2008년도 대한전기학회 하계학술대회 논문집 2008. 7. 16 - 18

모터 동역학을 포함한 이동 로봇의 추종 제어를 위한 동적 표면 제어

박봉석*, 최윤호**, 박진배*
연세대학교*, 경기대학교**

Dynamic surface control for trajectory tracking of mobile robots including motor dynamics

Bong Seok Park*, Yoon Ho Choi**, Jin Bae Park*
Yonsei University*, Kyonggi University**

Abstract - Almost all existing controllers for nonholonomic mobile robots are designed without considering the motor dynamics. This is because the presence of the motor dynamics increases the complexity of the system dynamics, and makes difficult the design of the controller. In this paper, we propose a simple controller for trajectory tracking of mobile robots including motor dynamics. For the simple controller design, the dynamic surface control methodology is applied and extended to multi-input multi-output systems (i.e., mobile robots) that the number of inputs and outputs are different. Finally, simulation results demonstrate the effectiveness of the proposed controller.

1. Introduction

Over the past twenty years, the control of mobile robots has been regarded as the attractive problem due to the nature of nonholonomic constraints. Many efforts have been devoted to the tracking control of nonholonomic mobile robots. However, most of the schemes have ignored the dynamics coming from electric motors which should be required to implement the robots in the real environment, that is, the robot kinematics or the robot dynamics have been only considered.

The backstepping technique has been widely used as one of representative methods for controlling nonholonomic mobile robots considering kinematics and dynamics [1],[2],[3]. However, the backstepping design procedure has the problem of ''explosion of complexity'' caused by the repeated differentiations of virtual controllers. That is, the complexity of controller grows drastically as the order of the system increases. When the model of the electrically driven mobile robots is considered, this problem of the backstepping design would become more serious. Swaroop et al. [4] proposed a dynamic surface control (DSC) technique to solve this problem by introducing a first-order filtering of the synthesized virtual control law at each step of the backstepping design procedure. The DSC idea was extended to uncertain single-input single-output (SISO) [5],[6] and multi-input multi-output (MIMO) systems [7]. Despite these efforts using the DSC technique, the DSC method is still not applied to MIMO systems (i.e., mobile robots) that have more degrees of freedom (DOFs) than the number of inputs under nonholonomic constraints.

Accordingly, we propose a simple controller for trajectory tracking of nonholonomic mobile robots including motor dynamics. For the simple control system design, we apply the DSC technique to nonholonomic electrically driven mobile robots, the MIMO systems that have more DOFs than the number of inputs under nonholonomic constraints. Based on Lyapunov stability theorem, we prove that the steady-state errors converge to zero.

2. Model of Mobile Robots With Nonholonomic Constraints

The kinematics and dynamics of nonholonomic mobile robots are described by the following differential equations:

$$\dot{q} = J(q)z = 0.5r\begin{bmatrix}\cos\theta & \cos\theta \\ \sin\theta & \sin\theta \\ R^{-1} & R^{-1}\end{bmatrix}\begin{bmatrix}v_1 \\ v_2\end{bmatrix} \tag{1}$$

$$M\dot{z} + C(\dot{q})z + Dz = \tau \tag{2}$$

where $q = [x\, y\, \theta]^T \in R^3$; x, y are the coordinates of the center of mass of the vehicle, and θ is the angle between the heading direction and the x axis, $z = [v_1\, v_2]^T \in R^2$; v_1 and v_2 represent the angular velocities of right and left wheels. R is the half of the width of the mobile robot and r is the radius of the wheel,

$$M = \begin{bmatrix}m_{11} & m_{12} \\ m_{12} & m_{11}\end{bmatrix}, C(\dot{q}) = 0.5R^{-1}r^2 m_c d\begin{bmatrix}d_{11} & 0 \\ 0 & d_{22}\end{bmatrix}$$

$$m_{11} = 0.25R^{-2}r^2(mR^2 + I) + I_\omega, m_{12} = 0.25R^{-2}r^2(mR^2 - I)$$

$$m = m_c + 2m_\omega, I = m_c d^2 + 2m_\omega R^2 + I_c + 2I_m, \tau = [\tau_1\, \tau_2]^T.$$

In these expressions, d is the distance from the center of mass P_c of the mobile robot to the middle point P_0 between the right and left driving wheels. m_c and m_ω are the mass of the body and wheel with a motor, respectively. I_c, I_ω, and I_m are the moment of inertia of the body about the vertical axis through P_c, the wheel with a motor about the wheel axis, and the wheel with a motor about the wheel diameter, respectively. The positive terms $d_{ii}, i = 1,2$, are the damping coefficients. $\tau \in R^2$ is the control torque applied to the wheels of the robot.

Property 1: The inertia matrix M is symmetric and positive definite.

In addition, the dynamic model of dc motors can be represented as follows:

$$\tau_m = K_T i_a,\ u = R_a i_a + L_a \dot{i}_a + K_E \dot{\theta}_m \tag{3}$$

where $\tau_m \in R^2$ is the torque generated by dc motor, $K_T = diag(k_{t1}, k_{t2})$ is the motor torque constant, $i_a \in R^2$ is the current, $u \in R^2$ is the input voltage, $R_a = diag(r_{a1}, r_{a2})$ is the resistance, $L_a = diag(l_{a1}, l_{a2})$ is the inductance, $K_E = diag(k_{e1}, k_{e2})$ is the back electromotive force coefficient, and $\dot{\theta}_m \in R^2$ is the angular velocity of the dc motor. Here, $diag(\cdot)$ denotes the diagonal matrix.

The relationship between the dc motor and the mobile robot wheel can be written as

$$N = \frac{\dot{\theta}_m}{z} = \frac{\tau}{\tau_m} \tag{4}$$

where $N = diag(n_1, n_2)$ is the gear ratio. Using (4), the dynamic model of dc motors (3) can be written as

$$\tau = NK_T i_a,\ u = R_a i_a + L_a \dot{i}_a + NK_E z. \tag{5}$$

Let us define the state variables as $x_1 = q, x_2 = z$, and $x_3 = i_a$. Then, (1), (2), and (5) can be expressed in the following state-space form:

$$\dot{x}_1 = J(x_1)x_2 \tag{6}$$

$$\dot{x}_2 = M^{-1}(-C(\dot{x}_1)x_2 - Dx_2 + NK_T x_3) \tag{7}$$

$$\dot{x}_3 = L_a^{-1}(u - R_a x_3 - NK_E x_2) \tag{8}$$

where $x_1 = [x_{11}\, x_{12}\, x_{13}]^T$, $x_2 = [x_{21}\, x_{22}]^T$, and $x_3 = [x_{31}\, x_{32}]^T$.

The control objective is to design a simple control law u for nonholonomic electrically driven mobile robots (6)-(8) to track the desired trajectory generated by the following reference robot:

$$\begin{aligned}\dot{x}_r &= v_r \cos\theta_r \\ \dot{y}_r &= v_r \sin\theta_r \\ \dot{\theta}_r &= \omega_r\end{aligned} \tag{9}$$

where x_r, y_r, and θ_r are the position and orientation of the reference robot. v_r and ω_r are the linear and angular velocities of the reference robot. respectively.

Assumption 1: The reference signal $z_r = [v_r\ \omega_r]^T$ is bounded, and $v_r > 0$.

Remark 1: In Assumption 1, $v_r > 0$ means that this paper is only focused on a simple controller design for the trajectory tracking problem of mobile robots incorporating motor dynamics. That is, the case of $v_r = 0$ is not considered.

3. Main Result

3.1 Controller Design

In this section, we develop a simple control system for nonholonomic electrically driven mobile robots. To design the control system using the DSC technique, we proceed step by step. Finally, we choose an actual control law u to derive the convergence of the error surfaces.

3.2 Stability Analysis

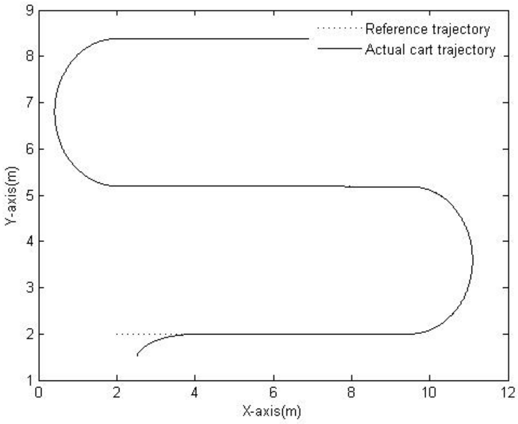
Generally, the stability analysis of the DSC system is more complicated than that of the backstepping control system because the extra first-order filters must be considered. In this section, we prove the asymptotic stability of the solution of the proposed control system.

4. Simulations

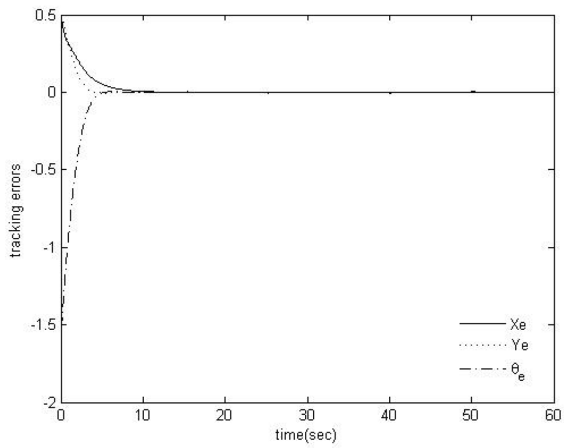
In this section, we perform the simulation for the tracking control of the nonholonomic electrically driven mobile robot to demonstrate the validity of the proposed control method. The physical parameters for the mobile robot are chosen as $R=0.75, d=0.3, r=0.15, m_c=30$, $m_\omega=1, I_c=15.625, I_\omega=0.005, I_m=0.0025$, and $d_{11}=d_{22}=5$. The parameters for the motor dynamics are chosen as $R_a = diag(1.6, 1.6)$, $L_a = diag(0.048, 0.048), K_E = diag(0.19, 0.19), K_T = diag(0.2613, 0.2613)$ and $N = diag(62.55, 62.55)$. The controller parameters for the proposed control systems are chosen as $k_1 = 3.5, k_2 = 3, k_3 = 5, k_4 = 3$, $k_5 = 1, \tau_2 = \tau_3 = 0.01$. The reference velocities v_r, ω_r for generating the reference trajectory are chosen as follows:

$$0 \le t < 15 : v_r = 0.5, \omega_r = 0$$
$$15 \le t < 25 : v_r = 0.5, \omega_r = 0.314$$
$$25 \le t < 40 : v_r = 0.5, \omega_r = 0$$
$$40 \le t < 50 : v_r = 0.5, \omega_r = -0.314$$
$$50 \le t < 60 : v_r = 0.5, \omega_r = 0$$

The initial postures for the reference robot and the actual robot are $(x_r, y_r, \theta_r) = (2, 2, 0)$ and $(x, y, \theta) = (2.5, 1.5, \pi/2)$, respectively. The simulation results are shown in Figs. 1 and 2. Fig. 1 shows the tracking result. In Fig. 2, all state errors converge to zero quickly in less than a few seconds.



<Fig. 1> Trajectory tracking result of the proposed controller.



<Fig. 2> Tracking errors x_e, y_e, θ_e.

5. Conclusion

In this paper, a simple controller for nonholonomic electrically driven mobile robots has been proposed. The dynamics, the kinematics, and the motor dynamics of mobile robots have been considered. The DSC technique has been extended to design the controller for trajectory tracking of mobile robots including motor dynamics. From the Lyapunov stability theory, we have proved that the steady-state error can converge to zero. Finally, from the simulation results, it has been shown that the proposed controller has good tracking performance.

6. Acknowledgement

This work was supported by the Brain Korea 21 Project in 2008.

[Reference]

[1] T. Fukao, H. Nakagawa, and N. Adachi, ''Adaptive tracking control of a nonholonomic mobile robot'', IEEE Trans. Robot. Automat., vol. 16, no. 5, pp. 609-615, 2000.
[2] W. Dong, and K. D. Kuhnert, ''Robust adaptive control of nonholonomic mobile robot with parameter and nonparameter uncertainties'', IEEE Trans. Robotics, vol. 21, no. 2, pp. 261-266, 2005.
[3] R. Fierro, and F. L. Lewis, ''Control of a nonholonomic mobile robot: Backstepping kinematics into dynamics'', Journal of Robotic System, vol. 14, no. 3, pp. 149-163, 1997.
[4] D. Swaroop, J. K. Hedrick, P. P. Yip, and J. C. Gerdes, ''Dynamic surface control for a class of nonlinear systems'', IEEE Trans. Autom. Control, vol. 45, no. 10, pp. 1893-1899, 2000.
[5] D. Wang, and J. Huang, ''Neural network-based adaptive dynamic surface control for a class of uncertain nonlinear systems instrict-feedback form'', IEEE Trans. Neural Network vol. 16, no. 1, pp. 195-202, 2005.
[6] P. P. Yip, and J. K. Hedrick, ''Adaptive dynamic surface control: A simplified algorithm for adaptive backstepping control of nonlinear systems'', International Journal of Control, vol. 71, no. 5, pp. 959-979, 1998.
[7] S. J. Yoo, J. B. Park, and Y. H. Choi, ''Adaptive dynamic surface control of flexible-joint robots using self-recurrent wavelet neural networks'', IEEE Trans. Syst., Man, Cybern. B, Cybern., vol. 36, no. 6, pp. 1342-1355, 2006.