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PID 제어기의 주파수응답 기반 다목적 설계도구

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Frequency Response Based Multi-objective Design Tool for PID Controller

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Abstract - This paper presents a Matlab toolbox for proportional-integral-derivative (PID) controller design. By means of the tool, the complete set of controllers simultaneously satisfying multiple design specifications such as stability and robust stability margins can be obtained directly from the only frequency response data on the plant.

1. INTRODUCTION

In much of industrial practice, the available information about plant is not an analytical model but time or frequency response [1]. Recently, a new method of synthesizing PID controllers based on the frequency response data and the number of unstable poles of the plant has been developed [2]. The algorithm provides the complete set of PID controllers stabilizing and achieving several meaningful performance criterions for a single-input single-output (SISO), linear time invariant (LTI) plant. The frequency response data can be reliably determined experimentally and the number of unstable poles can also be known from the physical consideration of the process.

Recently, a numerical algorithm of all stabilizing PID controllers for non-parametric model has been implemented by the Matlab codes [3]. In this paper, a Matlab toolbox to deal with several performance requirements, such as gain and phase margins, and H_∞ margin, is presented. A numerical example will be given to illustrate the use of the toolbox.

2. THEORETICAL BACKGROUND

Consider a feedback control system given by an LTI plant and a PID controller as shown in Fig. 1.

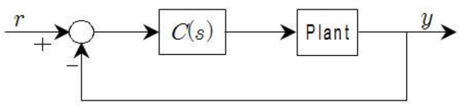

Fig. 1. A feedback control system.

The PID controller is expressed as

$$C(s)=\frac{K_d s^2+K_p s+K_i}{s}. \tag{1}$$

We first make some assumptions that the only information available for design are

I. knowledge of the frequency response data, i.e.,

$$G(j\omega)=G(\omega)e^{j\phi(\omega)}=G_r(\omega)+jG_i(\omega),\quad \text{for } \omega\in[0,\ \omega_{\max}], \tag{2}$$

II. knowledge of the number of unstable poles of the plant: p_r.

The algorithm for obtaining all stabilizing PID set has been reported in [2-3]. The design process can be divided into the following two steps in a large way.

1. Find a feasible range of K_p. It is given by

$$K_p=-\frac{\cos\phi(\omega)}{G(\omega)}. \tag{3}$$

2. Compute the complete set of stabilizing (K_i, K_d) set using (4) for each fixed K_p over the range above.

$$\left[K_i-K_d\omega_t^2+\frac{\omega_t\sin\phi(\omega_t)}{G(\omega)}\right]\cdot i_t>0,\quad \text{for } t=0,1,\cdots,l \tag{4}$$

where ω_t is the solution of (3) and $i_t\in(-1,0,1)$ is the admissible strings of integers.

The algorithm seeking all stabilizing PID controller can be extended to the multi-objective design problem by which one finds the PID set satisfying gain and phase margins, and H_∞ margin. Then the problem of achieving performance specifications is reduced to the problem of simultaneous stabilization of the plant $G(j\omega)$ and the families of the following real and complex plants.

A. The equivalent plant for a gain margin specification:

$$G_K^c(j\omega)=\{KG(j\omega):\ K\in[K^-,\ K^+]\}, \tag{5}$$

B. The equivalent plant for a phase margin specification:

$$G_\Theta^c(j\omega)=\{e^{-j\Theta}G(j\omega):\ \Theta\in[\Theta^-,\ \Theta^+]\}, \tag{6}$$

C. The equivalent plant for a H_∞ norm specification on the sensitivity function with $\|W_S(j\omega)S(j\omega)\|_\infty<\gamma_S$:

$$G_S^c(j\omega)=\left\{\left[\frac{1}{1+\frac{1}{\gamma_S}e^{j\theta}W_S(j\omega)}\right]G(j\omega):\ \theta\in[0,\ 2\pi]\right\}, \tag{7}$$

D. The equivalent plant for a H_∞ norm specification on the complementary sensitivity function with $\|W_T(j\omega)T(j\omega)\|_\infty<\gamma_T$:

$$G_T^c(j\omega)=\left\{\left[1+\frac{1}{\gamma_T}e^{j\theta}W_T(j\omega)\right]G(j\omega):\ \theta\in[0,\ 2\pi]\right\}. \tag{8}$$

Let the complete set of controllers stabilizing $G(j\omega)$ and the equivalent plants for a gain, phase, H_∞ margins (5), (6), and (7) or (8) be S, S_K, S_Θ, S_H. Thus the admissible set that satisfying all given specifications is obtained as the intersection of them.

$$S^*=S\cap S_K\cap S_\Theta\cap S_H. \tag{9}$$

3. DEVELOPMENT OF A MATLAB TOOLBOX

3.1 Design procedure

The Matlab implementation of the numerical algorithm for calculating all stabilizing set S is described in [3]. The design procedure for each performance criterion is similar. The only difference is that $G(j\omega)$ and $G_K(j\omega)$ are real but $G_\Theta(j\omega)$, $G_S(j\omega)$, and $G_T(j\omega)$ are complex.

Now, we present how the design algorithm is implemented by Matlab software. The design flow of multi-objective PID controllers is shown in Fig. 2. It is noted that the PID controller parameter K_p and (K_i, K_d) can be determined independently as in (3) and (4). Hence, the K_p range for multiple design specifications is calculated by the intersection of each feasible ranges of K_p. Moreover, the admissible (K_i, K_d) set is also obtained from (9).

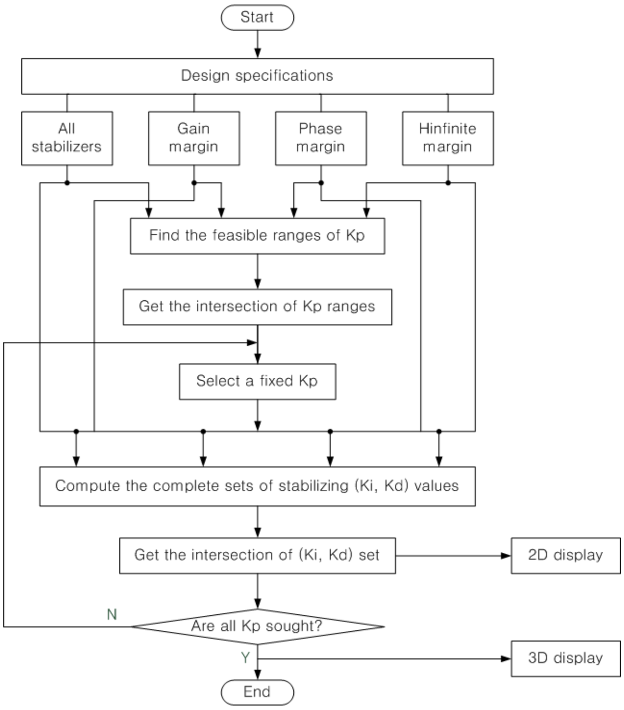

Fig. 2. Multi-objective PID controller design flow.

3.2 An illustrative example

Consider a stable plant in [2]. Its bode diagram is as Fig. 3 and the number of right half plane (RHP) poles is $p_r = 0$.

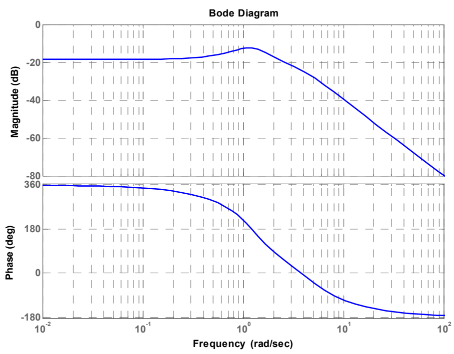

Fig. 3. Bode diagram of the plant.

Let us find the complete set of the stabilizing PID controllers that simultaneously satisfies the following specifications:

- Gain margin: $K \in [0,\ 6dB]$,
- Phase margin: $\Theta \in [0,\ 60\deg]$,
- H_∞ margins: $\| W(s)T(s) \| < 1$ with $W(s) = \dfrac{s+0.1}{s+1}$.

The Matlab scripts for this problem are shown as follows.

```
>>load FreqData; pr=0;
>>Gain=linspace(n,6,25); Theta=linspace(0,60,25);
>>wNs=[1 0.1]; wDs=[1 1]; wSys=tf(wNs,wDs); gamma=1;
>>[Kp,Ki,Kd]=PIDtool(FreqData,pr,Gain,Theta,wSys,gamma);
```

The feasible range of K_p for given performance criterions is obtained as $K_p \in (-4.2586,\ 2.1217)$. When a specific value of $K_p = 1$ is selected, the admissible regions of $(K_i,\ K_d)$ are shown in Fig. 4. And Fig. 5 gives the complete set of the stabilizing PID controllers for multiple design specifications. Moreover, a selection of the controller inside the shaded region in Fig. 4 enables us to display various characteristic of the designed system as shown in Fig. 6.

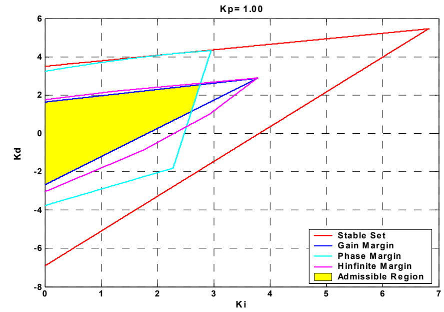

Fig. 4. The admissible $(K_i,\ K_d)$ set for fixed $K_p = 1$.

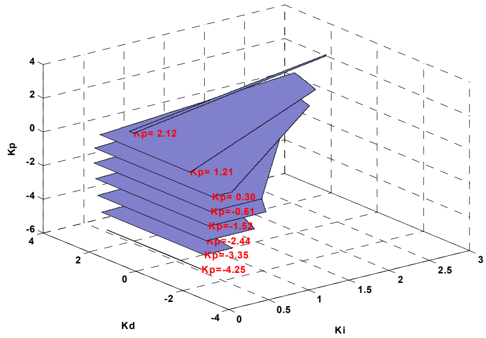

Fig. 5. The admissible $(K_i,\ K_d)$ set for fixed $K_p = 1$.

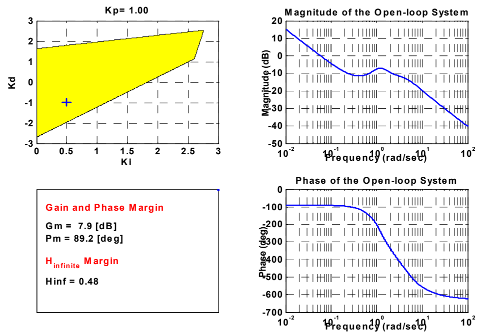

Fig. 6. Performance data for selected PID controller.

4. CONCLUSION

A frequency response based multi-objective design toolbox for PID controller has been implemented with Matlab software. The theoretical approach has been started from [2]. The set of the entire stabilizing PID controllers which meets gain and phase margins, and H_∞ margin can be displayed as 2 or 3 dimensional graphics so as to be useful for computer aided design.

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