선박내 접수탱크 진동에 대한 실험/이론적 연구

Experimental and analytical study on hydroelastic vibration of tank

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Key Words : Hydroelsatic vibration (유탄성진동), Added Mass(부가수 질량), Rayleigh-Ritz Method(Rayleigh-Ritz 방법), Modal test(모드 실험), Assumed mode method(진동파형가정 방법)

ABSTRACT

In this paper, a experimental and theoretical study is carried out on the hydroelastic vibration for a rectangular bottom and side plate of tank. It is assumed that the tank wall is clamped along the plate edges. The fluid velocity potential is used for the simulation of fluid domain and to obtain the added mass due to plate vibration. It is assumed that the fluid is imcompressible and inviscid. Assumed mode method is utilized to the plate model and hydrodynamic force is obtained by the proposed approach. The coupled natural frequencies are obtained from the relationship between kinetic energies of a wall including fluid and the potential energy of the wall. The theoretical result is compared with the three-dimensional finite element method. In order to verify the result, modal test was carried out for bottom/side plate of tank model by using impact hammer. It was found the fundamental natural frequency of bottom plate is lower than that of side plate of tank and theoretical result was in good agreement with that of commercial three-dimensional finite element program.

1. Introduction

This paper deals with the free vibration analysis of a rectangular plate in contact with various bounded fluids. This work arises as a part of a project developing a local vibration analysis and sloshing analysis program of a tank in a ship. In the many part of a ship, there exist so many tank structures contacting with water or oil. If there structures exhibit excessive vibration, it takes a lot of cost to improve the vibration situation because of required welding and special painting jobs. It is therefore very important to predict precise vibration characteristic for the structures at the design stage, however it is not easy to evaluate vibration characteristic of the structures in contact with a fluid by ship designer.

The plate vibration in contact with fluid has recently been studied. Zhou[1] analytically calculated the natural frequency of a rectangular plate in contact with water on one side, such as dams and floodgates. Bartlett[2] and S.H Choi[3] investigated simple added mass effect by using experiment and Finite Element Method. Bauer[4] treated partially covering the free liquid surface of a two-dimensional rectangular container of infinite width. Jeong[5][6] derived a formulation on a plate using normalized admissible functions satisfying all the plate boundary conditions. However, the identical two plates coupled with a bounded fluid without free surface was dealt with. Lee[7] derived an expression of the fluid domain using the velocity potential imposing the simple free surface boundary condition. On the other hand, as for a stiffened plate in air, Han[8] calculated the natural frequency of a stiffened plate having concentrated masses using the Receptance method. The theoretical study is performed on the natural frequency of a stiffened plate of a tank by K.S Kim[9][10][11], and the result is compared with the three-dimensional finite element method.

This paper describes an application of the hydroelasticity theory to the fluid-structure interaction problems for bottom/side plate of a rectangular tank. Tank wall is modeled using the Assumed Mode methods and the velocity potential method to assess the added mass. Then the natural frequency of bottom/side plate of vertical and horizontal tank is calculated theoretically and compared with experimental results.

2. Theory

Formulation of a stiffened plate

Figure 1 shows a plate with stiffener. For the modeling of a rectangular plate, assumed mode method is adopted and mode shapes of beam are used.

$$w(x,y,t)=\sum_{m=1}^{p}\sum_{n=1}^{q}A_{mn}(t)X_m(x)Y_n(y) \qquad (1)$$

where,

$X_m(x), Y_n(y)$ are mth and nth mode shapes of a beam for x and y direction respectively, which is spatial coordinate of a plate, and $A_{mn}(t)$ is the time dependent generalized coordinate.

Using the assumed mode method, kinetic and potential energy can be calculated. This approach can be extended to the stiffened plate. In that case, kinetic and potential energy of the plate itself can be expressed as follows

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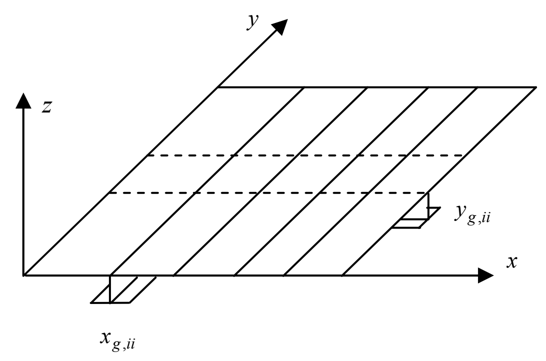

Fig.1 A plate with stiffeners and coordinate system

2.1.1 Kinetic energy of stiffened plate

$$T_p(t) = \frac{1}{2}\int_0^a\int_0^b \rho h\, \dot{w}(x,y,t)^2 dxdy$$

$$T_g(t) = \frac{1}{2}\rho\sum_{ii=1}^{n_g} A_{x,ii}\int_0^a \dot{w}(x,y_{g,ii},t)^2 dx + I_{px,ii}\int_0^a\left(\frac{\partial\dot{w}(x,y_{g,ii},t)}{\partial y}\right)^2 dx\Bigg] dy$$

$$T(t) = T_p + T_g \qquad (2)$$

V_p, V_g are kinetic energy of plate and stiffener.

a , ρ , A_{xii}, I_{pxii} and $y_{g,ii}$ are stiffener length, density, sectional area, polar moment of inertia, y-axis location of the ii^{th} stiffener.

2.1.2 Potential energy

$$V_p(t) = \frac{1}{2}\int_0^b\int_0^a D_E\left(\begin{matrix}\left(\frac{\partial^2 w}{\partial x^2}\right)^2 + \left(\frac{\partial^2 w}{\partial y^2}\right)^2 \\ +2\upsilon\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2(1-\upsilon)\left(\frac{\partial^2 w}{\partial xy}\right)^2\end{matrix}\right) dxdy,$$

$$V_g(t) = \frac{1}{2}\sum_{ii=1}^{n_g}\left[\begin{matrix} EI_{x,ii}\int_0^a\left(\frac{\partial^2 w(x,y_{g,ii},t)}{\partial x^2}\right)^2 dx \\ +GI_{px,ii}\int_0^a\left(\frac{\partial^2 w(x,y_{g,ii},t)}{\partial x\partial y}\right)^2 dx\end{matrix}\right]$$

$$V(t) = V_p + V_g \qquad (3)$$

V_p, V_g are potential energy of plate and stiffener.

D_E , υ are bending rigidity of plate, passion ratio and EI_x , GJ_x , n_g are equivalent bending rigidity, torsional rigidity of x-axis stiffener and number of stiffeners respectively.

Natural frequency of the stiffened panel can be obtained from Eq. (4) applying Lagrange equation to kinetic and potential energies.

$$[K]q + [M]\ddot{q} = 0, \qquad (4)$$

It should be noted that non-zero off-diagonal terms exists in Eq. (4) because the admissible functions are not the eigen-mode of the considered problem.

Hydrodynamic modeling

Figure 2 shows a rectangular tank filled with liquid and the coordinate system. a , b , H and L are the breadth, height of the plate, liquid filling level and length of the tank, respectively. The length of the tank is the length of two opposite sides between the face A and the face B in the y direction. The bottom plate is taken into account for the formulation. However, application to other plates will be very similar and straightforward. As for a side plate, the detail formulation is described in Reference [9].

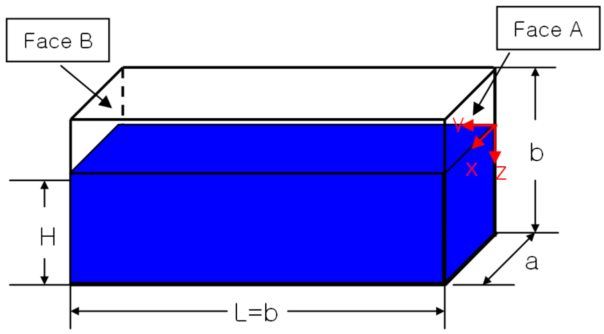

Fig.2 A rectangular tank filled with liquid

Velocity potential is used for the estimation of hydrodynamic force assuming incompressible and irrotational flow. Consequently, Laplace equation is the governing equation as shown in Eq. (5).

$$\nabla^2\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 \qquad (5)$$

Velocity potential can be obtained from Laplace equation with adequate boundary conditions, which are rigid wall boundary condition (normal velocity of fluid at tank walls = 0), elastic wall boundary condition (normal velocity of fluid at a tank wall is the same as velocity of the plate) and free surface boundary condition on disturbed free surface.

In general, free surface boundary condition is expressed as in Eq. (6).

$$\frac{\partial^2\Phi}{\partial t^2} + g\frac{\partial\Phi}{\partial z} = 0 \quad \text{at} \quad z = 0 \qquad (6)$$

However, it should be noted that because natural frequency of elastic wall is relatively high compared with water wave frequency, high frequency approximation can be applied to the general free surface boundary condition, which is simplified as Eq. (7).

$$\phi = 0 \quad \text{at} \quad z = 0 \qquad (7)$$

Force applied to the wall can also be calculated with the Bernoulli's equation (8) and velocity potential.

$$p = -\rho \frac{\partial \Phi}{\partial t}\bigg|_{z=0} \tag{8}$$

Consequently, force can be obtained as Eq. (9).

$$F_s = \int_0^H \int_0^a p \, dxdy \tag{9}$$

Because it is known that added mass is proportional to the acceleration of the wall, the added mass can be obtained as Eq. (10).

$$F_s = \sum_{r=1}^{\infty} A_{sr} \ddot{a}_r(t) \tag{10}$$

$$M_{add,sr} = A_{sr}$$

Natural frequency of stiffened plate in contact with liquid

Natural frequency of stiffened plate can be calculated with the structure modal mass, modal stiffness and added mass.

$$[K_{\text{modal}}]\, a_{add} - [\Lambda_{add}][M_{\text{modal}} + M_{add}]\, a_{add} = 0,$$

$$[\Lambda_{add}] = [\varpi^2_{add}] \tag{11}$$

3. Comparison of theory and FEM

Based upon the proposed methodology, a couple of numerical calculations are carried out and compared with NASTRAN, which is a three dimensional finite element program. Figure 3 shows a rectangular steel tank filled with water. The thickness of the wall is 1.5 mm. Calculation was carried out for bottom, vertical side cases and horizontal side plate at the 100% water filling level of tank depth.

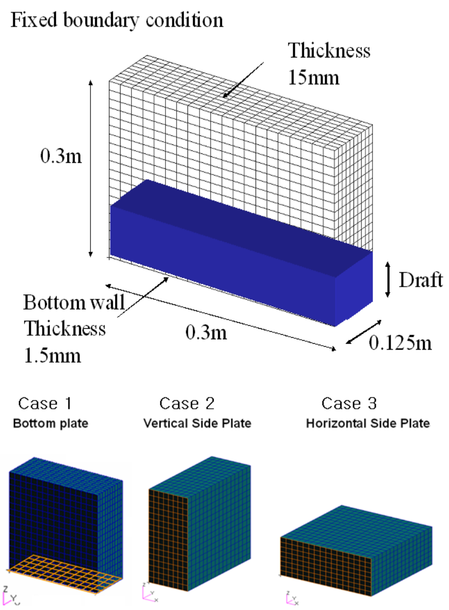

Fig.3 A rectangular tank filled with water

The fixed boundary condition was imposed on all edges and walls except wall of interest. Table 1 shows the natural frequencies obtained by the proposed theory and NASTRAN.

Table 1 Comparison of natural frequencies for theory and FEM

Mode	Bottom plate			Vertical side plate			Horizontal side plate		
	Test	Nas	Err. (%)	Test	Nas	Err. (%)	Test	Nas	Err. (%)
1	143	142	0.7	164	166	1.3	235	237	1.1
2	284	281	0.9	301	302	0.3	319	320	0.4
3	435	433	0.5	450	453	0.6	457	459	0.6

It is found that the proposed theory agrees very well with the NASTRAN result within a 1.3% discrepancy range as shown in Table 1. Figure 4 shows the mode shapes of the bottom, vertical side and horizontal side plate of the tank respectively.

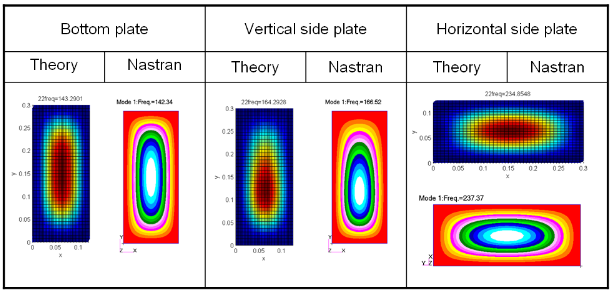

Fig. 4 The mode shapes of bottom, vertical side and horizontal side plate for theory and FEM

4. Comparison of test and FEM

Modal test was carried out and compared with FEM considering mass of accelerometer for simple tank model which is shown Figure 5. The tank is designed to be able to conduct the experiments for bottom, vertical side and horizontal side plate of tank filled with fluid. Table 2 show the natural frequencies obtained from modal test data (Figure 6) and NASTRAN. It is found that the discrepancy of test and FEM results is within a 9% for 98% tank filling rate. The related mode shapes are shown in Figure 7.

Table 2 Comparison of natural frequencies for test and FEM

Mode	Bottom plate			Vertical side plate			Horizontal side plate		
	Test	Nas	Err. (%)	Test	Nas	Err. (%)	Test	Nas	Err. (%)
1	155	141	9.0	167	166	0.6	224	235	4.8
2	273	274	0.4	301	293	2.8	300	311	3.6
3	419	427	1.9	449	441	1.8	441	450	2.0

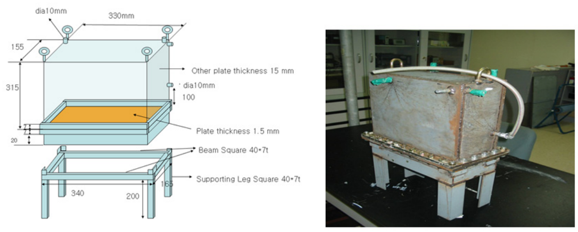

Fig.5 A rectangular tank for modal test

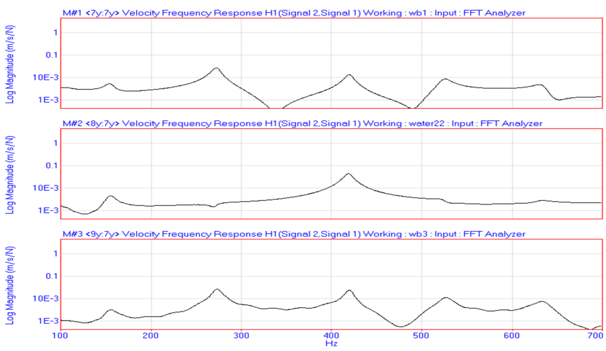

Fig. 6 The frequency response function of bottom plate

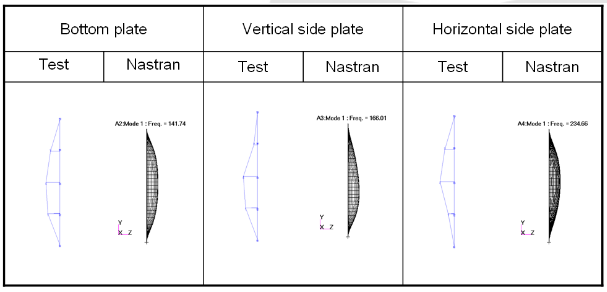

Fig. 7 The mode shapes for test and FEM

The figure 8 and figure 9 show comparison of the natural frequencies for bottom, vertical side and horizontal side plate of tank, which are obtained from the modal test. The natural frequency of bottom plate is 8% lower than that of vertical side plate and 45% lower than that of horizontal plate. Therefore, when vibration analysis of wetted plate is carried out, the fluid model contacted with plate is paid special attention at design stage of tank.

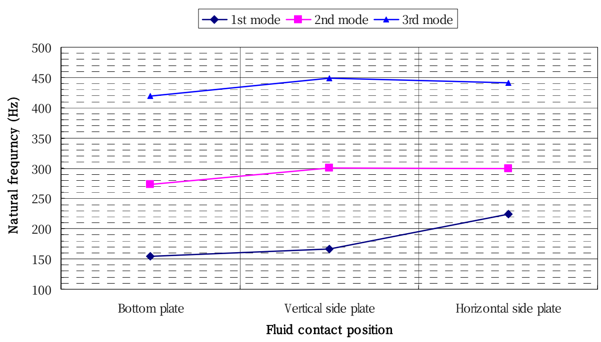

Fig. 8 Comparison of natural frequency for bottom, vertical side and horizontal side plate of tank

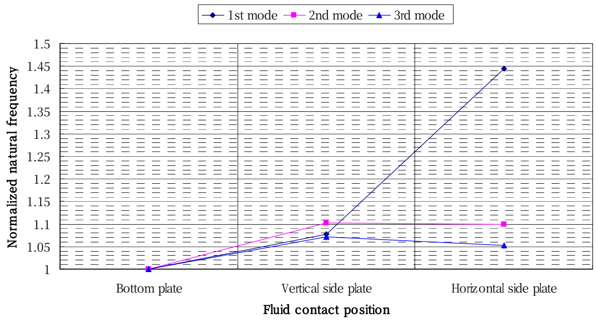

Fig. 9 Comparison of normalized natural frequency for bottom, vertical side and horizontal plate of tank (Divided bottom plate natural frequency)

5. Conclusion

An analytical method to estimate the natural frequency of a bottom and side wetted plate is developed. Through out the tests, it is found that this approach can properly deal with the fluid-structure interaction of a tank. The calculation results show very good agreement with the three-dimensional finite element method. In addition, model test was carried out and compared with FEM. It is found that FEM results are in good agreement with the test results within allowable experimental error range. According to test results, the natural frequency of bottom plate is lower than that of vertical side or that of horizontal plate. Therefore, when the vibration analysis of wetted plate is carried out, the fluid model contacted with plate is paid special attention at design stage of tank.

Some improvement can be expected when hydrodynamic and stiffened plate model is slightly revised.

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