

매입형 영구자석 동기전동기의 인덕턴스 산정방법 비교

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Comparison of Inductance Calculation Methods in Interior Permanent Magnet Synchronous Motor

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**Abstract** - The purpose of this paper is to investigate and compare the inductance evaluation methods of interior permanent magnet synchronous motors (IPMSM). Three major finite element methods are discussed. Their detail calculation processes will be presented as well as their fundamental principles. Not only the results, but also their solving method, computation time and complexity also will be compared. Finally, the calculated results will be verified with an experiment.

1. Introduction

The dominant influences in the correct prediction of the steady-state characteristics for the IPMSM are the d- and q-axis armature inductances and flux linkage of permanent magnet (PM) [1]. The flux linkage of permanent magnet can be easily and accurately calculated under no-load condition. Due to the nonlinear electromagnetic characteristics including the saturation and cross-coupling effect in the rotor of IPMSM [3], however, the d- and q-axis armature inductances of IPMSM become much difficult to be evaluated accurately.

So far, the finite element analysis (FEA) is the most trustable method for motor parameter calculation. Among the proposed FEA methods, three kinds can be classified. They are frozen permeabilities method (FPM) [1], vector control method (VCM) [2], and differential flux linkage method (DFM) [3], respectively. First, the principle and calculation process of each method will be investigated, then the calculated results of all three methods then will be compared and analyzed. The experiment method will be applied to verify the simulation results and reveal the accuracy of each calculation method. The final conclusions of this paper will be helpful to guide motor design and drive researchers to choose proper method to evaluate d- and q-axis inductances of IPMSM as particular situation.

2. Inductance of Evaluation Method

2.1 Frozen Permeabilities Method

The operating point of permanent magnet (PM) varies with the load. It means the permeability of PM will be different for the different excited current. In addition, the PM permeability influences the distribution and saturation of flux linkage which generated by excited armature windings, while the distribution and saturation of flux linkage also affect the operating point of PM. Therefore, the PM cannot be canceled before the inductance analysis.

Making use of the flexibility of FEA, [1] proposed a method to calculate the pure flux linkages due to the exciting current by removing the magneto-motive force of PM. The procedure of this method is:

Step-1, the nonlinear calculation is processed for each load condition, i.e. certain current and current vector angle. And store the permeability of each element including stator, rotor and PM.

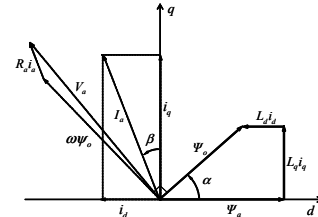
Step-2, by using the stored permeabilities, the stored field energy of motor is calculated by a linear calculation with different current and current vector angle.

Step-3, the d- and q-axis inductances are calculated by (1).

$$L_{d/q} = \frac{2}{3} \left( \frac{2W_s}{i^2} \right) \quad (1)$$

where  $W_s$  is the stored filed energy,  $i$  is d- and q-axis currents.

It can be seen that excited current is the initial condition of FEA.



<Fig. 1> Vector diagram of IPMSM

Thus, the 2D magneto-static field analysis can be used in this method. Its governing equation is expressed in (2). It can be seen that this is a typical Poisson equation.

$$\nabla \times \left( \frac{1}{\mu} (\nabla \times A) \right) = J \quad (2)$$

where  $A$  is the magnetic vector potential,  $\mu$  is the isotropic permeability, and  $J$  is the excited current density of the stator winding. When the model is meshed into about 6500 elements, the computation time this method spent is less than twenty minutes in a computer with Intel Core Due CPU.

2.2 Vector Control Method.

The vector diagram of IPMSM in steady-state can be described in Fig. 1. In the solid-line part, it can be seen that there are following relationships as described in (4)

$$\begin{aligned} L_d &= \frac{\psi_o \cos \alpha - \psi_a}{i_d} \\ L_q &= \frac{\psi_o \sin \alpha}{i_q} \end{aligned} \quad (3)$$

where  $\psi_a$  is the flux linkage generated by permanent magnet in no-load condition,  $\psi_o$  is the flux linkage generated by permanent magnet and excited armature current,  $L_d$  is d-axis inductance, and  $L_q$  is the q-axis inductance, respectively. The detail calculation procedure of this method can be found in Fig. 2. It is obvious that this method also uses the magneto-static field FEA. But in practice, the computation time needs about several-ten minutes with same computer before, because of the twice nonlinear analysis, FFT, and  $\alpha$  angle searching.

2.3 Differential Flux-linkage Method

As mentioned before, the operating point of PM varies with the load condition. [3] proposed a method to eliminate the PM effect without removing the magneto-motive force like FPM. This method regards the operating points of PM under two near load conditions as the approximately same. Thus, the difference of the flux linkages from two near load condition is totally effect of inductance. Additionally, in order to eliminate the q-axis flux linkage when the d-axis inductance is calculating, the q-axis armature current should be constant, vice versa. Then, the d- and q-axis inductance calculation equations can be calculated by dividing the difference of two near load currents as expressed in (4). The detail procedure of this method has been shown in Fig. 3.

$$L_d = \left( \frac{\psi_{d1} - \psi_{d2}}{i_{d1} - i_{d2}} \right)_{i_q = \text{const}}$$

$$L_q = \left( \frac{\psi_{q1} - \psi_{q2}}{i_{q1} - i_{q2}} \right) \Big|_{i_d = \text{const}} \quad (4)$$

Because the desired variable is the flux linkage, this method can be processed in both magneto-static field FEA and coupled field-circuit FEA, and also can be realized in laboratory experiment. The computation time of this method in magneto-static field FEA is more than that of VCM because more nonlinear calculation steps.

### 2.4 Experiment Method

As investigation before, the VCM and DFM also can be realized in experiment. Due to the limitation of hardware, this paper uses the VCM to verify the calculation results. However, it is impossible to obtain the flux linkage in no-load and load condition directly. The flux linkage in the no-load condition is measured in zero d-axis current control, and calculated by (5)

$$\omega \psi_a = v_{q0} - R_a i_{q0} \quad (5)$$

where  $v_{q0}$  is the q-axis voltage generated by PI controller in steady state,  $i_{q0}$  is the measured q-axis current. And the d- and q-axis inductances are calculated by (6)

$$L_q = \frac{v_q - R_a i_q - \omega \psi_a}{\omega i_d}$$

$$L_d = \frac{v_d - v_{d0} - R_a i_d}{-\omega(i_q - i_{q0})} \quad (6)$$

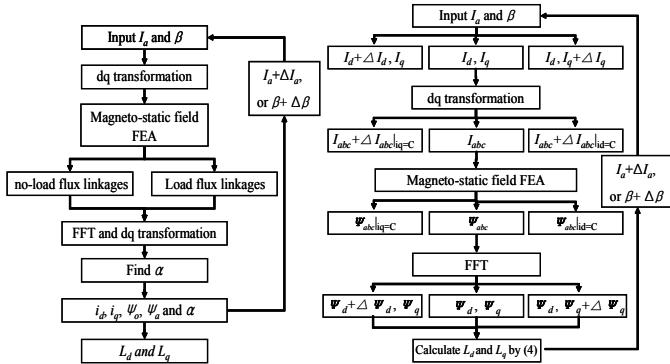
where  $v_{d0}$  is the d-axis voltage in no-load condition. The  $v_{q0}$ ,  $v_{d0}$  supply the energy losing in the mechanical losses under no-load condition. That is why they do not exist in theory equations.

## 3. Analysis Model and Results

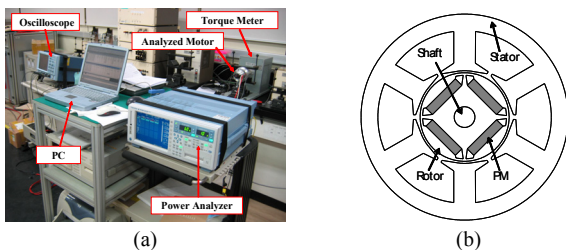
The experiment devices are shown in Fig. 4 (a). In this paper, the analyzed motor is a spindle-type IPMSM. Its cross-section are shown in Fig. 4 (b). The detail dimensions and specification of this motor is shown in Table I. The calculated and measured d- and q-axis inductances are shown in Fig. 5.

## 4. Discussion and Conclusion

The d- and q-axis inductances calculated by the previously mentioned methods and measured by the VCM experiment are compared and shown in Fig. 5 (a) and (b), respectively. Due to the



**<Fig. 2> Flowchart of vector control method** **<Fig. 3> Flowchart of differential flux-linkage method**



**<Fig. 4> Experiment verification: (a) experiment devices, (b) cross-section of analysis motor**

limitation of hardware and the characteristics of the tested motor, the experiment only is processed from 20° through 60° current vector angle. The inductances calculated by FPM have no large difference with different excited current (6Arms and 9Arms). That is because this method calculates the inductance by total flux linkage including fundamental and harmonics. The saturation effect hence is not significant. The inductances calculated by DFM are smaller than those of VCM. It is because that the essential of the inductances calculated by DFM is the incremental inductance, while the inductances calculated by VCM are the apparent inductance. Due to the VCM calculates the inductances at rotation state, the incremental inductance is smaller than the apparent inductance.

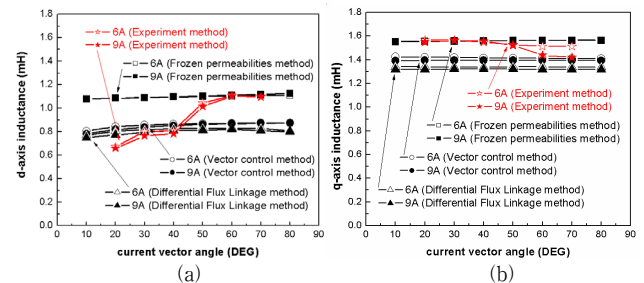
In addition, before 45° current vector angle the measured d-axis inductance is more similar with those calculated by DFM, while it becomes close to those calculated by FPM after 45°. According to the calculation principle, both the VCM and DFM handle the operating point approximately. When the current vector angle is small, the demagnetizing effect is not significant, and hence the approximate handling may be correct. This time, the fundamental effect is more important. But when the demagnetizing effect is stronger, the approximation is available no more. In Fig. 5 (b), the measured q-axis has decrement after 45° current vector angle. This is because of the cross-coupling saturation effect of d-axis flux linkage. This comparison results shows that it may not use only one method to evaluate both d- and q-axis inductances, and even use hybrid method to solve inductance for different current vector angle.

### [Reference]

- [1] J. Y. Lee, S. H. Lee, G. H. Lee, and J. P. Hong, "Determination of parameters considering magnetic nonlinearity in an interior permanent magnet synchronous motor," *IEEE Trans. Magn.*, Vol. 42, No. 4, Apr. 2006.
- [2] G. H. Kang, J. P. Hong, G. T. Kim, and J. W. Park, "Improved parameter modeling of interior permanent magnet synchronous motor based on finite element analysis," *IEEE Trans. Magn.*, Vol. 36, No. 4, Jul. 2000.
- [3] B. Stumberger, G. Stumberger, D. Dolinar, etc., "Evaluation of saturation and cross-magnetization effects in interior permanent-magnet synchronous motor," *IEEE Trans. Ind. Appl.*, Vol. 39, No. 5, Sept./Oct. 2003.

**<Table I> Specification of analysis model**

Parameter	Value	Unit
stator outer radii/ rotor outer radii	79 / 34.5	mm
airgap length/ stack length	0.8 / 35	mm
volume of PM	16×3.5×34	mm <sup>3</sup>
remanent flux density of PM (@20°C)	1.2	T
No. of turn in series connected	60	-
No. of parallel circuits	2	-



**<Fig. 5> Calculated and measured Inductances: (a) d-axis inductance; (b) q-axis inductance**

**<Table II> Comparison of computation Process**

Method	FPM	VCM	DFM
Computation time	fast	slow	very slow
Complexity	simple	complex	very complex