BER Analysis for Decode-and-Forward Relaying in Rayleigh Fading Channel

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Abstract

In this paper, a bit error rate (BER) study is presented for decode-and-forward (DF) relaying for user cooperation in independent and identically distributed Rayleigh fading channels.

I. Introduction

For the DF relaying, relays demodulate and decode the transmitted signal from the source before encoding again and retransmitting it to the destination. At the destination, the receiver can employ a variety of diversity combining techniques to benefit from the multiple signal replicas available from the relays and the source.

The performance of the DF relaying protocol is often evaluated by an outage probability [1] and bit error rate (BER) [2] especially when the statistics of the channels between the source, relays, and destination are assumed to be independent and peformance of the DF relaying was investigated for a single relay when the communication between the source and the destination is unavailable.

In this paper, hence, considering an arbitrary between the source and the destination, we present a BER study of DF relaying with M-ary PSK in i.i.d. Rayleigh fading channels.

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II. System Model

A source node is communicating with a destination node through intermediate N relay nodes. The complex channel coefficients between the source and the destination or the ith relay are denoted by h_{SD} or h_{SR_i}, respectively, and the complex channel coefficient between the ith relay and the destination is represented by h_{RD_i}. Every channel between the nodes is assumed mutually independent Rayleigh distributed. Thus, the channel powers, denoted by $\alpha_0 = |h_{SD}|^2$, $\alpha_{1,i} = |h_{SR_i}|^2$ and $\alpha_{2,i} = |h_{R_iD}|^2$ where $i = 1,\cdots,N$ are independent and exponentially distributed random variables whose means are λ_0, $\lambda_{1,i}$, $\lambda_{2,i}$, respectively. From the assumption of identically distributed fading channels, let $\lambda_{1,i} = \lambda_1$ and $\lambda_{2,i} = \lambda_0 = \lambda_2$ for $i = 1,\cdots,N$. We assume that the average transmit signal-to-noise ratios (SNRs) for the source and relays are equal, denoted by ρ.

A time-division channel allocation scheme with $N+1$ time slots is adopted in order to realize orthogonal channelization [1]. In the first time slot, the source broadcasts its signal to the destination and all relays. During the following N time slots, then the relays that belong to a decoding set C_D decode and forward the source message to the destination in a predetermined order.

Ⅲ. BER for DF Relaying

Hereafter, the elements in the sets C and C_D are expressed as only the indices of relays. Since C_D is a random set, using the total probability law the BER of DF relaying is written as eq. (1) where where $|C_D|$ denotes the cardinality of C_D.

$$\overline{P_b} = \sum_{r=0}^{N} \binom{N}{r} \overline{B_D}(|C_D|=r)\Pr\{|C_D|=r\}, \quad (1)$$

$\Pr\{C_D\}$ denotes the probability that the decoding set C_D exists for M-ary PSK, and $\overline{B_D}(C_D)$ denotes the BER for the combined signal obtained by using maximal ratio combining (MRC) after the destination receives PSK-modulated signals from the source through the members of the decoding set. Assuming that a modulated symbol is transmitted over a time slot, the probability for the decoding set C_D in the i.i.d. fading channels is obtained by

$$\Pr\{|C_D|=r\} = (1-\overline{S})^r(\overline{S})^{N-r}, \quad (2)$$

where $\overline{S}$ denotes the error rate of modulated symbols transmitted from the source to a relay and, for M-ary PSK constellations, is given by

$$\overline{S} = 1-(1-\overline{B})^{\log_2 M}, \quad (3)$$

where $\overline{B}$ represents the BER of modulated symbols received by a relay and is given in [3].

Let $\gamma(C_D) = \left(\rho\alpha_0 + \sum_{i\in C_D}\rho\alpha_{2,i}\right)/\log_2 M$ denote the MRC output SNR for the signals received at the destination. By taking the expectation with respect to the i.i.d. channels, then the moment generating function (MGF) of $\gamma(C_D)$ is given by

$$M_{\gamma(C_D)}(s) = \left(\frac{\log_2 M}{\rho\lambda_2 s + \log_2 M}\right)^{|C_D|+1}, \quad (4)$$

We can easily find the probability density function (PDF) of $\gamma(C_D)$ by taking the inverse Laplace transform of the MGF in (4):

$$f_{\gamma(C_D)}(\gamma) = \left(\frac{\log_2 M}{\rho\lambda_2}\right)^{|C_D|+1}\frac{\gamma^{|C_D|}}{|C_D|!}exp\left(-\frac{\gamma\log_2 M}{\rho\lambda_2}\right), \quad (5)$$

The BER of the MRC-combined signal with M-ary PSK at the destination is obtained by using the PDF of $\gamma(C_D)$ in (5) and the analysis in [3].

Ⅳ. Numerical Results

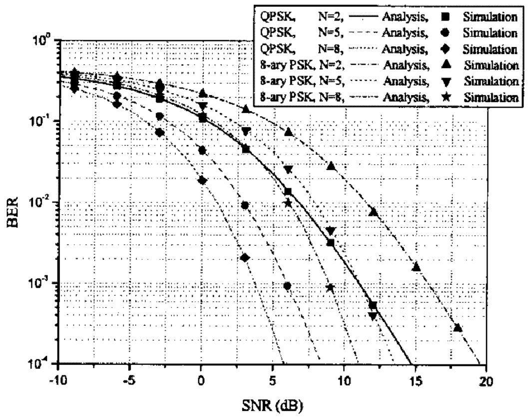

Fig. 1. BER of DF relaying for M-ary PSK, when N=2, 5 and 8.

In the simulation, we set $\lambda_1 = \lambda_2 = 1$. The figure shows exact matches between the results from the analysis and the simulation. The BER performance improves as the number of cooperative relays goes up. However, an increase in the number of relays requires an increase in the number of orthogonal channels to achieve spatial diversity.

References

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