree of freedom. The square matrix in above assembled matrix is called the transfer matrix as state vector at 0 is transferred to the state vector at 3 through this matrix. Thus, it is possible to progress through the structure so that the state vector at the far end is related to the state vector at the starting end.

By taking above matrix as an example, the natural frequency of a single lumped mass beam model can be obtained by multiplying a number of matrix elements by considering the boundary condition of the beam. Considering the boundary condition of fixed-free beam ,the determinant of square matrix of element (3,3),(3,4),(4,3) and (4,4) will produce the predicted natural frequency. While, for fixed-fixed beam, the natural frequency can be obtained through the determinant of element (1,3),(1,4),(2,3) and (2,4). The determinant for each cases produces a second degree polynomial function and the real positive root of the function is the value of the natural frequency.

By applying the same approach, 6 lumped mass beam model can be expressed as a single transfer matrix by assembling previous matrices. As the obtained matrix consists of complicated numerical figures, numerical interpolation method is used to determine the natural frequencies corresponding to their mode numbers for both different boundary conditions of beams.

3. Numerical Calculation Result

3.1 Lumped Mass Model Numerical Calculation Result

Before modifying the parameter of mass in each point mass, the obtained natural frequencies are firstly compared with the exact solution. Table 1 shows the properties of beam that used in this numerical calculation.

Beam properties (Aluminum)	
Elastic Modulus	$70 \times 10^9\ Pa$
Moment of inertia	$2.13 \times 10^{-10}\ m^4$
Density	$2700\ kg/m^3$
Area	$1.6 \times 10^{-4}\ m^2$
Length	$0.3m$
Mass	$0.1296\ kg$

Table 1.Properties of Beam

The obtained natural frequencies using iterative calculation transfer matrix method are then compared with the exact solutions. Table 3 and table 4 show the obtained results for fixed-free and fixed-fixed beam using Transfer Matrices Method which are compared with the exact solution.

Natural Frequency	Exact Solution(Hz)	3lumped mass(Hz)	4lumped mass(Hz)
First	36.524	37.48(1.026)	37.05(1.014)
Second	228.92	251.14(1.097)	240.84(1.052)
Third	640.972	807.17(1.259)	688.95(1.074)

Natural Frequency	5lumped mass(Hz)	6lumped mass(Hz)
First	36.86(1.009)	36.76(1.006)
Second	236.41(1.033)	234.07(1.02)
Third	673.56(1.05)	663.92(1.03)

Table 2.Natural Frequencies of the Original Beam (Fixed-Free Beam)

Natural Frequency	Exact Solution(Hz)	3lumped mass(Hz)	4lumped mass(Hz)
First	232.436	232.896(1.002)	232.633(1.001)
Second	640.719	717.788(1.12)	640.068(0.999)
Third	1256.067	900.639(0.717)	1384.18(1.101)

Natural Frequency	5lumped mass(Hz)	6lumped mass(Hz)
First	232.366(0.9997)	232.366(0.9997)
Second	639.802(0.999)	640.068(0.999)
Third	1240.347(0.987)	1248.776(0.994)

Table 3.Natural Frequencies of the Original Beam (Fixed-Fixed Beam)

The number in the bracket represents the ratio of natural frequencies obtained from Transfer Matrices Method and exact solution. It is obvious to conclude that, as the number of lumped mass increase, the obtained natural frequencies approach the exact solution but as the number of mode increase, the rate of change gradually increase but still can be considered accurate as in the calculation of natural frequencies for infinite number of lumped mass, the first few obtained natural frequencies are reliable. Thus, the applied transfer matrix method is reliable to determine the value of natural frequencies which is then used to predict the dynamic behavior of fixed-free and fixed-fixed beam due to changes of mass.

3.2 Numerical Calculation Result due to Changes of Mass (fixed-free beam)

In this six-lumped-mass beam model, the total mass of beam is equally distributed. In order to investigate the optimal station that mostly influences the value of natural frequency, four different points are selected and 25%,50% and 75% of the original mass of the respective point mass are purposely added and reduced accordingly.

Figure 2 and figure 3 show the percentage rate of change for the first three natural frequencies for fixed-free beam at different station due to mass increment and mass reduction at different station respectively. Each station represents the right end and left end (m1 and m6) location of the beam while the other two represent the central station of the beam (m3 and m4).

Theoretically, it is known as the masses increase, the natural frequencies decrease and vice versa. In this study, the mass is purposely changed at different station to determine the most influenced station that will affect the value of natural frequencies. From the obtained result, it is clear, as the modified mass station moves away from the fixed-part of the beam, the percentage rate of change of the natural frequencies increased gradually. Thus, the changes of mass at the right end free part of the beam produce the highest percentage rate for fixed-free beam.

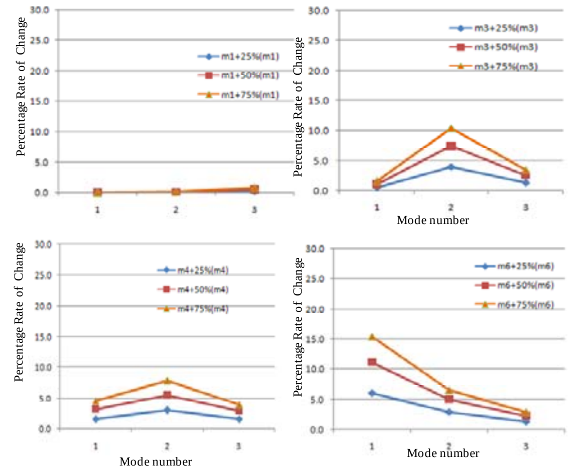

Fig. 2. Percentage Rate of Change (natural frequencies) due to Mass Increment

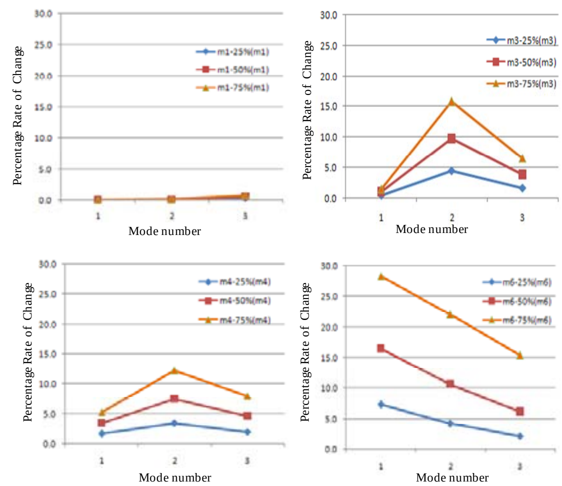

Fig. 3.Percentage Rate of Change (natural frequencies) due to Mass Decrement

3.3 Numerical Calculation Result due to Changes of Mass (fixed-fixed beam)

The boundary condition of a beam structure is also a factor that influences the percentage rate of change of natural frequencies. Figure 4 and figure 5 show the percentage rate of change for the first three natural frequencies for fixed-fixed beam at different station due to mass increment and mass reduction at different station respectively.

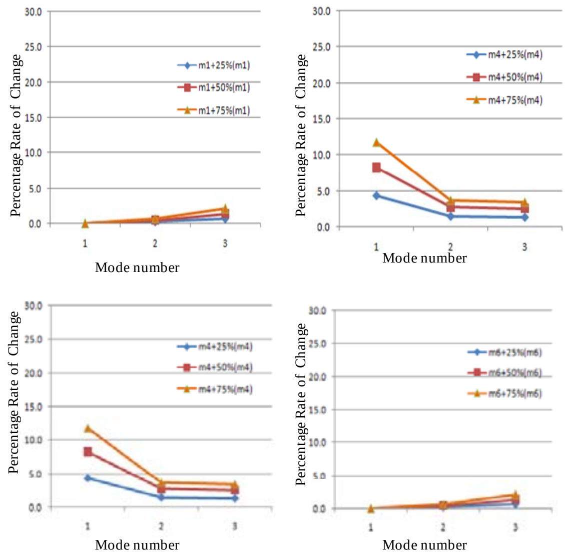

Fig. 4.Percentage Rate of Change (natural frequencies) due to Mass Increment

For fixed-fixed beam, the mass is purposely modified at the same station as done on fixed-free beam. Due to its boundary condition, the pattern of changing percentage rate at different modified mass locations is different compared to fixed-free case. It can be clearly seen, the percentage rates of change are high when the mass is modified at the central station of the beam and approximately low at the near fixed-part of the beam. In addition, the percentage rate is exactly similar at station m3 and m4 due to their location at the center of beam.

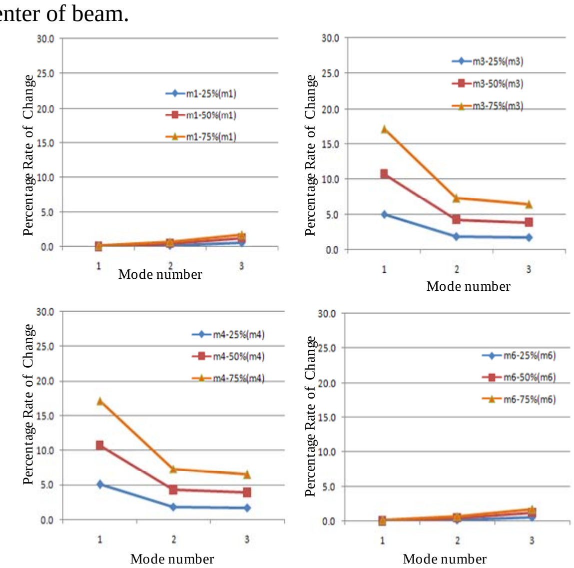

Fig. 5.Percentage Rate of Change (natural frequencies) due to Mass Decrement

4. Conclusion

The conclusions for this study can be drawn as follow

(1) The changes of natural frequencies are more influenced by the boundary condition of beam compared to the changes of mass. But at the same time ,the changes of natural frequencies are more influenced by the decrement of mass than the increment of mass.

(2) For fix-free beam, the changes of natural frequencies are more sensitive at free end part of beam and for fixed-fixed beam, the changes of natural frequencies are more sensitive at the center part of beam.

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